Electric Current ,Resistance and Resistivity

(1) Introduction

In our previous few chapters of electrostatics,we have discussed various terms and characterstics related to charge at rest
Now in this chapter we will study about the moving charges,phenomenom related to them and various effects related to charge in motion
Consider two metallic conducting balls charged at different potential are hanged using a non conducting insulating wires .Since air is an insulator ,no charge transfer takes place
Now if we join both the metallic wire using a conducting metallic wire then charge will flow from metallic ball at higher potential to the one at lower potential.
This flow of charge will stop when the two balls would be at the same potentials.
If somehow we could maintian the potential between the metallic balls,we will get constant flow of the charge in metallic wire,connecting the two conducting balls
This flow of charge in metallic wire due to the potential dicference between two conducters used is called electric current about which we would be dicussing in this chapter.

(2)Electric current and Current density

Electric Current
We already had a brief idea about the electric current which wed defined as the state of motion of the electric charge .Now we are going to study about the electric current in details
Quantitatively electric current is defined as the time rate of flow of the net charge of the area of crosssection of the conducter i.e

Electric current = Total charge flowing / time taken

if q is the amount of charge flowing through the conducter in t sec,The current through the conducter is given by

$I=q/t$ (1)

SI unit of the current is Ampere(A) named so in the honour of french scientist Andee marie Ampere(1775-1836).Now,

1 Ampere= 1 Coulumb/ 1 sec=1 Cs^{-1}

Thus current through any conducter is said to be 1 ampere,if 1 C of charge is flowing through the conducter in 1 sec

Small amount of currents are accordingly expressed in milliamperes ($1mA=10^{-3}$ A) or in micro ampere ($1 mA=10^{-6}$ A)

Direction of electric current is in the direction of the flow of positive charged carriers and this current is known as conventional current.

Direction of the flow of electron in conductor gives the direction of electronic current. Direction of conventional current is opposite to that of electronic current

Electric current is a scalar quantity .Although electric current represent the direction of the flow of positive charged carrier in the conductor,still current is treated as scalar quantity as current in wires in a circuit does not follows the laws of vector addition

Current density

The current density at a point in the conductor is defined as the current per unit cross-section area.Thus if the charge is flowing per unit time uniformaly over the area of crosss-section A of the conductor,then current density J at any point on that area is defined as

$J=I/A$ -(2)

It is the characterstic property of point inside the conductor nor of the conductor as a whole

Direction of current density is same as the direction of conventional current

Note that current density is a vector quantity unlike electric current

Unit of current density is Ampere/meter^2 (Am^{-2})

EMF and Electric Measurement

(6)Grouping of the cell's

A limited ammount of current can be drawn from a single cell or battery
There are situations where single cell fails to meet the current requirement in a circuits
To overcome the problem cells can be grouped in series and in parallel combinations or mixed grouping of cells is done in order to obtain a large value
of electric current

(A) Series combination

Figure below shows the two cells of emf's E_1 and E_2 and internal resistance r_1 and r_2 respectively connected in series combination through external resistance

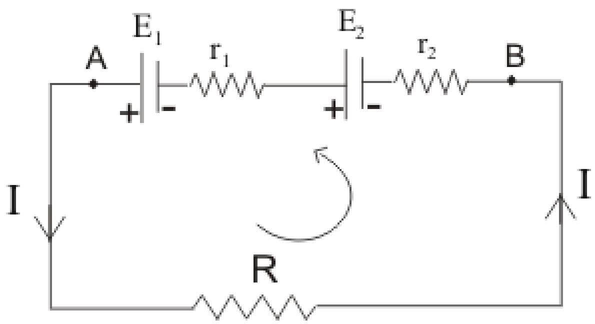


Figure 9a. Cells grouped in series

Points A and B in the circuit acts as two terminals of the combination
Applying kirchoff's loop rule to above closed circuit

$-Ir_2-Ir_1-IR+E_1+E_2=0$

or

$I=E_1+E_2/R+(r_1+r_2)$

Where I is the current flowing through the external resistance R
Let total internal resistance of the combination by $r=r_1+r_2$ and also let $E=E_1+E_2$ is the total EMF of the two cells
Thus this combination of two cells acts as a cell of emf $E=E_1+E_2$ having total internal resistance $r=r_1+r_2$ as shown above in the figure

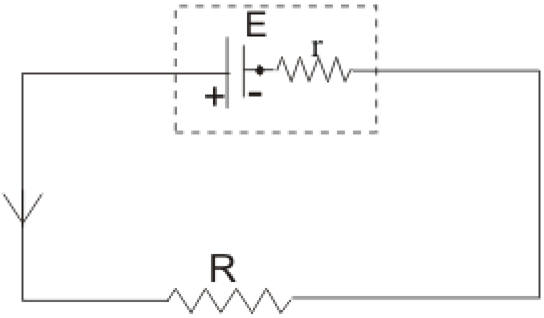


Figure 9b. Equivalent cell of emf E and equivalent internal resistance r

(B) Parallel combinations of cells

Figure below shows the two cells of emf E_1 and E_2 and internal resistance r_1 and r_2 respectively connected in parallel combination through external resistance

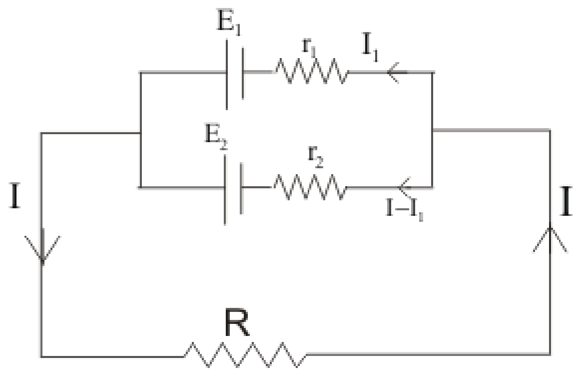


Figure 10. Cells grouped in parallel combination

Applying kirchoff's loop rule in loop containing E_1 ,r_1 and R,we find

E_1-IR-I_1r_1=0 -------------------------(1)

Similarly applying kirchoff's loop rule in loop containing E_2 ,r_2 and R,we find

E_2-IR-(I-I_1)r_2=0 -------------------------(2)

Now we have to solve equation 1 and 2 for the value of I,So multiplying 1 by r_2 and 2 by r_1 and then adding these equations results in following equation

$IR(r_1+r_2)+r_2r_1I-E_1r_2-E_2r_1=0$

which gives

$$I = \frac{E_1r_2 + E_2r_1}{R(r_1 + r_2) + r_2r_1}$$

We can rewrite this as

$$I = \frac{\frac{E_1r_2 + E_2r_1}{r_1 + r_2}}{R + \frac{r_1r_2}{r_1 + r_2}} = \frac{E}{R + r} \qquad (11)$$

where

$$E = \frac{E_1r_2 + E_2r_1}{r_1 + r_2}$$

and

$$r = \frac{r_1r_2}{r_1 + r_2}$$

E is the resulting EMF due to parallel combination of cells and r is resulting internal resistance.

