Electric Charge and Electric Field

1. Electric Charge

Electric charge is a fundamental property like mass, length etc associated with elementary particles for example electron, proton and many more.

Electric charge is the property responsible for electric forces which acts between nucleus and electron to bind the atom together.

Charges are of two kinds
(i) negative charge
(ii) positive charge

Electrons are negatively charged particles and protons, of which nucleus is made of, are positively charged particles. Actually nucleus is made of protons and neutrons but neutrons are uncharged particles.

electric force between two electrons is same as electric force between two protons kept at same distance apart i. e., both set repel each other but electric force between an electron and proton placed at same distance apart is not repulsive but attractive in nature.

Conclusion
(a) Like charges repel each other

(b) Unlike charges attract each other

Assignment of negative charge on electron and positive charge on proton is purely conventional , it does not mean that charge on electron is less than that on proton.

Importance of electric forces is that it encompasses almost each and every field associated with our life; being it matter made up of atoms or molecules in which electric charges are exactly balanced or adhesive forces of glue associated with surface tension, all are electric in nature.

Unit

Charge on a system can be measured by comparing it with the charge on a standard body.

SI unit of charge is Coulomb written as C.

1 Coulomb is the charge flowing through the wire in 1 second if the electric current in it is 1A.

Charge on electron is -1.602×10^{-19} C and charge on proton is positive of this value.

2. Basic properties of electric charge

(i) Additivity of charges

Charges adds up like real numbers i. e., they are Scalars more clearly if any system has n number of charges q_1, q_2, q_3, q_n then total charge of the system is

$q = q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots\ldots q_n$

Proper sign have to be used while adding the charges for example if

$q_1 = +1C$

$q_2 = -2C$

q_3 = +4C

then total charge of the system is

$q = q_1 + q_2 + q_3$

q = (+1) + (-2) + (+4) C

q = (+3) C

(ii) Charge is conserved

Charge of an isolated system is conserved.

Chage can not be created or destroyed but charged particles can be created or destroyed.

(iii) Quantization of charge

All free charges are integral multiples of a unit of charge e, where $e = -1.602 \times 10^{-19}$ C i. e., charge on an electron or proton.

Thus charge q on a body is always denoted by

q = ne

where n = any integer positive or negative

Many such solid materials are known which on rubbing attract light objects like light feather, bits of papers, straw etc.

Explaination of appearance of electric charge on rubbing is simple.

Material bodies consists of large number of electrons and protons in equal number and hence is in neutral in their normal state. But when the body is rubbed for example when a glass rod is rubbed with silk cloth, electrons are transferred from glass rod to silk cloth. The glass rod becomes positively charged and the silk cloth becomes negatively charged as it recieves extra electrons from the glass rod.

In this case rod after rubbing, comb after passing through dry hairs becomes electrified and these are the example of frictional electricity.

4. Coulumb's law

Coulomb's law is the law of forces between electric charges.

Statement

" It states that two stationary point charges q_1 and q_2 repel or attract each other with a force F which is directly proportional to the product of charges and inversly proportional to the square of distance between them."

This dependence can be expressed by writing

$$F\alpha \frac{q_{1}q_{2}}{r^{2}} \qquad (1)$$

These forces are attractive for unlike charges and repulsive for like charges .

We now try to express Coulomb's law in vector form for more clearity of magnitude and direction of forces.

Consider two point charges q_1 and q_2 at points with position vector $\mathbf{r_1}$ and $\mathbf{r_2}$ with respect to the origin

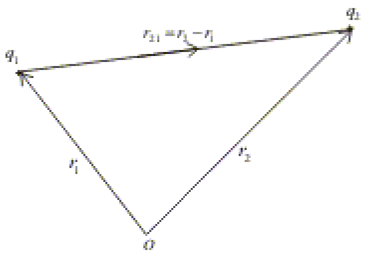


Figure 1.

vector $\mathbf{r_{21}} = \mathbf{r_2} - \mathbf{r_1}$ is the difference between $\mathbf{r_2}$ and $\mathbf{r_1}$ and the distance of separation r is the magnitude of vector $\mathbf{r_{21}}$.

pointwise it can be written as

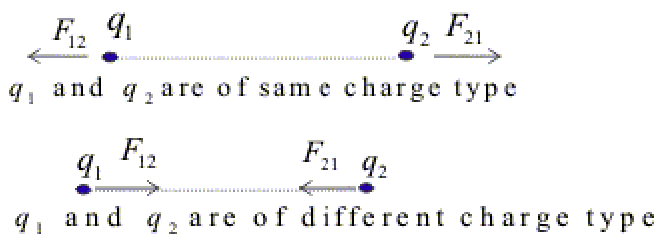
$\mathbf{r_1}$ = position vector of charge q_1 with respect to origin

$\mathbf{r_2}$ = position vector of charge q_2 with respect to origin

$\mathbf{r_{21}}$ = vector from 1 to 2 ($r_2 - r_1$)

$\mathbf{r_{12}} = -\mathbf{r_{21}}$ = vector from 2 to 1 ($\mathbf{r_1} - \mathbf{r_2}$)

$\mathbf{r} = \mathbf{r_{12}} = \mathbf{r_{21}}$ = distance between 1 and 2.



Coulomb's law can then be expressed as

$$\boldsymbol{F_{21}}=\frac{kq_{1}q_{2}\boldsymbol{r_{21}}}{r{3}}$$ (2a)

and, $\mathbf{F_{12}}$ = force on q_1 due to q_2

$$\boldsymbol{F_{12}}=\frac{kq_{1}q_{2}\boldsymbol{r_{12}}}{r_{3}}=-\boldsymbol{F_{21}}$$ (2b)

Special Case

for simplicity we can choose q_1 being placed at origin

$\mathbf{r_1} = 0$

and if we write $\mathbf{r_2} = \mathbf{r}$ the position vector of q_2 then

5. Principle Of Superposition

Coulomb's law gives the electric force acting between two electric charges.

