Electric Potential

## 1. Introduction

We already have an introduction of work and energy while studying mechanics.
We know that central forces are conservative in nature i.e., work done on any particle moving under the influence of consrevative forces does not depend on path taken by the particle but depends on initial and final positions of the particle.

Electrostatic force given by Coulumb's law is also a central force like gravitational force and is conservative in nature.

For conservative forces, work done on particle undergoing displacement can be expressed in terms of potential energy function.
In this chapter we will apply work and energy considerations to the electric field and would develop the concept of electric potential energy and electric potential.

## 2. Electric potential energy

Conside a system of two point charges in which positive test charge q' moves in the field produced by stationary point charge $q$ shown below in the figure.


Figure 1

Charge $q$ is fixed at point $P$ and is displaced from point $R$ to $S$ along a radial line $P R S$ shown in the figure.
Let $r_{1}$ be the distance between points $P$ and $R$ and $r_{2}$ be the distance between $P$ and $S$.

$$
\begin{equation*}
F=\frac{1}{4 \pi \varepsilon_{0}} \frac{q q}{r^{2}} \tag{1}
\end{equation*}
$$

If $q$ ' moves towards $S$ through a small displacement $d r$ then work done by this force in making the small displacement dr is
$d W=F \cdot d r$
$d W=\frac{1}{4 \pi \varepsilon_{0}} \frac{q q^{\prime}}{r^{2}} d r$
Total work done by this force as test charge moves from point $R$ to $S$ i.e., from $r_{1}$ to $r_{2}$ is,
$W=\int_{r_{1}}^{r_{2}} F d r=\int_{r_{1}}^{r_{2}} \frac{1}{4 \pi \varepsilon_{0}} \frac{q q^{\prime}}{r^{2}} d r$
or
$W=\frac{q q^{\prime}}{4 \pi \varepsilon_{0}}\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)$
Thus for this particular path work done on test charge q' depends on end points not on the path taken. Work done W in moving the test charge $q^{\prime}$ from point $R$ to $S$ is equal to the change in potential energy in moving the test charge $q^{\prime}$ from point $R$ to $S$. Thus,
$W=U\left(r_{1}\right)-U\left(r_{2}\right)$
where
$U\left(r_{1}\right)=\frac{q q^{\prime}}{4 \pi \varepsilon_{0} r_{1}}$
is the potential energy of test charge $q^{\prime}$ when it is at point $R$ and
$U\left(r_{2}\right)=\frac{q q^{\prime}}{4 \pi \varepsilon_{0} r_{2}}$
is the potential energy of test charge $q^{\prime}$ when it is at point $S$.
Thus potential energy of test charge $q$ ' at any distance $r$ from charge $q$ is given by
$U=\frac{q q^{\prime}}{4 \pi \varepsilon_{0} r}$
Equation 5 gives the electric potential energy of a pair of charges which depends on the separation between the charges not on the location of charged particles.
If we bring the test charge $q$ ' from a very large distance such that $r_{2}=\infty$ to some distance $r_{1}=r$ then we must do work against electric forces which is equal to increase in potential energy as given by equation 5 .

## Electric Potential

## 3. Electric Potential

We now move towards the electric potential which is potential energy per unit charge.
Thus electrostatic potential at any point of an electric field is defined as potential energy per unit charge at that point.
Electric potential is represented by letter V .
$\mathrm{V}=\mathrm{U} / \mathrm{q}^{\prime}$ or $\mathrm{U}=\mathrm{q}^{\prime} \mathrm{V}$
Electric potential is a scalar quantity since both charge and potential energy are scalar quantities.
S.I. unit of electric potential is Volt which is equal to Joule per Coulumb. Thus,

1 Volt $=1 \mathrm{JC}^{-1}$
In equation 4 if we divide both sides by $q$ ' we have
$\frac{W}{q^{\prime}}=\frac{U\left(r_{1}\right)}{q^{\prime}}-\frac{U\left(r_{2}\right)}{q^{\prime}}=V\left(r_{1}\right)-V\left(r_{2}\right)$
where $V\left(r_{1}\right)$ is the potential energy per unit charge at point $R$ and $V_{2}$ ) is potential energy per unit charge at point $S$ and are known as potential at points $R$ and $S$ respectively.
Again consider figure 1. If point $S$ in figure 1 would be at infinity then from equation 7
$V\left(r_{1}\right)-V(\infty)=\frac{W}{q^{\prime}}$
Since potential energy at infinity is zero therefore $\mathrm{V}(\infty)=0$. Therefore
$V\left(r_{1}\right)=\frac{W}{q^{\prime}}$
hence electric potential at a point in an electric field is the ratio of work done in bringing test charge from infinity to that point to the magnitude of test charge.
Dimensions of electric potential are $\left[\mathrm{ML}^{2} \mathrm{~T}^{-3} \mathrm{~A}^{-1}\right]$ and can be calculated easily using the concepts of dimension analysis.