(7) Wheat stone bridge

Wheat stone bridge was designede by british physicist sir Charles F wheatstone in 1833

It is a arrangement of four resistors used to determine resistance of one resistors in terms of other three resistors

Consider the figure given below which is an arrangement of resistors and is knowns as wheat stone bridge

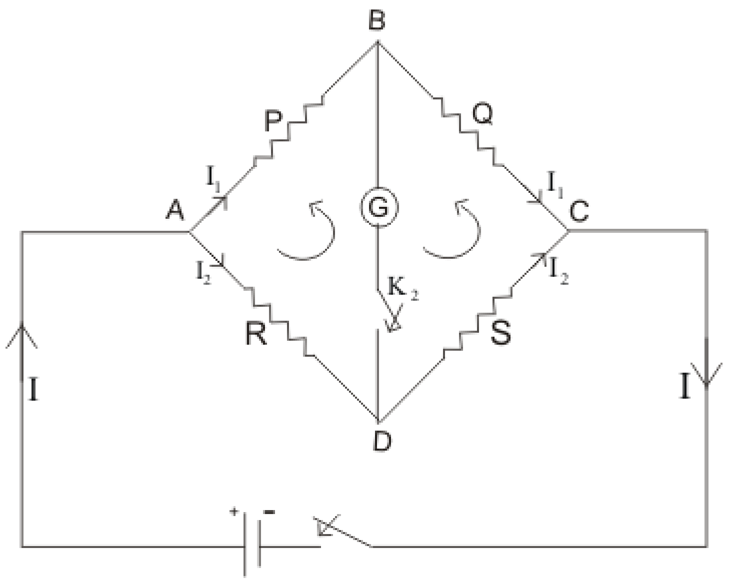


Figure 11. Wheatstone bridge circuit arrangement

Wheatstone bridge consists of four resistance P,Q,R and S with a battery of EMF E.Two keys K_1 and K_2 are connected across terminals A and C and B and D respectively

ON pressing key K_1 fisrt and then pressing K_2 next if galvanometer does not show any deflection then wheatstone bridge is said to be balanced

Galavanometer is not showing any deflection this means that no current is flowing through the galvanameter and terminal B and D are at the same

potential .THus for a balanced bridge

$V_B=V_D$

Now we have to find the condition for the balanced wheatstone bridge .For this applying kirchoff's loop rule to the loop ABDA ,we find the relation

$-I_2R+I_1P=0$

or $I_1P=I_2R$ --(a)

Again applying kirchoff's rule to the loop BCDB

$I_1Q-I_2S=0$

or $I_1Q=I_2S$ --(b)

From equation a and b we get

$I_1/I_2=R/P=S/Q$

or

$P/Q=R/S$ (12)

equation 12 gives the condition for the balanced wheatstone bridge

Thus if the ratio of the resistance R is known then unknown resistance S can easily be calculated

One important thing to note is that when bridge is balanced positions of cell and galvanometer can be exchanged without having any effect on the balance of the bridge

Sensitivity of the bridge depends on the relative magnitudes of the resistance in the four arm of the bridge is

EMF and Electric Measurement

(8) Meterbridge (slide wire bridge)

Meter bridge is based on the principle of wheatstone bridge and it is used to find the resistance of an unknown conductor or to compare two unknown
resistance
Figure below shows a schematic diagram of a meter bridge

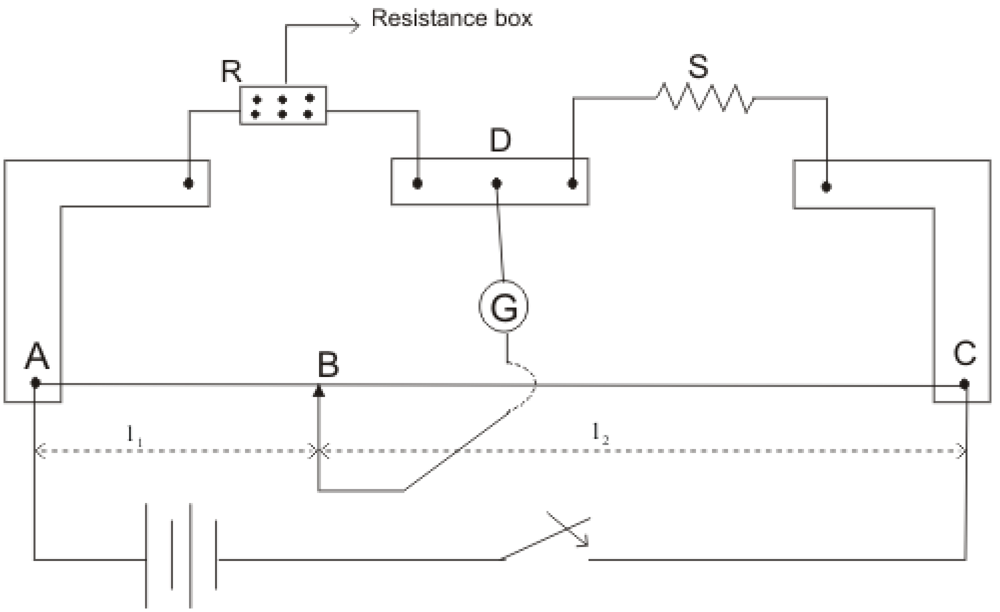


Figure 12. A meter bridge

In above figure AC is a 1m long wire made of maganin or constanan having uniform area of cross-section
This wire is stretched along a scale one a wooden base
Ends A and C of the wire are screwed to two L shaped copper strips as shown in figure
A resistance box R and an unknown resistance S are connected as shown in figure
One terminal of galvanometer is connected to point D and another terminal is joined to a jockey that can be slided on a bridge wire
when we adjust the suitable resistance of value R in the resistance box and slide this jockey along the wire then a balance point is obtained sat at point B
Since the circuit now is the same as that of wheatstone bridge ,so from the condition of balanced wheatstone bridge we have
P/Q=R/S
Here resistance P equals
$P=\rho l_1/A$

where ρ is the resistivity of the material of the wire and A is the area of cross-section of wire

Now $P/Q=(\rho l_1/A)(A/\rho l_2)=l_1/l_2$

(9) Potentiometer

Potentiometer is an accurate instruments used to compare emf's of a cells,Potential difference between two points of the electric wire

Potentiometer is based on the principle that potential drop across any portion of th wire of uniform crossection is proportional to the length of that portion of thw wire when a constant current flows through the wire

Figure below shows the construction of a potentiometer which consists of a number of segments of wire of uniform area of cross-section stretched on a wooden board between two copper strips .Meter scale is fixed parallel to the lenght of the wire

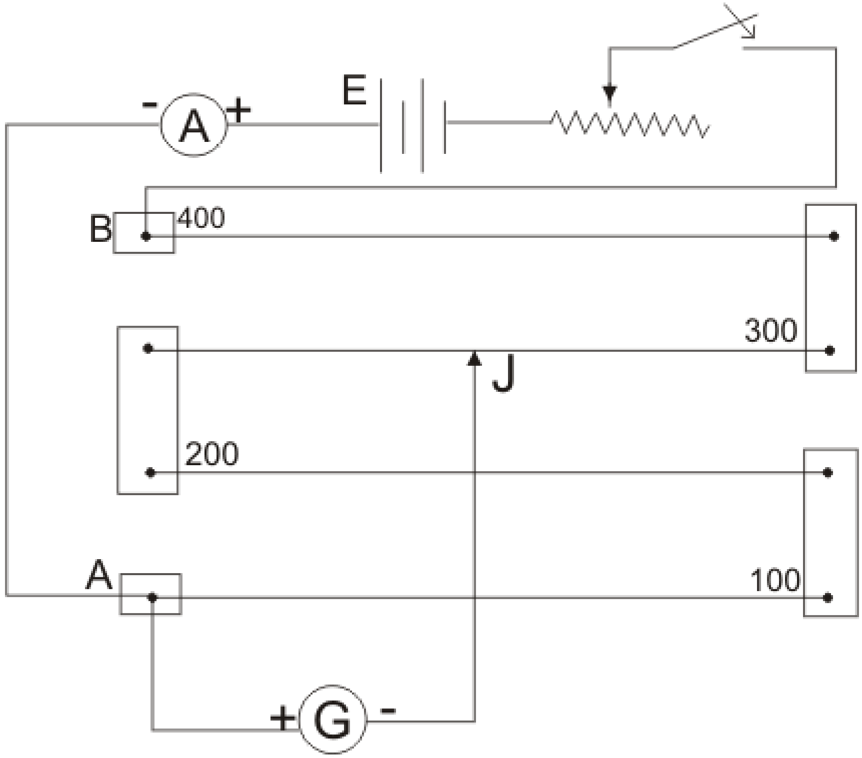


Figure 13. Potentiometer

A battery is connected across terminals A and B through a rehestat so that a constant currents flows through the wire

Potentiometer is provided with a jockey J with the help of which contact can be made at any point on the wire

Suppose A and ρ are the area of cross-section and resistivity of the material of the wire the resitance

R=ρl/A ----------------------------(i)

where l is the lenght of the wire

If I is the current flowing through the wire then from Ohm's Law,

V=IR ------------------------------(ii)

Where V is the potential differene across the position of the wire of length l

Thus ,from (i) and (ii)

V=IR=I(ρl/A)=kl

where K=ρI/A

=> V is proportional to l when current I is constant

K=V/l is also known as potential gradient which is the fall of potential per unit length of wire

Senstivity of a potentiometer depends on its potential gradient .If the potential gradient of a potentiometer is small then the potentiometer is more sensitive and hence more accurate

Thermal Effect of Current

(1) Introduction

We have already learned the electric current and the physics behind it in the previous chapter

We have also discussed the mechanism of flow of current in a conducter but not the physical consquences of flow of elctric current are related to other forms of energy

In this chapter we will study about causes and consequences of electric current

In the nutshell ,what we will study in this chapter is the connection between the electricity and thermal energy.