Principle of superposition gives the method to find force on a charge when system consists of large number of charges.

According to this principle when a number of charges are interacting the total force on a given charge is vector sum of forces exerted on it by all other charges.

This principle makes use of the fact that the forces with which two charges attract or repel one another are not affected by the presence of other charges.

If a system of charges has n number of charges say q_1, q_2, ..., q_n, then total force on charge q_1 according to principle of superposition is

$\mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{F}_{1n}$

Where $\mathbf{F}_{12}$ is force on q_1 due to q_2 and $\mathbf{F}_{13}$ is force on q_1 due to q_3 and so on.

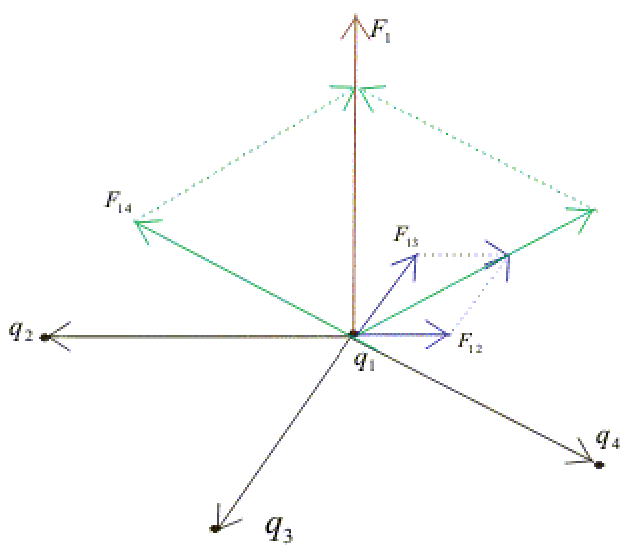


Figure 2. Force due to system of multiple charges

$\mathbf{F_{12}}$, $\mathbf{F_{13}}$, ... $\mathbf{F_{1n}}$ can be calculated from Coulomb's law i. e.

$$\boldsymbol{F_{12}}=\frac{kq_{1}q_{2}\boldsymbol{\widehat{r}_{12}}}{4\pi \epsilon _{0}r_{12}^{2}}$$

to,

$$\boldsymbol{F_{1n}}=\frac{kq_{1}q_{2}\boldsymbol{\widehat{r}_{1n}}}{4\pi \epsilon _{0}r_{1n}^{2}}$$

The total force F_1 on the charge q_1 due to all other charges is the vector sum of the forces $\mathbf{F_{12}}$, $\mathbf{F_{13}}$, ... $\mathbf{F_{1n}}$.

$\mathbf{F_1 = F_{12} + F_{13} +}$ ....

$$F_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \ldots\ldots\ldots\ldots \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

$or,$

$$F_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$$

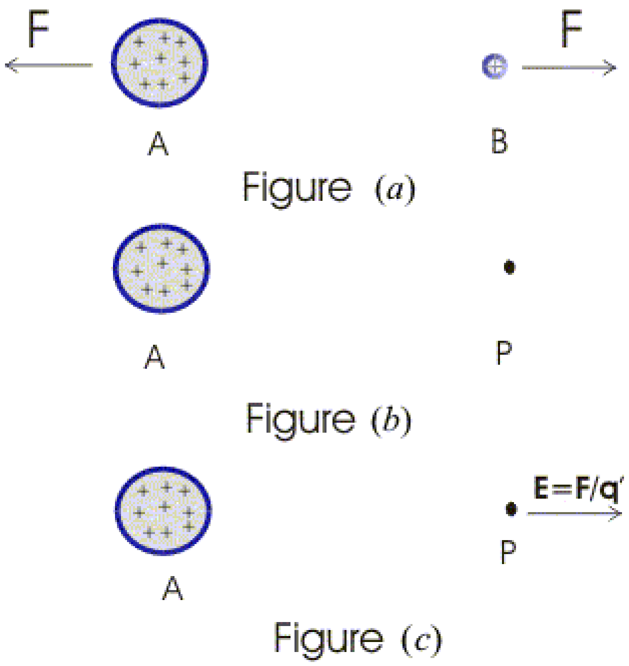
The vector sum is obtained by parellogram law of addition of vector.

Similarly force on any other charge due to remaining charges say on q_2, q_3 etc. can be found by adopting this method.

6. Electric Field

Electrical interaction between charged particles can be reformulated using the concept of electric field.

To understand the concept consider the mutual repulsion of two positive charged bodies as shown in fig (a)



Now if remove the body B and label its position as point P as shown in fig (b), the charged body A is said to produce an electric field at that point (and at all other points in its vicinity)

When a body B is placed at point P and experiences force F, we explain it by a point of view that force is exerted on B by the field not by body A itself.

The body A sets up an electric field and the force on body B is exerted by the field due to A.

An electric field is said to exists at a point if a force of electric origin is exerted on a stationary charged (test charge) placed at that point.

If F is the force acting on test charge q placed at a point in an electric field then electric field at that point is
$\mathbf{E} = \mathbf{F}/q$
or $\mathbf{F} = q\mathbf{E}$

Electric field is a vector quantity and since F = qE the direction of E is the direction of F.

Unit of electric field is ($N.C^{-1}$)

Q. Find the dimensions of electric field
Ans. [$MLT^{-3}A^{-1}$]

7. Calculation of Electric Field

In previous section we studied a method of measuring electric field in which we place a small test charge at the point, measure a force on it and take the ratio of force to the test charge.

Electric field at any point can be calculated using Coulomb's law if both magnitude and positions of all charges contributing to the field are known.