## 4. Electric potential due to a point charge

Consider a positive test charge +q is placed at point O shown below in the figure.


## Figure 2

We have to find the electric potential at point $P$ at a distance $r$ from point $O$.
If we move a positive test charge $q$ ' from infinity to point $P$ then change in electric potential energy would be
$U_{P}-U_{\infty}=\frac{q q^{\prime}}{4 \pi \varepsilon_{0} r}$
Electric potential at point $P$ is
$V_{p}=\frac{U_{p}-U_{\infty}}{q^{\prime}}=\frac{q}{4 \pi \varepsilon_{0} r}$
Potential V at any point due to arbitrary collection of point charges is given by
$V=\frac{1}{4 \pi \varepsilon_{0}} \sum_{i=1}^{n} \frac{q_{i}}{r_{i}}$
here we see that like electric field potential at any point independent of test charge used to define it.
For continous charge distributions summation in above expressin will be replaced by the integration
$V=\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}$
where $d q$ is the differential element of charge distribution and $r$ is its distance from the point at which $V$ is to be calculated.

Electric Potential

## 5. Relation between electric fiels and electric potential

Consider the electric field E due to a point charge +q at point O in a radially outward direction shown below in the figure.


Figure 3
Suppose $R$ and $S$ are two points at a distance $r$ and $r+d r$ from point $O$ where $d r$ is vanishingly small distance and $V$ is electric potential at point $R$.
Now force on any test charge $\mathrm{q}^{\prime}$ at point R in terms of electric field is

## F=q'E

Work done by the force in displacing test charge from $R$ to $S$ in field of charge $q$ is
$d W=F \cdot d r=q$ 'E $\cdot d r$
and, change in potential energy is
$d U=-d W=-q^{\prime} E \cdot d r$
Change in electric potential would be
$d V=d U / q$
or dV = -E•dr
From equation 11 electric field is
$\mathrm{E}=-(\mathrm{dV} / \mathrm{dr})$
the quantity $\mathrm{dV} / \mathrm{dr}$ is the rate of change of potential with the distance and is known as potential gradient. Negative sign in equation 12 indicates the decrease in electric potential in the direction of electric field. For cartesian coordinate system
$E=E_{x} i+E_{y} j+E_{z} k$
and,
dr=dxi+dyj+dzk
from equation 11
$\mathrm{dV}=-\mathrm{E} \square \mathrm{dr}$
or, $d V=-\left(E_{x} d x+E_{y} d y+E_{z} d z\right)$
Thus components of E are related to corresponding derivatives of V in the following manner
$\mathrm{E}_{\mathrm{x}}=\mathrm{dV} / \mathrm{dx}$
$\mathrm{E}_{\mathrm{y}}=\mathrm{dV} / \mathrm{dy}$

## 6. Equipotential surfaces

Surface over which the electric potential is same everywhere is called an equipotential surface.
Equipotential surfaces are graphical way to represent potential distribution in an electric field.

We can draw equipotential surfaces through a space having electric field.

For a positive charge, electric field would be in radially outward direction and the equipotential surfaces would be concentric spheres with centers at the charge as shown below in the figure.


Figure 4

Since electric potential remains same everywhere on an equipotential surface from this it follows that PE of a charged body is same at all points on this surface.This shows that work done in moving a charged body between two points on an equipotential surface would be zero.

At every point on equipotential surface electric field lines are perpandicular to the surface. This is because potential gradient along any direction parallel to the surface is zero i.e., E=-dV/dr=0
so component electric parallel to equipotential surface is zero.

Electric Potential

## 7. Potential due to an electric dipole

We already know that electric dipole is an arrangement which consists of two equal and opposite charges +q and $-q$ separated by a small distance $2 a$.
Electric dipole moment is represented by a vector pof magnitude $2 q$ a and this vector points in direction from $-q$ to $+q$.
To find electric potential due to a dipole consider charge $-q$ is placed at point $P$ and charge $+q$ is placed at point $Q$ as shown below in the figure.