(2) Heating effect of current

In previous chapter while discussing electric energy and power ,we learned that $I\Delta V$ amount of energy is lost per second when a current I flows through a potential ΔV and this energy appears in the form of heat energy

Due to the conversion of electric energy into heat energy the conductor becomes hot .This effect is known as Joule's Heating and this heating is thermodynamics irreversible.

Cause Behind Joule's Heating:-

Explanation behind the Joule's heating is that when a potential difference ΔV is maintained between the ends of a conductor,the free electrons in the conductor are accelerated towards the higher potential end of the conductor

In their way electrons frequently collided withe the positive ions of the conductor due to which their velocity decreased

This the energy electrons gained on account of acceleration is transferred to the positive lattice ions or atoms and electrons then returns to their equilibrium distribution of velocities

Thus ,lattice ions receives energy randomly at the average rate of $I\Delta V$ per unit time

Ions spends this energy by vibrating about their mean positions resulting in rise in the temperature of the conductor

This way Joule's heating nothing but the conversion of electrical energy into heat energy

(3) Thermoelectricity

We know that currents flows in a conductor whenever there is a electric potential difference betweens the ends of the conductor

If there is a temperature difference between the ends of the conductor then thermal energy flows from hotter end to the colder ends

Thermal energy flows may also be carries by the electrons in the conductor and hence resulting the presence of electric current

At the hotter end of the conductor electrons have slightly higher kinetic energy and hence they move faster

So there is net flow of current towards the end of the conductor with lower temperature.Thus an electric current exists in the conductor due to the difference in the temperature of two ends of the conductor

This phenonmenon due to which electricity is produced when two ends of the conductor are kept at different temperature is known as thermoelectricty

Thermal Effect of Current

(4)Seeback effect

Seeback effect was first discovered by Thomas John seaback

It stated that when two different conductor are joined to form a circuit and the two junctions are held at the different temperature then an emf is developed which results in the flow of the electric current through the circuit.Arrangement is shown as below in figure

Maginitude of thermo-electric emf depends upon the nature of the two metals and on the temperature difference between terminals

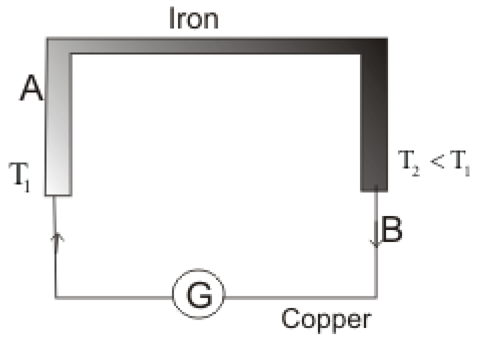


Figure 1. Thermocouple of iron and copper and direction of current is from Cu to Fe through hot junction

Seaback effect is reversible i.e, if the hot and cold junctions are reversed the direction of thermoelectric current is alse reversed

Seaback investigated thermo-electric properties of a large number of metals and arranged them in a series known as thermo-electric series or seaback series and is given as follows

Bi,Ni,Co,Pt,Cu,Mn,Hg,Pb,Sn,Au,Ag,Zn,Cd,Fe,As,Sb,Te

When any two of these metals in the series is used to form a thermocouple ,the thermo emf is greater when two metals used are farther apart in the circuit

Figure 1 shows the thermocouple of Cu and Fe.The current in this couple flows from Cu to Fe through the hot junction

The thermo emf of this couple is only 1.3 milivolt for a temperature difference of 100 C between the hot and cold junction

(5) variation of thermo-emf with temperature

To study the effect of difference of temperature of the two junctions consider a thermo-couple of two dis-similar metals A and B

Now consider that cold junction is at temperature 0 C and the temperature of the hot junction is raised gradually

It is found in experiment that thermo emf varies with the temperature of the hot junction

Figure below shows the graph of variation of thermo emf in the circuit with the variation of temperature of the hot junction.

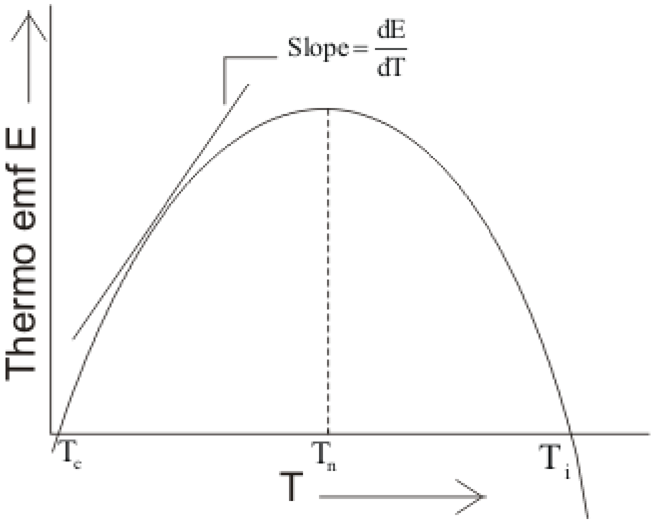


Figure 2. Variation of thermo emf with temperature of hot junction

From graph it is clear that thermoemf is zerp when both the junctions are at the same temperature 0 C and gradually increases as the temperature of the hot junction increases

The temperature of the hot junction at which thermoemf in the thermocouple becomes maximum is called neutral temperature (T_n) for that thermocouple

For a given thermo-couple of two metals neutral temperature has a fixed value

On further increasing the temperature of the hot junction,after T_n has reached,The thermo-emf decreases and becomes equal at a particular temperature called temperature of inversion T_i.

Beyond T_i ,if the temperature of hot junction is still increased ,the thermo-emf again started to increase but in reverse direction

Temperature of inversion T_i is as much above the neutral temperature as neutral temperature is above the temperature of the cold junction.Thus mathimatically

T_n-T_c=T_i -T_n

or T_n=(T_i+T_c)/2

Hence neutral temperature is the mean of the temperature of inversion and temperature of the cold junction

Thermo-emf is the property of each material and can be easily measured for a junction of two dissimilar metals at different temperatures

Thermo-emf of number of thermocouples is given by the simple relation

$E=\alpha T+\beta T^2$

where T is the temperature difference between the two junction and α and β are the parameters of the materoal used

Also the rate of change of thermo-emf with temperature i.e dE/dT is called thermo-power or seaback coefficent S .Mathematically

$S=dE/dT$

Thermal Effect of Current

(6) Peltier Effect

Peltier effect is named after his discover Jean Peltier who in 1934 discovered a thermo-electric effect which is converse of Seaback effect

Peltier discovered that "when an electric current is passed through two disimilar conductor connected to form a thermo-couple ,heat is evolved at one junction and absorbed at the other end.The absorption and evolution of heat depends on the direction of flow of current