To find the magnitude of electric field at a point P, at a distance r from the point charge q, we imagine a test charge q'to be placed at P. Now we find force on charge q' due to q through Coulomb's law.
$$\boldsymbol{F}=\frac{kqq_{'}}{4\pi \epsilon _{0}r^{2}} $$
electric field at P is $$\boldsymbol{E}=\frac{kqq_{'}}{4\pi \epsilon _{0}r^{2}} $$
The direction of the field is away from the charge q if it is positive

Electric field for either a positive or negative charge in terms of unit vector r directed along line from charge q to point P is $$\boldsymbol{F}=\frac{kqq_{'}\boldsymbol{\widehat{r}}}{4\pi \epsilon _{0}r^{2}} $$
r = distance from charge q to point P.

When q is negative , direction of E is towards q, opposite to r.

Electric Field Due To Multiple Charges

Consider the number of point charges q_1, q_2,.......... which are at distance r_{1P}, r_{2P},.................. from point P as shown in fig

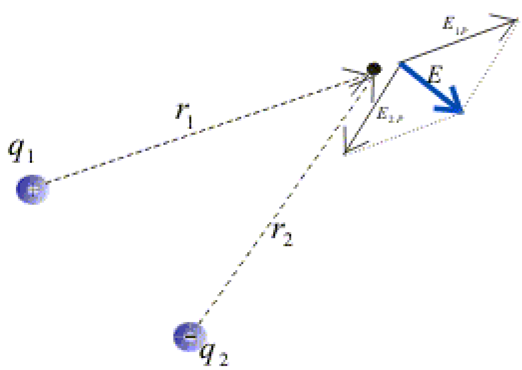


Figure:- figure shows the resultant electric field due to two point charges at point P

The resultant electric field is the vector sum of individual electric fields as
E = E_{1P} + E_{2P} + ...

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \dots\dots\dots\right)$$

This is also a direct result of principle of superposition discussed earlier in case of electric force on a single charge due to system of multiple charges.

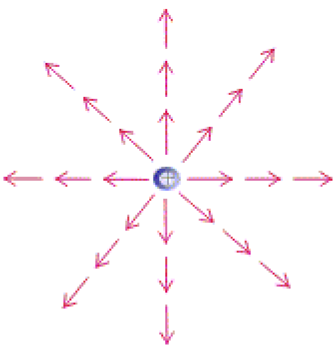
E is a vector quantity that varies from one point in space to another point and is determined from the position of square charges.

8. Electric Field Lines

For a single positive point charge q, electric field is

$$\boldsymbol{F}=\frac{kqq_{'}\boldsymbol{\widehat{r}}}{4\pi \epsilon _{0}r^{2}} $$

now to get feel of this field one can sketch a few representative vectors as shown in fig below



Since electric field varies as inverse of square of the distance that points from the charge the vector gets shorter as you go away from the origin and they always points radially outwards.

Connecting up these vectors to form a line is a nice way to represent this field .

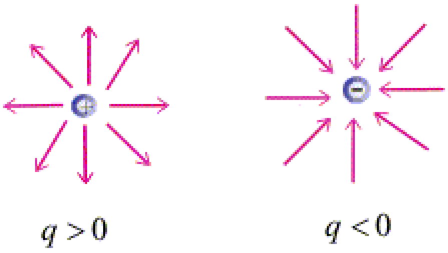
The magnitude of the field is indicated by the density of the field lines.

Magnitude is strong near the center where the field lines are close togather, and weak farther out, where they are relatively apart.

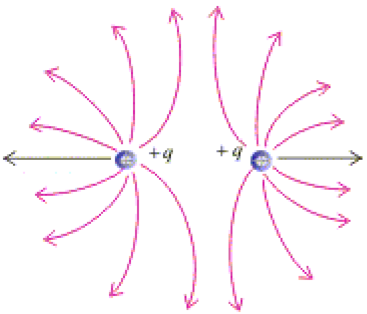
So, electric field line is an imaginary line drawn in such a way that it's direction at any point is same as the direction of field at that point.

An electric field line is, in general a curve drawn in such a way that the tangent to it ateach point is the direction of net field at that point.

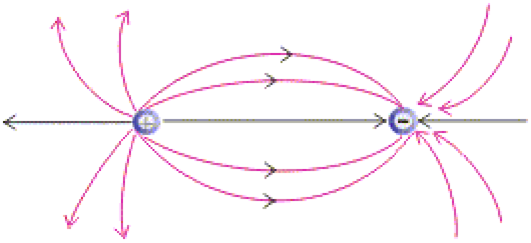
Field lines of a single position charge points radially outwards while that of a negative charge are radially inwards as shown below in the figure.



Field lines around the system of two positive charges gives a different picture and describe the mutual repulsion between them.



Field lines around a system of a positive and negative charge clearly shows the mutual attraction between them as shown below in the figure.

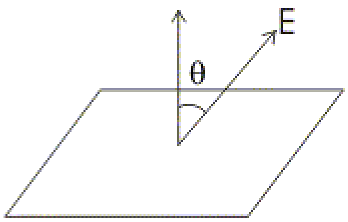


Some important general properties of field lines are
1.Field lines start from positive charge and end on a negative charge.
2.Field lines never cross each other if they do so then at the point of intersection there will be two direction of electric field.
3.Electric field lines do not pass through a conductor , this shows that electric field inside a conductor is always zero.
4.Electric field lines are continuous curves in a charge free region.

9. Electric Flux

Consider a plane surface of area ΔS in a uniform electric field E in the space.

Draw a positive normal to the surface and θ be the angle between electric field E and the normal to the plane.



Electric flux of the electric field through the choosen surface is then
$\Delta\varphi = E \, \Delta S \cos\theta$

Corresponding to area ΔS we can define an area vector ΔS of magnitude ΔS along the positive normal. With this definition one can write electric flux as
$\Delta\varphi = \mathbf{E} \cdot \mathbf{\Delta S}$

direction of area vector is always along normal to the surface being choosen.

Thus electric flux is a measure of lines of forces passing through the surface held in the electric field.