Figure 5

Since electric potential obeys superposition principle so potential due to electric dipole as a whole would be sum of potential due to both the charges $+q$ and $-q$. Thus
$V=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q}{r_{1}}-\frac{q}{r_{2}}\right)$
where $r_{1}$ and $r_{2}$ respectively are distance of charge $+q$ and $-q$ from point $R$.
Now draw line PC perpandicular to RO and line QD perpandicular to RO as shown in figure. From triangle POC $\cos \theta=O C / O P=O C / a$
therefore $\mathrm{OC}=\mathrm{a} \cos \theta$ similarly $\mathrm{OD}=\mathrm{a} \cos \theta$
Now,
$r_{1}=Q R \square R D=O R-O D=r-\cos \theta$
$V=\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{1}{r-a \cos \theta}-\frac{1}{r+a \cos \theta}\right)=\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{2 a \cos \theta}{r^{2}-a^{2} \cos ^{2} \theta}\right)$
since magnitude of dipole is
$|p|=2 q a$
$V=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{p \cos \theta}{r^{2}-a^{2} \cos ^{2} \theta}\right)$
If we consider the case where $r \gg a$ a then
$V=\frac{p \cos \theta}{4 \pi \varepsilon_{0} r^{2}}$
again since $p \cos \theta=\mathbf{p} \cdot \mathbf{r}^{\wedge}$ where, $\mathbf{r}^{\wedge}$ is the unit vector along the vector OR then electric potential of dipole is
$V=\frac{\boldsymbol{p} \cdot \hat{\boldsymbol{r}}}{4 \pi \varepsilon_{0} r^{2}}$
for $r \gg a$
From above equation we can see that potential due to electric dipole is inversly proportional to $r^{2}$ not ad $1 / r$ which is the case for potential due to single charge.
Potential due to electric dipole does not only depends on $r$ but also depends on angle between position vector $r$ and dipole moment $\mathbf{p}$.

## 8. Work done in rotating an electric dipole in an electric field

Consider a dipole placed in a uniform electric field and it is in equilibrium position. If we rotate this dipole from its equllibrium position, work has to be done.
Suppose electric dipole of moment $\mathbf{p}$ is rotated in uniform electric field $\mathbf{E}$ through an angle $\theta$ from its equilibrium position. Due to this rotation couple acting on dipole changes.
If at any instant dipole makes an angle $\varphi$ with uniform electric field then torque acting on dipole is
$\mathrm{dW}=$ torque x angular displacement
$=p E \sin \varphi d \varphi$
Total work done in rotating the dipole through an angle $\theta$ from its equilibrium position is

$$
\begin{equation*}
W=\int_{0}^{\theta} p E \sin \varphi d \varphi=p E[-\cos \varphi]_{0}^{\theta}=p E(1-\cos \theta) \tag{21}
\end{equation*}
$$

This is the required formula for work done in rotating an electric dipole placed in uniform electric field through an angle $\theta$ from its equilibrium position.

## 9.Potential energy of dipole placed in uniform electric field

Again consider equation 20 which gives the work done in rotating electric dipole through an infinetesimly small angle $d \varphi$ is
$\mathrm{dW}=\mathrm{pE} \sin \varphi \mathrm{d} \varphi$
which is equal to the change in potential energy of the system
$\mathrm{dW}=\mathrm{dU}=\mathrm{pE} \sin \varphi \mathrm{d} \varphi$
If angle $d \varphi$ is changed from $90^{\circ}$ to $\theta$ then in potential energy would be
$W=\int_{0}^{\theta} p E \sin \varphi d \varphi=p E(-\cos \varphi)_{0}^{\theta}=p E(1-\cos \theta)$
We have choosen the value of $\varphi$ going from $\pi / 2$ to $\theta$ because at $\pi / 2$ we can take potential energy to be zero (axis of dipole is perpandicular to the field). Thus $\mathrm{U}\left(90^{\circ}\right)=0$ and above equation becomes

$$
U(\theta)-U\left(90^{\circ}\right)=\int_{\infty \infty}^{\theta} p E \sin \varphi d \varphi=p E(-\cos \varphi)_{\infty}^{\theta}=-p E \cos \theta=-p \cdot E
$$

## CBSE Class-12 Physics Quick Revision Notes Chapter-02: Electrostatic Potential and Capacitance

## - Electrostatic Potential at a Point:

It is the work done by per unit charge by an external agency, in bringing a charge from infinity to that point.