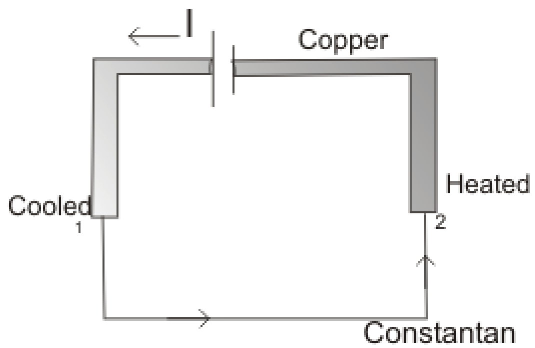


Figure 3. In copper constantan thermocouple if an electric current flows as shown then junction n1 gets cooled and 2 get heated

Peltier effect is entirely reversible in nature

Peltier Coefficent

Peltier coefficent is defined as the amount of heat energy absorbed or evolved due to peltier effect at the junction of two dissimilar metals when one coulumb of charge passes through the junction

Peltier coefficent is denoted by π

Value of Peltier Coefficent is different for different thermo-couple .Its value also depends upon the temperature of the junction

If q amount of charge passes through the junction then

Energy absorbed or evolved=πq

if V is the contact Potential difference

Then workdone=qV

Now heat absorbed=Workdone

So π=V

Hence Peltier coefficent (in J/C) at a junction is numerically equal to the contact v in(Volts)

(7) Thomson effect

Thomson effect is related to the emf that develops between two parts of the single metal when they are at different temperature

Thus thomson effect is the absorption or evolution of heat along a conductor when current passes through it when one end of the conductor is hot and another is cold

If two parts of the metal are at small temperature difference dT ,themn the electric potential difference is proportional to dT $dV \alpha dT$

or

$dV=\sigma dT$

where σ is the constant of proportinality and is known as thomson coefficent

Peltier coefficent and thomson coeficent are related to thermopower according to following relations

$\pi=Ts=T(dE/dT)$

and $\sigma =-T(ds/dT)=-T(d^2E/dT^2)$

We have seen that all the three effects are defined in terms of three coefficent namely seaback,peltier and thomson coefficent but the basic quantity is thermo-power which is the rate of change of thermo-emf with temperature

(8)Applications of thermoelectricity or thermo-electric effect are

To measure temperature using thermo electric thermometer

To detect heat radiation using thermopiles

Thermoelectric refregerator or generator

Electric Current ,Resistance and Resistivity

(3) Drift Velocity

Metallic conductors have large numbers of elctrons free to move about.These elctrons which are free to move are called conduction electrons
Thus valence electrons of atom becomes the conduction electrons of the metals
At room temperature,these conduction electrons moves randomly inside the conductor more or less like a gas molecule
During motion,these conduction electrons collide with ions(remaining positive charged atom after the valence electrons move away) again and again and there direction of motion changes after each and every collision.
As a results of these collisions atoms moves in a zig-zag path
Since in a conductor there are large number of elctrons moving randomly inside the conductor.Hence they have not net motion in any particular direction.Since the number of electrons crossing an imaginary area ΔA from left to right inside the conductor very nearly equals the number of electron crossing the same area element from right to left in a given interval of time leaving flow of electric current through that area nearly equals to zero
Now when we applied some P.D using a battery across the two ends of the conductor,then an electric field sets up inside the conductor
As a result of this electric field setup inside the conductor,conduction electron expeirence a force in direction opposite to electric field and this force accelerates the motions of the electrons
As a result of this accelerated motion electrons drifts slowly along the length of the conductor towards the end at higher potential
Due to this acceleration velocity of electron's increases only for short interval of time as each acclerated electrons suffers frequent collision with positive ions and looses their Kinectic energy
After each collision electrons starts fresh in random direction ,again get accelerated and loose their gained Kinetic energy in another collision
This extra velocity gained by the electrons is lost in subsequent collison and the processes continued till the electron reach positive end of the conductor
Under the effect of electric field inside the conductor ,free electrons have random thermal velocities due to the room tmeperature and small velocities with which they drift towards the positive end of the conductor.
if τ is the average time between two successive collisions and E is the strength of applied electric field then force on electron due to applied electric field is
$F=eE$

if m is the mass of electron ,then acceleration produced is given by

$a=eE/m$

Since electron is acclerated for an average time interval τ,additional velocity acquired by the electron is

$v_d=a\tau$

or $v_d=(eE/m)\tau$ (3)

This small velocity imposed on the random motion of electrons in a conductor on the application of electric field is known as drift velocity

This drift velocity is defined as the velocity with which free electrons gets drifted towards the positive end of the conductor under the influence of externally applied electric field

(4) Relation between drift velocity and electric current

Consider a conducting wire of lenghth L and having uniform cross-section area A in which electric field is present

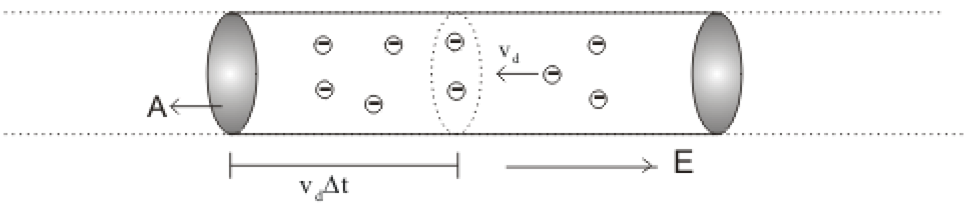


Figure 1:- Electrons moving opposite to electric field in a conductor

Consider in the wire that there are n free electrons per unit volume moving with the drift velocity v_d

In the time interval Δt each electron advances by a distance $v_d\Delta t$ and volume of this portion is $Av_d\Delta t$ and no of free electron in this portion is $nAv_d\Delta t$ and all these electrons crosses the area A in time Δt

Hence charge crossing the area in time Δt is

$\Delta Q = neAv_d \Delta t$

or

$I = \Delta Q/\Delta t = neAv_d$ (4)

This is the relation between the electric current and drift velocity

If the moving charge carriers are positive rather than negative then electric field force on charge carriers would be in a direction of electric fields direction and drift velocity would be in left to right direction opposite to what shown in fig-1

In terms of drift velocity current density is given as

$j = I/A = nev_d$ (5)

Electric Current ,Resistance and Resistivity

(5) Ohm's Law and Resistance

Ohm's law is the relation between the potential difference applied to the ends of the conductor and current flowing through the conductor.This law was expressed by George Simon Ohm in 1826

Statement of Ohm's Law

'if the physical state of the conductor (Temperature and mechanical strain etc) remains unchanged ,then current flowing through a conductor is always ditectly proportional to the potential difference across the two ends of the conductor

Mathematically

$V \alpha I$

or

$V=IR$ (6)

Where constant of proportionallity R is called the electric resistance or simply resistance of the conductor

Value of resistance depends upon the nature ,dimension and physically dimensions of the conductor

Ohm's Law can be deducted using drift velocity relation as given in equation -3 .Thus from the equation

$v_d=(eE/m)\tau$

but Now $E=V/l$

Therfore

$v_d=(eV/ml)\tau$

Also $I=neAv_d$

Substituting the value of v_d in I relation

$I=(ne^2A\tau/ml)\ V$ (7)

or $V/I=(ml/ne^2A\tau)=R$ a constant for a given conductor

Thus

$V=IR$

Mathematical expression of Ohm's Law

From Ohm's Law

$V=IR$ or $R=V/I$ (8)

Thus electric resitance is the ratio of potential difference across the two ends of conductor and amount of current flowing through the conductor

electric resistance of a conductor is the obstraction offered by the conductor to the flow of the current through it.

SI unit of resistance is ohm (Ω) where

1 Ohm=1 volt/1 Ampere

or $1\Omega = 1VA^{-1}$

Dimension of resistance is $[ML^2T^{-3}A^{-2}]$

(6) Resistivity and conductivity

In terms of drift velocity ,electric current flowing through a conducting wire of length L and uniform area of cross-section A
is

$I = dQ/dt = neAv_d = (ne^2A\tau/ml) V$

The above can be rearranged to give the ohm's law i.e,

V=IR

where $R = (ml/ne^2A\tau)$ Now $R = \rho l/A$ (9)

Where ρ is called the specific resistance or resistivity of the conductor

And $\rho = m/ne^2\tau$ (10)

From equation (9) ,we can see that resistance of the wire is proportional to its length and inversly proportional to its cross-sectional area.