Special Cases
If E is perpendicular to the surface i. e., parallel to the area vector then θ = 0 and
$\Delta\varphi = E \, \Delta S \cos 0$

If θ = π i. e., electric field vector is in the direction opposite to area vector then
$\Delta\varphi = - E \, \Delta S$

If electric field and area vector are perpendicular to each other then θ = π/2 and Δφ = 0

Flux is an scaler quantity and it can be added using rules of scaler addition.

For calculating total flux through any given surface , divide the surface into small area elements. Calculate the flux at each area element and add them up.

Thus total flux φ through a surface S is

$$\varphi \;\square\; \Sigma \mathbf{E}.\Delta \mathbf{S}$$

This quantity is mathematically exact only when you take the limit $\Delta S \rightarrow 0$ and the sum in equation 3 is written as integral

$$\varphi = \int \Sigma \mathbf{E}.d\mathbf{S}$$

9. Electric Dipole

Electric dipole is a pair of equal and opposite charges, +q and -q, separated by some distance 2a.
Total charge of the dipole is zero but electric field of the dipole is not zero as charges q and -q are separated by some distance and electric field due to them when added is not zero.

(A)Field of an electric dipole at points in equitorial plane

We now find the magnitude and direction of electric field due to dipole.

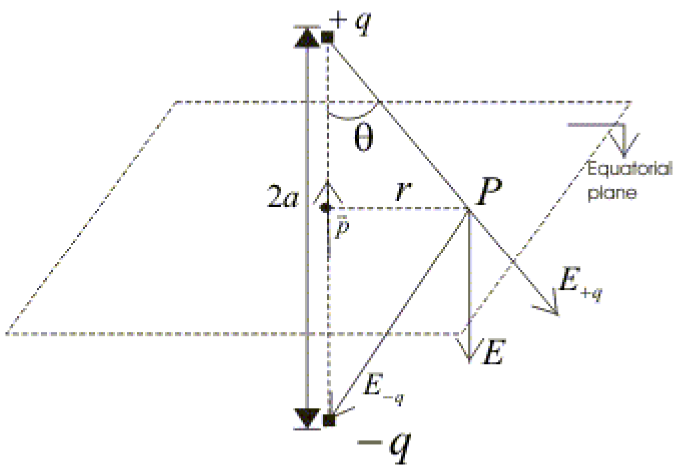


Figure:-Electric field of dipole at points in equatorial plane

P point in the equitorial plane of the dipole at a distance r from the centre of the dipole. Then electric field due to +q and -q are $$\boldsymbol{E_{-q}}=\frac{-q\boldsymbol{\widehat{P}}}{4\pi \epsilon _{0}(r^{2}+a^{2})}$$ (1a) $$\boldsymbol{E_{+q}}=\frac{q\boldsymbol{\widehat{P}}}{4\pi \epsilon _{0}(r^{2}+a^{2})}$$ (1b)

and they are equal

Pˆ = unit vector along the dipole axis (from -q to +q)

From fig we can see the direction of E_{+q} and E_{-q}. Their components normal to dipole cancel away and components along the dipole add up.

Dipole moment vector points from negative charge to positive charge so in vector form.

$\mathbf{E} = -(\mathbf{E}_{+q} + \mathbf{E}_{-q}) \cos\theta$

$$E = -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2+a^2}+\frac{1}{r^2+a^2}\right]\frac{a}{\sqrt{r^2+a^2}}$$

or,

$$E = -\frac{2qa}{4\pi\varepsilon_0 (r^2+a^2)}\hat{p}$$

(2)

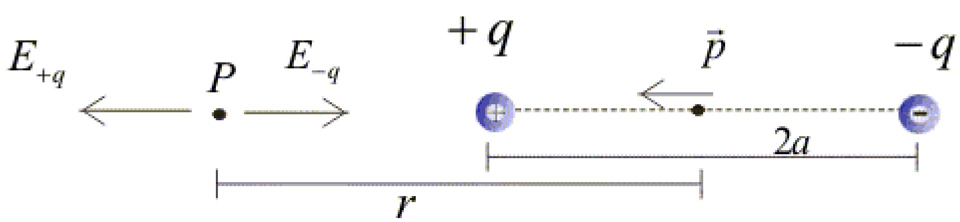
At large distances (r>>a), above equation becomes

$$\boldsymbol{E}=\frac{-2qa\boldsymbol{\widehat{P}}}{4\pi \epsilon _{0}r^{3}}$$

(3)

and they are equal

(B) Field of an electric dipole for points on the axis



Let P be the point at a distance r from the centre of the dipole on side of charge q. as shown in the fig.

$$\boldsymbol{E_{-q}}=\frac{-q\boldsymbol{\widehat{P}}}{4\pi \epsilon _{0}(r+a)^{2}}$$

(4a)

P^ = unit vector along the dipole axis (from -q to +q)

also

$$\boldsymbol{E_{+q}}=\frac{q\boldsymbol{\widehat{P}}}{4\pi \epsilon _{0}(r-a)^{2}}$$

(4b)

Total field at P is

$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q}$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2}-\frac{1}{(r+a)^2}\right]\hat{p}$$

or,

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)}\right]\hat{p}$$

(5)

for r>>a

$$\boldsymbol{E}=\frac{4qa\boldsymbol{\widehat{P}}}{4\pi \epsilon _{0}r^{3}} $$

(6)

For equation (3) and (6) charge q and dipole separation 2a appear in combination qa. This leads us to define dipole moment vector **P** of electric dipole. Thus, electric dipole moment **P** = q × 2a $\hat{\mathbf{P}}$ (7)

Unit of dipole moment is Coulomb's meter (Cm).