- Electrostatic Potential due to a Charge at a Point:

$$
V(r)=\frac{2}{4 \pi \varepsilon_{0}} \frac{Q}{r}
$$

- The electrostatic potential at a point with position vector $r$ due to a point dipole of dipole moment p place at the origin is

$$
V(r)=\frac{1}{4 \pi \varepsilon_{0}} \frac{\vec{p} \cdot \vec{r}}{r^{2}}
$$

The result is true also for a dipole (with charges $-q$ and $q$ separated by $2 a$ ) for $r \gg a$.

- For a charge configuration $\mathrm{q}_{1}, \mathrm{q}_{2}, \ldots \ldots . \mathrm{q}_{\mathrm{n}}$ with position vectors $\mathrm{r}_{1}, \mathrm{r}_{2}, \ldots . \mathrm{r}_{\mathrm{n}}$, the potential at a point $P$ is given by the superposition principle,

$$
V=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q_{1}}{r_{1 p}}+\frac{q_{2}}{r_{2 p}}+\ldots . .+\frac{q_{n}}{r_{n p}}\right)
$$

where $\mathrm{r}_{1 \mathrm{p}}$ is the distance between $\mathrm{q}_{1}$, and P , as and so on.

- Electrostatics Potential Energy Stored in a System of Charges:

It is the work done (by an external agency) in assembling the charges at their locations.

- Electrostatic Potential Energy of Two Charges $q_{1}, q_{2}$, at $\mathbf{r}_{1}, \mathbf{r}_{\mathbf{2}}$ :

$$
U=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r_{12}}
$$

where $\mathrm{r}_{12}$ is distance between $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$

- Potential Energy of a Charge $q$ in an External Potential: $\mathrm{V}(\mathrm{r})=\mathrm{qV}(\mathrm{r})$
- Potential Energy of a Dipole of Dipole Moment p in a Uniform Electric Field: $\mathrm{E}=-\mathrm{p} . \mathrm{E}$.
- Equipotential Surface:

An equipotential surface is a surface over which potential has a constant value.
a) For a point charge, concentric spheres centered at a location of the charge are equipotential surfaces.
b) The electric field $E$ at a point is perpendicular to the equipotential surface through the point.
c) $E$ is in the direction of the steepest decrease of potential.

- Capacitance C of a System of Two Conductors Separated by an Insulator: It is defined as,

$$
C=\frac{Q}{V}
$$

where $Q$ and $-Q$ are the charges on the two conductors $V$ is the potential difference between them.

- Capacitance is determined purely geometrically, by the shapes, sizes, and relative positions of the two conductors.
- Capacitance C of a parallel plate capacitor (with vacuum between the plates):

$$
C=\varepsilon_{0} \frac{A}{d}
$$

where $A$ is the area of each plate and $d$ the separation between them.

- For capacitors in the series combination:

The total capacitance C is

$$
\frac{1}{C}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}+\ldots
$$

- For capacitors in the parallel combination:

The total capacitance C is

$$
C=C_{1}+C_{2}+C_{3}+\ldots \ldots
$$

where $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$... are individual capacitances

- The energy $U$ stored in a capacitor of capacitance $C$, with charge $Q$ and voltage $V$ :

$$
U=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{1}{2} \frac{Q^{2}}{C}
$$

- The electric energy density (energy per unit volume) in a region with electric field:

$$
(1 / 2) \varepsilon_{0} E^{2}
$$

- The potential difference between the conductor (radius $r_{o}$ ) inside $\&$ outside spherical shell (radius $R$ ):

$$
\phi\left(r_{0}\right)-\phi(R)=\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{1}{r_{0}}-\frac{1}{R}\right)
$$

which is always positive.

- When the medium between the plates of a capacitor filled with an insulating substance:
Changes observed are as follows:
i. Polarization of the medium gives rise to a field in the opposite direction.
ii. The net electric field inside the insulating medium is reduced.
iii. Potential difference between the plates is thus reduced.
iv. Capacitance C increases from its value when there is no medium (vacuum). where K is the dielectric constant of the insulating substance.


## - Electrostatic Shielding:

A conductor has a cavity with no charge inside the cavity, then no matter what happens outside the conductor. Even if there are intense electric fields outside the conductor, the cavity inside has, shielding whatever is inside the cavity from whatever is outside the cavity. This is called electrostatic shielding.