Thus resistance of a long and thin wire will greater then the resistance of short and thick wire of the same material

Now from equation (9)

$R = \rho l/A$ (11)

And from ohm law R=V/I

Therefore

$\rho = (V/I)(A/L)$

$= (V/L) / (I/A)$

$= E/J$ (12)

Where E=V/L is the electric field at any point inside the wire and J=I/A is current density at any point in the wire.

Unit of resistivity is ohm-meter.

Thus from equation (12) ,electric resistivity can also be defined as the ratio of electric field intensity at any point in the conductor and the current density at that point.

The greater the resistivity of the material ,greater would be the field needed to establish a given current densisty

Perfect conductor have zero resistivities and for perfect insulators resistivity would be infinite

Metals and alloys have lowest resistivities and insulators have high resistivities and exceeds those of metals by a factor of 10^{22}

The reciprocal of resistivity is called conductivity and is represented by σ

Unit of conductivity is ohm^{-1}meter^{-1}($\Omega^{-1}m^{-1}$) and

σ is defined as

$\sigma=1/\rho$

Since $\rho=E/J$

or $\sigma=J/E$

or $J=\sigma E$ (13a)

The above relation can also be written in vector form as both J and E are vector quatities where vector **J** being directed towards **E**

$\mathbf{J}=\sigma\mathbf{E}$ (13b)

Electric Current ,Resistance and Resistivity

(7) variation of resistivity with temperature

Resistance and hence resistivity of conductor depends on mubers of factors
One of the most important factors is dependence of resistance of metals on temperature
Resistivity of the metallic conductor increases with increase on temperature
when we increase the temperature of the metallic conductor,its constituent atoms vibrate with greater amplitudes then usual.This results
to the more frequent collison between ions and electrons
As a result average time between the two successive collision decreases resulting the decrease in drift velocity
Thus increase collison with the increase in tempearture results in increase resistivity
For small temperature variations ,resistivity of the most of the metals varies according to the following relations

$$\rho(T)=\rho(T_0)[1 + \alpha(T-T_0)] \qquad (14)$$

Where $\rho(T)$ and $\rho(T_0)$ are the resistivies of the material at temperature T and T_0 respectively and α is the constant for given materail and is known as coefficient of resistivity.
Since resistance of a given conductors depends on the length and crosssectional area of the conductor through the relation

$R=\rho l/A$

Hence temperature variation of the resistance can be given as

$$R=R(T_0)[1 + \alpha(T-T_0)] \qquad (15)$$

Resistivity of alloys also increase with temperature but this increase is much small as compared to metals
Resistivies of the non-metals decreases with increase in temperature .This is because at high temperature more electrons becomes avialable for conduction as they set themselves loose from atoms and hence temperature coefficient of resistivity is negative for non-metals
A similar behavior occurs in case of semi-conductors .temperature coefficient of resistivity is negative for semi-conductors and its value is often large for a semi-conductor materials

(8) Current Voltage relations

We know that current through any electrical device such as resistors depends on potential defference between the terminals
Devices obeying ohm's law follow a linear relationship between current following and potential applied where current is directly proportional to voltage applied .Graphical relation between V and I is shown below in figure

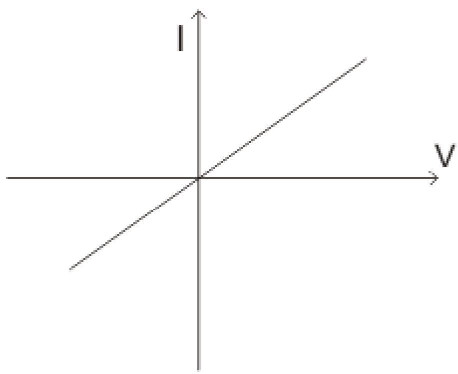


Figure 2:- IV relation for resistors obeying Ohm’s Law

Graph for a resistor obeying ohm's law is a straight line through the origin having some finite slope
There are many electrical devices that does not obey the ohm's law and current may depends on voltage in more complicated ways.Such devices are called non-ohmic devices for examples vaccum tubes,semiconductor diodes ,transistors etc
Consider the case of a semi conductor junction diode which are used to convert alternating current to direct current and are used to perform variety of logic functions is a non=ohmic device
Graphical voltage relation for a diode is shown below in the figure

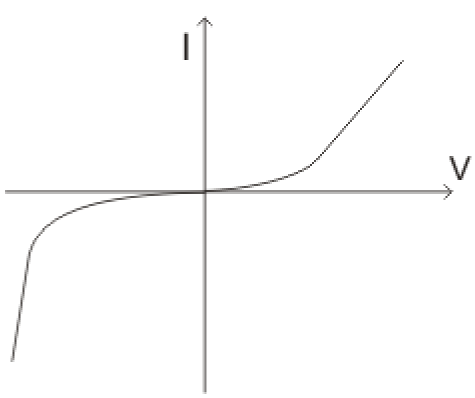


Figure 3:- IV relation for semiconducting diode

Figure clearly shows a non linear depeence of current on voltage and diode clearly does not follow the ohm's w
When a device does not follow obey ohm's law,it has non linear voltage -current relation and the quantity V/I is no longer a constant however ratio is still known as resistance which now varies with current
In such cases we define a quantity dV/dI known as dynamic resistance which expresses the relation between smaal change in current and resulting change in voltage
Thus for non-ohmic electrical devices resistance is not constant for different values of V and I

Electric Current ,Resistance and Resistivity

(9) Colour code of carbon resistors

Commercially resistors of different type and values are avilable in the market but in electronic circuits carbon resistors are more frequently used
In carbon resistors value of resistance is indicated by four coloured bands marked on its surface as shown below in figure

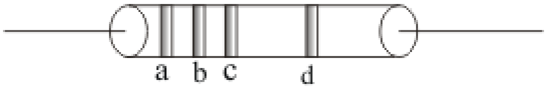


Figure 4:- Carbon resistor with strips

The first three bands a,b.c determine the value of the resistance and fourth band d gives the tolerance of the resistance
The colour of the first and second band respectively gives the first and second significant figure of the resistance and third band c gives the power of the ten by which two significant digits are multiplied for obtainng the value of the resistance
value of different colurs for making bands in carbon resistors are given below in the table

Colur	Figure(first and second band)	Multiplier(for third band)	tolerance

Black	0	1	-
Brown	1	10	-
Red	2	10^2	-
Orange	3	10^3	-
Yellow	4	10^4	-
Green	5	10^5	-
Blue	6	10^6	-
Violet	7	10^7	-
Gray	8	10^8	-
white	9	10^9	-
Gold	-	10^{-1}	5%
Silver	-	10^{-2}	10%
no Colour	-	-	20%

For example in a given resistor let first strip be brown ,second strip be red and third be orange and fourth be gold then resitance of the resitor would be
$12X\ 10^3$ +/- 5%

Electric Current ,Resistance and Resistivity

(10)Combination of Resistors

We have earlier studied that sevral capacitors can be connected in series or parallel combination to form a network. In same way sevral resistor may be combined to form a network.
Just like capacitors resistors can be grouped in series and parallel.
Equivalent resistance of the combination of any number of resistors is a single resistance which draw same current as the combination of different resistances draw when the same potential difference is applied across it.