In terms of electric dipole moment, field of a dipole at large distances becomes

(i) At point on equitorial plane (r>>a)

$E = -P/4\pi\varepsilon_0 r^3$

$$\boldsymbol{E}=\frac{-\boldsymbol{P}}{4\pi \epsilon _{0}(r)^{3}} $$

(ii) At point on dipole axis (r>>a)

$$\boldsymbol{E}=\frac{2\boldsymbol{P}}{4\pi \epsilon _{0}(r)^{3}} $$

Note:-

(i) Dipole field at large distances falls off as $1/r^3$

(ii) Both the direction and magnitude of dipole an angle between dipole moment vector **P** and position vector **r**

(C) Dipole in a uniform external field

Consider a dipole in a uniform electric field **E** whose direction makes an angle θ with dipole axis (line joining two charges)

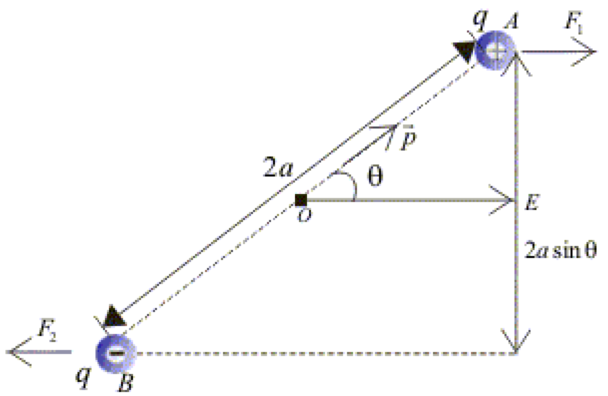


Figure a:- torque on dipole is τ=pEsinθ

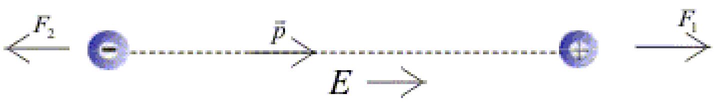


Figure b:- Dipole in equilibrium in uniform electric field.

Force $\mathbf{F_1}$ of magnitude q**E**, acts on positive charge in direction of electric field and a force $\mathbf{F_2}$ of same magnitude acts on negative charge but it acts in direction opposite to F_1.

Resultant force on dipole is zero, but since two forces do not have same line of action they constitute a couple.

We now calculate torque (**r** × **F**) of these forces about zero.

Torque of F_1 about O is

$\boldsymbol{\tau_1} = \mathbf{OB} \times \mathbf{F_1}$

$= q\,(\mathbf{OB} \times \mathbf{E})$

Torque of F_2 about O is

$\boldsymbol{\tau_2} = \mathbf{OA} \times \mathbf{F_2}$

$= -q(\mathbf{OA} \times \mathbf{E})$

$= q(\mathbf{AO} \times \mathbf{E})$

net torque acting on dipole is

$\boldsymbol{\tau} = \boldsymbol{\tau_1} + \boldsymbol{\tau_2}$

$= q(\mathbf{OB} + \mathbf{AO}) \times \mathbf{E}$

$= q\,(\mathbf{AB} \times \mathbf{E})$

AB = 2a and **p** = 2q**a** (dipole moment)

$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

Direction of torque is perpendicular to the plane containing dipole axis and electric field.

Effect of torque is to rotate the dipole to a position in which dipole moment **p** is parallel to **E** the electric field vector is shown above in figure b and for uniform electric field dipole is in equilibrium in this position.

magnitude of this torque is
$\tau = |\boldsymbol{\tau}| = pE \sin\theta$

Examples

Question 1
Two point charges q_1 and q_2 are located with points having position vectors $\mathbf{r_1}$ and $\mathbf{r_2}$
(1) Find the position vector $\mathbf{r_3}$ where the third charge q_3 should be placed so that force acting on each of the three charges would be equal to zero.
(2) Find the amount of charge q_3

Question 2
Consider a thin wire ring of radius R and carrying uniform charge density λ per unit length.
(1) Find the magnitude of electric field strength on the axis of the ring as a function of distance x from its centre.
(2) What would be the form of electric field function for $x >> R$.
(3) Find the magnitude of maximum strength of electric field.

Question 3
Two equally charged metal balls each of mass m Kg are suspended from the same point by two insulated threads of length l m long. At equilibrium, as a result of mutual separation between balls, balls are separated by x m. Determine the charge on each ball.

Guass 's Law

1. Introduction

Gauss's law was suggested by Kark Fredrich Gauss(1777-1855) who was german scientist and mathematecian.

Gauss's law is basically the relation between the charge distribution producing the electrostatic field to the behaviour of electrostatic field in space.

Gauss's law is based on the fact that flux through any closed surface is a measure of total amount of charge inside that surface and any charge outside that surface would not contribute anything to the total flux.

Before we look further to study Gauss's law in detail let's study electric field due to continous charge distributions.

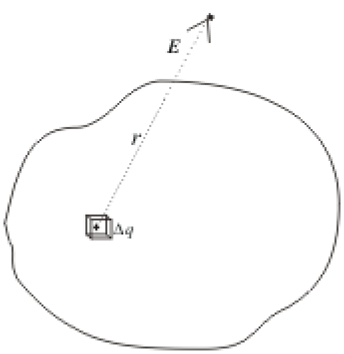
2. Electric field due to continous charge distributions

So far as in the previous chapter we have discussed force and field due to discrete charges.

We now assume that charges on a surface are located very close togather so that such a system of charges can be assumed to have continous distribution of charges.

In a system of closely spaced charges, total charge could be continously distributed along some line, over a surface or throughout a volume.

First divide the continous charge distribution into small elements containing Δq amount of charge as shown in fig 1



Electric field at point A due to element carrying charge Δq is

$$\Delta \boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r^2}\hat{\boldsymbol{r}} \qquad (1)$$

where r is the distance of element under consideration from point A and **r**ˆis the unit vector in the direction from charge element towards point A.

Total electric field at point A due to all such charge elements in charge distribution is

$$\boldsymbol{E} \cong \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{\boldsymbol{r}}_i \qquad (2)$$

where index i refers to the i_{th} charge element in the entire charge distribution.

Since the charge is distributed continously over some region , the sum becomes integral. Hence total field at A within the limit Δq→0 is,

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\lim_{\Delta q_i \to 0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{\boldsymbol{r}}_i = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{\boldsymbol{r}} \qquad (3)$$

and integration is done over the entire charge distribution.