(A) Resistors in Series
Resisors are said to be connected in series combinaton. If same current flows through each resistor when same potential difference is applied across the combination.
Consider the figure given below

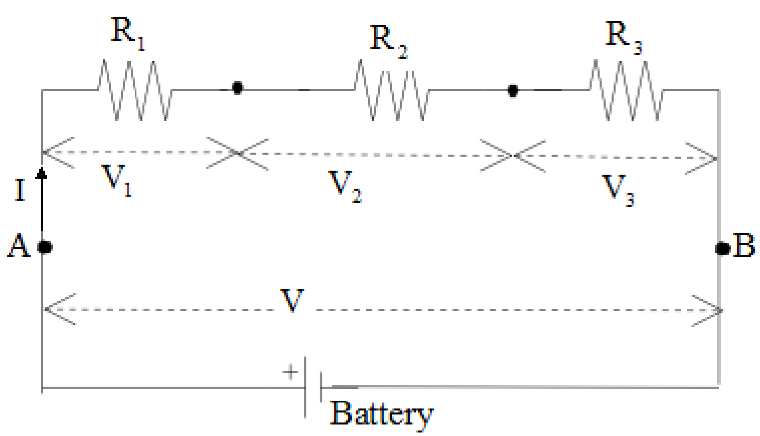


Figure 5:- Series combination of three resistors connected to a battery

In figure given above three resistors if resistance R_1, R_2 and R_3 are connected ibn series combination.
If battery is connected across the series combination so as to mintain potential difference V between points A and B, the current I would pass through each resistor.
If V_1, V_2 andV_3 is the potential difference across each resistor R_1, R_2 and R_3 respectively, then according to Ohm's Law,

V_1=IR_1

V_2=IR_2

V =IR

3 3

Since in series combination current remains same but potential is divieded so,

$V=V_1+V_2+V_3$

or, $V=I(R_1+R_2+R_3)$

If $R_{eq}$is the resistance equivalent to the series combination of R_1, R_2 and R_3 then ,

$V=IR_{eq}$

where, $R_{eq}=R_1+R_2+R_3$

Thus when the resistors are connected in series, equivalent resistance of the series combination is equal to the sum of individual resistances.

Value of esistance of the series combination is always greater then the value of largest individual resisnces.

For n numbers of resistors connected in series equivalent resistance would be

$R_{eq}=R_1+R_2+R_3+.........................+R_n$

(B) Resistors in parallel

Resistors are said to be connected in parallel combination if potential difference across each resistors is same.

Thus , in parallel combination of resistors potential remains the same but current is divided.

Consider the figure given below

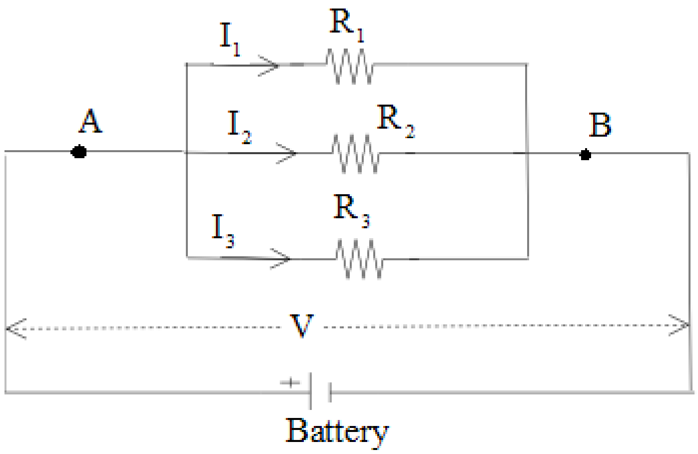


Figure 6:- Parallel combination of three resistors connected to a battery

Battery B is connected across parallel combination of resistors so as to maintain potential difference V across each resistors.Then total current in the circuit would be

$I=I_1+I_2+I_3$ (16)

Since potential difference across each resistors is V. Therefore, on applying Ohm's Law

$V=I_1R_1=I_2R_2=I_3R_3$

or,

$$I_1=\frac{V}{R_1}, I_2=\frac{V}{R_2}, I_3=\frac{V}{R_3}$$

From equation (16)

$$I=\frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3}=V\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

If R is the equivalent resistance of parallel combination of three resistors heaving resistances R_1, R_2 and R_3 then from Ohm's Law

$V=IR_{eq}$

or,

$$I=\frac{V}{R_{eq}} \qquad (17)$$

Comparing equation (16) and (17) we get

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

For resistors connected in parallel combination reciprocal of equivalent resistance is equal to the sum of reciprocal of individual resistances.

Value of equivalent resistances for capacitors connected in parallel combination is always less then the value of the smallest resistance in circuit.

If there are n number of resistances connected in parallel combination, then quivalent resistance would be reciprocal of

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots\dots\dots\dots\dots + \frac{1}{R_n}$$

EMF and Electric Measurement

(1) Introduction

In previous chapter we have already studied about electric current and resistance.
We know that a force must be applied on free charges of a conductor in order to maintain a continous current in the conductor.
Here a question arises how can we maintain this force in order to maintain a continous flow of current. You will find answer to this question while studying this chapter.
In this chapter we will learn about ElectroMotive Force(emf) and sources of emf (responsible for driving charge round the closed circuit). We'll also learn about electric circuits and measurements.

(2) ElectroMotive Force(emf)

Consider a conductor lying in presence of electric field as shown below in the figure such that an electric field exists inside the conductor.

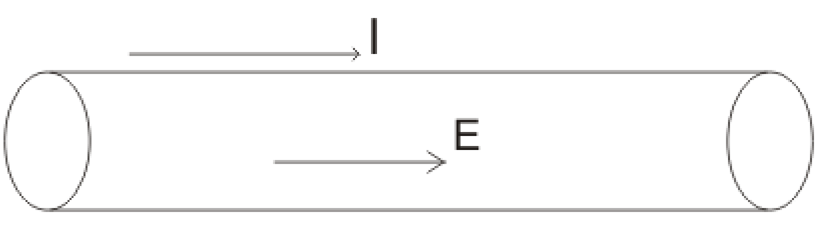


Figure 1. Current and Field E in a conductor

We know that when electric field exists in a conductur electric current begins to flow inside the conductor. Now a question arises what happens to the
charge carriers when they reach the ends of the conductor and would this current remains constant with the passage of time.
We can easily conclude that for an open ended conductor as shown in the figure , charges would accumulate at

the ends of the conductor resulting a change in electric field with the passage of time. Due to this electric current would not remain constant and would flow only for a very short interval of time , digrametically shown below in the figure.

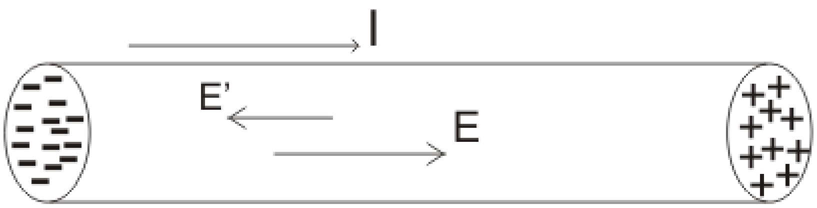


Figure 2a. Charges accumulates at the ends of the conductor and develops field E' opposite to E

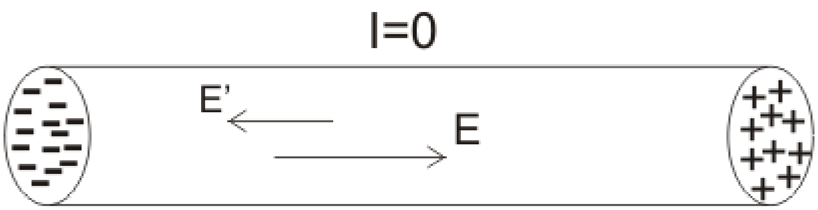


Figure 2b. Finally total field E_{tot}= E+E'=0 and current flowing in conductor becomes zero

Thus, in order to maintain a steady current throughout a conducting path the path must be in the form of a closed loop forming a complete circuit. Even this condition is not sufficient to maintain a steady current in the circuit.

This is because charge always moves in the direction of decreasing potential and electric field always does a positive work on the charge.

Now after travelling through a complete circuit when charge returns to a point where it has started, potential at that point must be same as the potential at that point in the begning of the journey but flow of current always involves loss of potential energy.

Hence we need some external source in the circuit in which maintains a potential difference at its terminals by increasing the potential energyof the electric charge.

Such a source make charge travel from lower potential to higher potential energy in direction opposit to the electrostatic force trying to push charge from higher potential to lower potential.

This force that makes charge move from lower potential to higher potential is called electro mative force (EMF).

The source or device which provide emf in a complete circuit is known as source of EMF and examples of such devices are generator, batteries, thermocuples etc.

The source of EMF are basically energy converters that convert mechanical, thermal, chemical or any other form of energy into electrical potential energy and transform it into the circuit to which the source of emf is connected.

Now we know that a source of emf or battery maintains a potential difference between its two terminals as shown in below figure

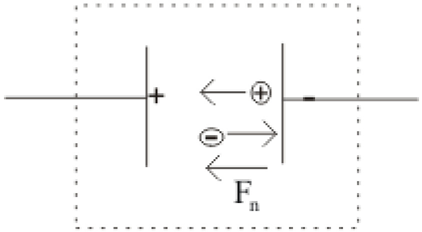


Figure 3. Internal structure of the battery

Generally a battery consist of two terminals one positive and other is negative.

Some internal force F_n generally non electric in nature is exerted on the charges of the material of the battery.

This non-electric force depends on the nature of source of EMF.

These Force(F_n) drives the positive charges of the material towards P and nigative charges of the material towards Q. This battery force

F_n is directed Q to P.

Positive charge accumulate on plate Pand negative charge accumulate on plate Q and a potential developes between plates P and Q. Thus an electric field would set up inside the battery from P to Q which excert an electric force on the charge of the material.

When a steady state is reached, the the electric force and battery force F_n would become equal and opposit mathematically,

$qE=F_n$ (1)

and after a steady state is reached no further accumulation of charhe takes place.

Workdone by battery force F_n in taking poisitive charge from terminal Q to terminal P would be

$W=F_nd$

where d is distance between plates P and Q.

Workdone by force F_n per unit charge is

$EMF=W/q=F_nd/q$ (2)

where the quantity E is known as E.M.F. of the battery.

For steady state

$ENF=qEd/q=ED=V$ (3)

where V_{eq}=Ed is the potential difference across the terminals of the battery when nothing is connected externally between P and (i.e. when circuit is open)

EMF and Electric Measurement

(3)Internal Resistance of Battery (or cell)

The resistance offered by medium in between plates of battery (electrolytes and electrodes of the cell) to the flow of current within the battery is called internal resistance of the battery.

Internal resistance of a battery usually d branch containing batteryenoted by r and in electric circuit its representation is shown below in the figure

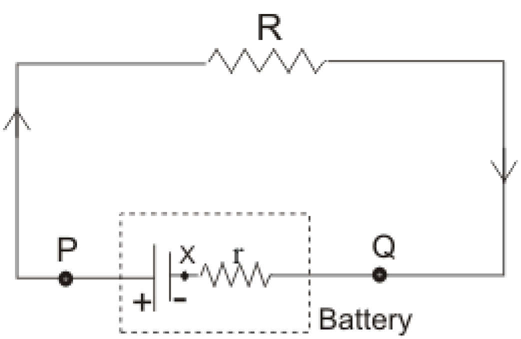


Figure 4. r Represents internal resistance of the battery

Internal resistance of a battery depends on factors like seperation between plates, plate area, nature of material of plate etc. For an ideal cell r=0 , but real batteries or sourcesof emf always has same finite internal resistance.

If P and Q are two terminals of the battery shown below in the figure

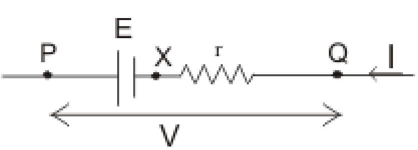


Figure 5. Terminal voltage of a cell

then potential difference between terminals P and Q is

$V_P=(V_P-V_x) - (V_Q - V_x) = E-Ir$

let $V_P-V_Q=V$

$V=E-IR$

now for I=0 and V=EMF

and this potential difference V is called the terminal difference of the cell or battery and defined as the emf of the battery when no current drawn from it.

For real battery equation(4) which gives $V=E-Ir$ whereI is the current in the branch containing battery.

From figure(4) potential difference across the external resistance R of the circuit would be equal to terminal potential difference of the cell. Thus

$V=IR$ also $V=E-Ir$

or, $IR=E-Ir$

which gives

I=E/(R+r) =Net EMF/Net resistance

From equation(4) we can calculate that when current is drawn from the battery terminal potential difference is less than the EMF of the battery.

(4)Electric Energy and Power

To understand the process of energy transfer in a simple circuit consider a simple circuit as shown in the figure given below

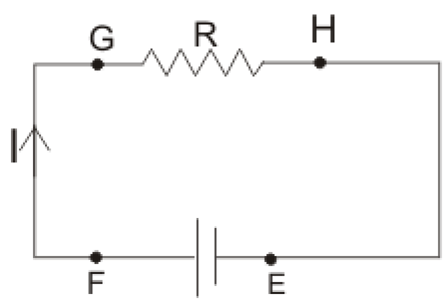


Figure 6. Circuit consisting of a resistance R

Positive terminal of the battery as we all know is always at higher potential.

Let ΔQ amount of charge begin to flow in the circuit from point E through the battery and resistor and then back to point E.

When Charge ΔQ moves from point E to point F through the battery electric potential potential energy of the system increased by the amount

ΔU=ΔQ V -(6) and the electric energy of the battery decreased by the same amount.

When charge ΔQ moves from point G to S through resistoe R, there comes a decrease in electric potential energy.

This loss in potential energy appears as the increased in thermal energy of the resistor.

Thermal energy of the resistor increases because when the charge moves through the resistor they loss there electrical potential energy by colliding with the atom in the resistor. This way electrical energy is transformed internal energy crossesponding to increase in vibrational motion of the atom of the resistor and this cause increase in temprature of the resistor.

The connecting wires are assumed to have negligible resistance and no energy transfer occur for the path FG and HE.

In time Δt charge ΔQ moves through the resistor i.e. from G to H. The rate at which it loss potential energy $\Delta U/\Delta t=(\Delta Q/\Delta t)\Delta V=I\Delta V$ where I is thecurrent in the resistor and ΔV is the potential difference across it.

This charge ΔQ regain its energy when it passes through the battery at the cost of conservation of chemical energy of electrolyte to the electrical energy.

This loss of potential energy as stated earlier appears as increased thermal energy of the resistor. If P represents the rate at which energy is delivered to the resistor then

$P=I\Delta V$

We know that $\Delta V=IR$ for a resistor hence alternative forms of equation(8) are

$P=I^2R=\Delta V^2/R$ where I is expressed in amperes, ΔV iv volts and resustance R in ohm(Ω)

SI unites of power is watt such that

1watt=1volt*1ampare

Bigger unit of electric power are Kilowatt(KW) and Megawatt(MW).