If a charge q is uniformly distributed along a line of length L, the linear charge density λ is defined by

$$\lambda = \frac{q}{L} \qquad (4)$$

and the unit of λ is Coulumb/meter(C/m).

For charge distributed non-uniformly over a line, linear charge density is

$$\lambda = \frac{dq}{dL} \qquad (5)$$

where dQ is the amount of charge in a small length element dL.

For a charge Q uniformly distributed over a surfsce of area A, the surface charge density σ is

$$\sigma = \frac{q}{A} \qquad (6)$$

and unit of surface charge density is C/m^2. For non uniform charge distributed over a surface charge density is

$$\sigma = \frac{dq}{dA} \qquad (7)$$

where dA is a small area element of charge dQ.

Similarly for uniform charge distributios volume charge density is

$$\rho = \frac{q}{V} \qquad (8)$$

and for non uniform distribution of charges

$$\rho = \frac{dq}{dV} \qquad (9)$$

and unit of volume charge distribution is C/m^3.

3. Gauss's Law

We already know about electric field lines and electric flux. Electric flux through a closed surface S is

$$\phi = \int_S \boldsymbol{E} \cdot \boldsymbol{da} \qquad (10)$$

which is the number of field lines passing through surface S.

Statement of Gauss's Law

“ELectric flux through any surface enclosing charge is equal to q/ε_0 , where q is the net charge enclosed by the surface”

mathematically,

$$\oint \boldsymbol{E} \cdot \boldsymbol{da} = \frac{q}{\varepsilon_0} \qquad (11)$$

where q_{enc} is the net charge enclosed by the surface and **E** is the total electric field at each point on the surface under consideration.

It is the net charge enclosed in the surface that matters in Gauss's law but the total flux of electric field **E**depends also on the surface choosen not merely on the charge enclosed.

So if you have information about distribution of electric charge inside the surface you can find electric flux through that surface using Gauss's Law.

Again if you have information regarding electric fluxthrough any closed surface then total charge enclosed by that surface can also be easily calculated using Gauss's Law.

Surface on which Gauss's Law is applied is known as Gaussian surface which need not be a real surface. Gaussian surface can be an imaginary geometrical surface which might be empty space or it could be partially or fully embedded in a solid body.

Again consider equation 11

$$\oint \boldsymbol{E} \cdot \boldsymbol{da} = \frac{q}{\varepsilon_0}$$

In left hand side of above equation **E·da** is scalar product of two vectors namely electric field vector **E** and area vector **da**. Area vector **da** is defined as the vector of magnitude |da| whose direction is that of outward normal to area element da. So,**da=n**ˆda where **n**ˆ is unit vector along outward normal to da.

$$\boldsymbol{E} \cdot \boldsymbol{da} = |\boldsymbol{E}||\boldsymbol{da}|\cos\theta \qquad (12)$$

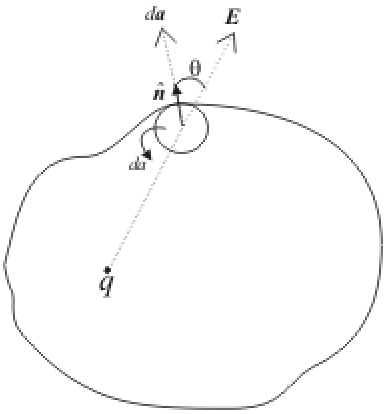


Figure 1

From above discussion we can conclude that,

(1) If both **E** and surface area da at each points are perpandicular to each other and has same magnitude at all points of the surface then vector **E** has same direction as that of area vector as shown below in the figure.

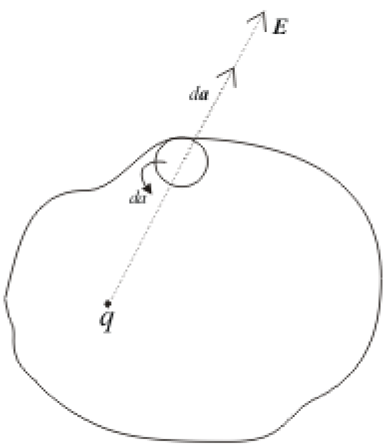


Figure 2

since **E** is perpandicular to the surface

$\int \boldsymbol{E} \cdot \boldsymbol{da} = \int |\boldsymbol{E}||\boldsymbol{da}| \cos 0^0 = EA$

(2) If **E** is parallel to the surface as shown below in the figure

Gauss's Law

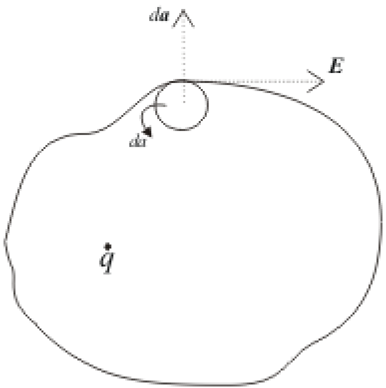


Figure 3

$$\int \boldsymbol{E} \cdot \boldsymbol{da} = \int |\,\boldsymbol{E}\,||\,\boldsymbol{da}\,| \cos 90^0 = 0$$

E at all points on the surface.

Guass 's Law

4. Applications of Gauss's Law

(A) Derivation of Coulumb's Law/h3>

Coulumb's law can be derived from Gauss's law.

Consider electric field of a single isolated positive charge of magnitude q as shown below in the figure.

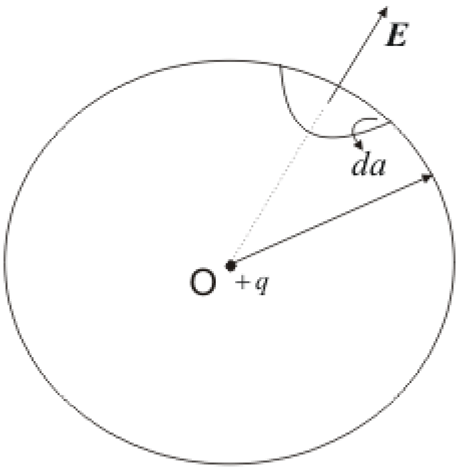


Figure 4

Field of a positive charge is in radially outward direction everywhere and magnitude of electric field intensity is same for all points at a distance r from the charge.

We can assume Gaussian surface to be a sphere of radius r enclosing the charge q.

From Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = E\oint da = \frac{q_{enc}}{\varepsilon_0}$$

since E is constant at all points on the surface therefore,

$$EA = \frac{q}{\varepsilon_0}$$

or,

$$E = \frac{q}{\varepsilon_0 A}$$

surface area of the sphere is A=4πr^2

thus,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Now force acting on point charge q' at distance r from point charge q is

$$F = q'E$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}$$

This is nothing but the mathematical statement of Coulumb's law.

(B) Electric field due to line charge

Consider a long thin uniformly charged wire and we have to find the electric field intensity due to the wire at any point at perpandicular distance from the wire.

If the wire is very long and we are at point far away from both its ends then field lines outside the wire are radial and would lie on a plane perpandicular to the wire.

Electric field intensity have same magnitude at all points which are at same distance from the line charge.

We can assume Gaussian surface to be a right circular cylinder of radius r and length l with its ends perpandicular to the wire as shown below in the figure.

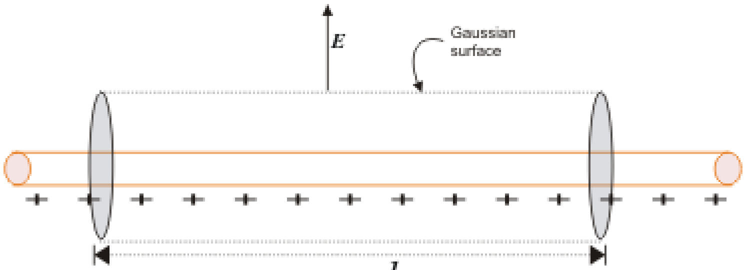


Figure 5. Cylindrical Gaussian surface for calculation of electric field due to line charge

λ is the charge per unit length on the wire. Direction of E is perpandicular to the wire and components of E normal to end faces of cylinder makes no contribution to electric flux. Thus from Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$$

Now consider left hand side of Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = E \oint da$$

Since at all points on the curved surface E is constant. Surface area of cylinder of radius r and length l is A=2πrl therefore,

$$\oint \mathbf{E} \cdot d\mathbf{a} = E(2\pi rl)$$

Charge enclosed in cylinder is q=linear charge density x length l of cylinder,
or, q=λl
From Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q}{\varepsilon_0}$$

$$or, E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi r\varepsilon_0}$$

$$\Rightarrow E \propto \frac{\lambda}{r}$$

Thus electric field intensity of a long positively charged wire does not depends on length of the wire but on the radial distance r of points from the wire.

Guass 's Law

(C) Electric field due to charged solid sphere

We'll now apply Gauss's law to find the field outside uniformly charged solid sphere of radius R and total charge q.

In this case Gaussian surface would be a sphere of radius r>R concentric with the charged solid sphere shown below in the figure.From Gauss's law

$$\oint \mathbf{E} \cdot \mathbf{da} = \frac{q_{enc}}{\varepsilon_0}$$

where q is the charge enclosed.

Charge is distributed uniformly over the surface of the sphere. Symmetry allows us to extract **E** out of the integral sign as magnitude of electric field intensity is same for all points at distance r>R.

Since electric field points radially outwards we have

$$\oint \mathbf{E} \cdot \mathbf{da} = E\oint da$$

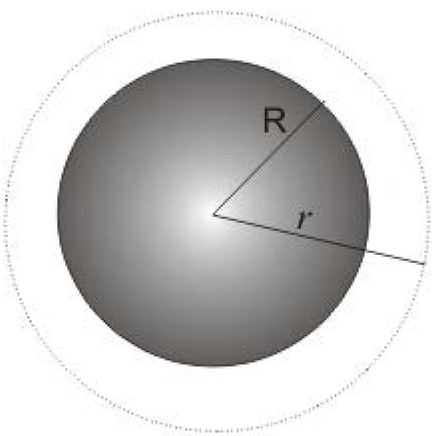


Figure 6

also as discussed magnitude of **E** is constant over Gaussian surface so,

$$E\oint da = E(4\pi r^2)$$

where $4\pi r^2$ is the surface area of the sphere.

Again from Gauss's law we have

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

THus we see that magnitude of field outside the sphere is exactly the same as it would have been as if all the charge were concentrated at its center.

(D) Electric field due to an infinite plane sheet of charge

Consider a thin infinite plane sheet of charge having surface charge density σ(charge per unit area).
We have to find the electric field intensity due to this sheet at ant point which is distance r away from the sheet.
We can draw a rectangular gaussian pillbox extending equal distance above and below the plane as shown below in the figure.

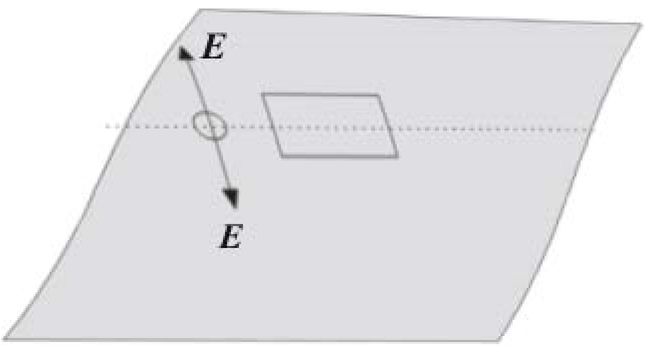


Figure 7

By symmetry we find that **E**on either side of sheet must be perpandicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.
No field lines crosses the side walls of the Gaussian pillbox i.e., component of **E** normal to walls of pillbox is zero.
We now apply Gauss's law to this surface

$$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$$

in this case charge enclosed is

$q = \sigma A$

where A is the area of end face of Gaussian pillbox.

E points in the direction away from the plne i.e., **E** points upwards for points above the plane and downwards for points below the plane. Thus for top and bottom surfaces,

$$\oint \boldsymbol{E} \cdot d\boldsymbol{a} = 2A\,|\,\boldsymbol{E}\,|$$

thus

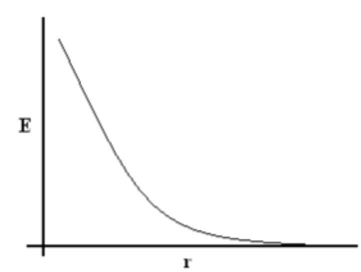
$2A|\mathbf{E}| = \sigma A/\varepsilon_0$

or,

$|\mathbf{E}| = \sigma/2\varepsilon_0$

Here one important thing to note is that magnitude of electric fied at any point is independent of the sheet and does not decrease inversly with the square of the distance. Thus electric field due to an infinite plane sheet of charge does not falls of at all.

- **Like Charges and Unlike Charges:**
  Like charges repel and unlike charges attract each other.
- **Conductors and Insulators:**
  Conductors allow movement of electric charge through them, insulators do not.
- **Quantization of Electric Charge:**
  It means that total charge (q) of a body is always an integral multiple of a basic quantum of charge (e)
  $$q = ne$$
  where n = 0, ±1, ±2, ±3, ….
- **Additivity of Electric Charges:**
  Total charge of a system is the algebraic sum of all individual charges in the system.
- **Conservation of Electric Charges:**
  The total charge of an isolated system remains uncharged with time.
- **Superposition Principle:**
  It is the properties of forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s).
- **The Electric Field E at a Point due to a Charge Configuration:**
  It is the force on a small positive test charges q placed at the point divided by a magnitude
  $$\frac{|q|}{4\pi\varepsilon_0 r^2}$$
  It is radially outwards from q, if q is positive and radially inwards if q is negative.
  E at a point varies inversely as the square of its distance from *Q*, the plot of *E* versus *r* will look like the figure given below.



- **Coulomb's Law:**
  The mutual electrostatic force between two point charges q_1 and q_2 is proportional to the product q_1q_2 and inversely proportional to the square of the distance r_{21} separating them.
  $$\vec{F}_{21}(\textit{force} \text{ on } q_2 \text{ due to } q_1) = \frac{k(q_1q_2)}{r_{21}^2}\hat{r}_{21}$$
  Where $\hat{r}_{21}$ is a unit vector in the direction from q_1 to q_2 and $k = \frac{1}{4\pi\varepsilon_0}$ is the proportionality constant.
- **An Electric Field Line:**
  It is a curve drawn in such a way that the tangent at each point on the curve gives the direction of electric field at that point.

- **Important Properties of Field Lines:**
  These are:
  (i) Field lines are continuous curves without any breaks.
  (ii) Two field lines cannot cross each other.
  (iii) Electrostatic field lines start at positive charges and end at negative charges – they cannot form closed loops.
- **Electric Field at a Point due to Charge q:**

$$\vec{E} = \frac{\vec{F}}{q}$$

- **Electric Field due to an Electric Dipole in its Equatorial Plane at a Distance r from the Centre:**

$$E = \frac{-p}{4\pi\varepsilon_0} \frac{1}{(a^2 + r^2)^{\frac{3}{2}}}$$

$$\cong \frac{-p}{4\pi\varepsilon_0}, \ for \ \text{r>>a}$$

- **Electric Field due to an Electric Dipole on the Axis at a Distance r from the Centre:**

$$E = \frac{2pr}{4\pi\varepsilon_0 (r^2 - a^2)^2}$$

$$\cong \frac{2p}{4\pi\varepsilon_0 r^3}, \ for \ \text{r>>a}$$

- **A Dipole Placed in Uniform Electric Field E experiences:**
  Torque $\vec{\tau}$,

$$\vec{\tau} = \vec{p} x \vec{E}$$

- **The Electric Flux:**
  $\phi = \int d\phi = \int \vec{E}.d\vec{s}$ is a ‘dot’ product, hence it is scalar.

  $\Delta\phi$ is positive for all values of $\theta < \frac{\pi}{2}$

  $\Delta\phi$ is negative for all values of $\theta > \frac{\pi}{2}$
- **Gauss’s Law:**
  The flux of electric field through any closed surface S is 1/ε0 times the total charge enclosed by S.

$$\phi = \int \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

- Electric field outside the charged shell is as though the total charge is concentrated at the centre. The same result is true for a solid sphere of uniform volume charge density.
- The electric field is zero at all points inside a charged shell.

- **Electric field E, due to an infinitely long straight wire of uniform linear charge density λ:**

  $$E = \frac{\lambda}{2\pi\varepsilon_0 r}.\hat{n}$$

  where r is the perpendicular distance of the point from the wire and is the radial unit vector in the plane normal to the wire passing through the point.

- **Electric field E, due to an infinite thin plane sheet of uniform surface charge density σ:**

  $$E = \frac{\sigma}{2\varepsilon_0}.\hat{n}$$

  Where $\hat{n}$ is a unit vector normal to the plane, outward on either side.

- **Electric field E, due to thin spherical shell of uniform surface charge density σ:**

  $$E = \frac{q}{4\pi\varepsilon_0 r^2}.\hat{r} \quad (r \geq R)$$

  $$E = 0 \quad (r < R)$$

  where r is the distance of the point from the centre of the shell and R the radius of the shell, q is the total charge of the shell & $q = 4\pi R^2\sigma$.

- Electric field E along the outward normal to the surface is zero and σ is the surface charge density. Charges in a conductor can reside only at its surface. Potential is constant within and on the surface of a conductor. In a cavity within a conductor (with no charges), the electric field is zero.