EMF and Electric Measurement

(5) Kirchoff's Rules

We have already analyzed simple circuit using ohm's laws and reducing these circuit to series and parallel combination of resistors

But we also come across circuits containing sources of EMF and grouping of resistors can be far more complex and can not be easily reduced to a single equivalent resistors

Such complex circuits can be anaylzed using two kirchoff's rules

(A) The junction Rule (or point rule)

This law states that "The algebric sum of all the currents entering juntion or any point in a circuit must be equal to the sum of currents leaving the junction"

Alternatively this rule can also be stated as " Algebric sum of the currents meeting at a point in a electric circuit is always zero i.e

$\Sigma I=0$ at any point in a circuit

This law is based on the law of conservation of charge

Consider a point P in an electric circuit at which current I_1,I_2,I_3 and I_4 are flowing through conductors in the direction shown below in the figure below

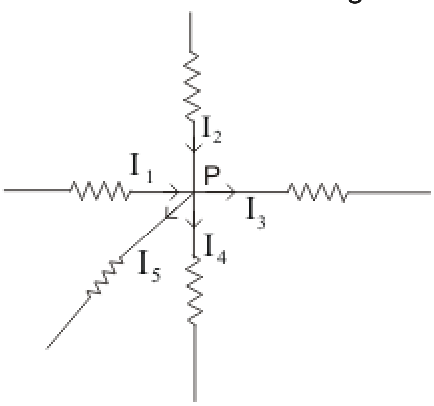


Figure 7. Current at a junction is zero

If we take current flowing towards the junction as positive and current away from the junction as negative,then from kirchoff's law

$I_1+I_2+(-I_3)+(-I_4)=0$

or,

$I_1+ I_2=I_3+ I_4$

From this law ,we conclude that netcharge coming towards a point must be equal to the net charge going away from this point in the same interval of time

(B) The Loop Rule (or Kirchoff's Voltage Law)

The rule states that " the sum of potential difference across all the circuit elements along a closed loop in a circuit is zero

$\Sigma V=0$ in a closed loop

Kirchoff's loop rule is based on the law of conservation of energy becuase total amount of energy gained and losed by a charge round a trip in a closed loop is zero

when applying this kirchoff's loop rule in any DC circuit,we first choose a closed loop in a circuit that we are analyzing

Next thing we have to decide is that whether we will traverse the loop in a clockwise direction or in anticlockwise direction and the answer is that ,the choice of direction of travel is arbitrary to reach the same point again

When traversing the loop ,we will be following convention to note down drop or rise in the voltage across the resistors or battery

i) If the resistor is being traversed in the direction of the current then change in PD across it is negative i.e -IR

ii)If the resistor is being traversed in the direction opposite to the current then change in PD across it is negative i.e IR

iii) If a source of EMF is traversed in the direction from -ve terminal to its positive terminal then change in electric potential is positive i.e E

iv)If a source of EMF is traversed in the direction from +ve terminal to its negative terminal then change in electric potential is negative i.e -E

We would now demonstrate the use of kirchoff's loop law in finding equations in simple circuit

Consider the circuit as shown below

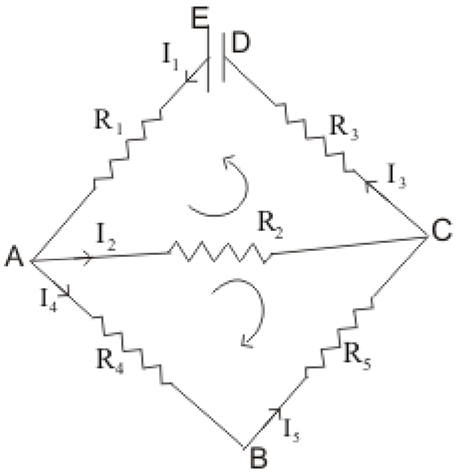


Figure 8. Circuit for explaining Kirchoff's loop rule

First consider loop ABDA.Lets traverse loop in anticlock wise direction.From kirchoff's loop law

$\Sigma V=0$

Neglecting internal resistance of the cell and using sign conventions stated previously we find

$-I_3R_3+E-I_1R_1-I_2R_2=0$

or

$I_1R_1+I_2R_2+I_3R_3=E$

And similarly if we traverse the loop ABCA in clock wise direction

$-I_2R_2+I_5R_5+I_4R_4=0$

or,

$I_5R_5+I_4R_4-I_2R_2=0$

- **Electrical Conductivity:**
  It is the inverse of specific resistance for a conductor whereas the specific resistance is the resistance of unit cube of the material of the conductor.
  $$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$
  Where σ is the conductivity and ρ is resistivity.
- **SI Unit of Conductivity:**
  The SI unit of conductivity is $mhom^{-1}$.
- **Current through a given area of a conductor:**
  It is the net charge passing per unit time through the area.
- **Current Density Vector:**
  The current density vector $\vec{J}$ gives current per unit area flowing through area ΔA when it is held normal to the direction of charge flow. Note that the direction of $\vec{J}$ is in the direction of current flow.
- **Current Density:**
  Current density j gives the amount of charge flowing per second per unit area normal to the flow.
  $$J = nqV_d$$
  where n is the number density (number per unit volume) of charge carriers each of charge q and vd is the drift velocity of the charge carriers. For electrons q = –e. If j is normal to a cross – sectional area A and is constant over the area, the magnitude of the current I through the area is neV_dA.
- **Mobility:**
  Mobility μ is defined to be the magnitude of drift velocity per unit electric field.
  $$\mu = \left(\frac{V_d}{E}\right)$$
  Now, $V_d = \frac{q\tau E}{m_q}$
  where q is the electric charge of the current carrier and mq is its mass.
  $$\therefore \mu = \left(\frac{q_\tau}{m_q}\right)$$
  Thus, mobility is a measure of response of a charge carrier to a given external electric field.

- **Resistivity:**
  Resistivity ρ is defined to be reciprocal of conductivity.

$$\rho = \frac{1}{\sigma}$$

  It is measured in ohm-metre (Qm).
- **Resistivity as a function of temperature:**
  It is given as,

$$\rho_T = \rho_0[1+\alpha(T-T_0)]$$

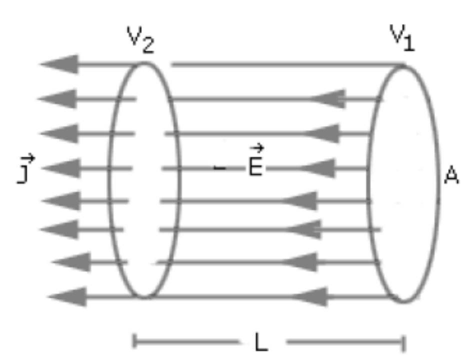
  Where α is the temperature coefficient of resistivity and ρ_T is the resistivity of the material at temperature T.
- **Ranges of Resistivity:**
  a) Metals have low resistivity: Range of ρ varies from 10^{-8} Ω m to 10^{-6} Ω m.
  b) Insulators like glass and rubber have high resistivity: Range of ρ varies from 10^{22} to 10^{24} times greater than that of metals.
  c) Semiconductors like Si and Ge lie roughly in the middle range of resistivity on a logarithmic scale.
- **Total resistance in Series and in Parallel**
  (a) Total resistance R of n resistors connected in series is given by R = R1 + R2 + ... + Rn
  (b) Total resistance R of n resistors connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}$$

- If the mass of a charge carrier is large, then for a given field $\vec{E}$, its acceleration will be small and will contribute very little to the electric current.
- **Electrical Conductivity:**
  When a conducting substance is brought under the influence of an electric field $\vec{E}$, free charges (e.g. free electrons in metals) move under the influence of this field in such a manner, that the current density $\vec{J}$ due to their motion is proportional to the applied electric field.

$$\vec{J} = \sigma\vec{E}$$

  where σ is a constant of proportionality called electrical conductivity. This statement is one possible form of Ohm's law.
- Consider a cylindrical material with cross sectional area A and length L through which a current is passing along the length and normal to the area A, then, since $\vec{J}$ and $\vec{E}$ are in the same direction,



$$J = \sigma E$$

$$JAL = \sigma ELA$$

  Where A is cross sectional area and L is length of the material through which a current is passing along the length, normal to the area A. But, JA = I, the current through the area A and EL = $V_1 - V_2$, the potential difference across the ends of the cylinder denoting V_1-V_2 as V,

$$V = \frac{IL}{\sigma A} = RI$$

Where $R = \frac{L}{\sigma A}$ is called resistance of the material. In this form, Ohm's law can be stated as a linear relationship between the potential drop across a substance and the current passing through it.

- **Measuring resistance:**

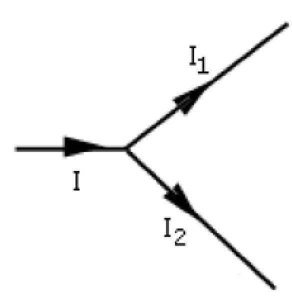
  R is measured in ohm ((Ω)), where $1\Omega = \frac{1V}{A}$

- **EMF:**
  Emf (Electromotive force) is the name given to a non-electrostatic agency. Typically, it is a battery, in which a chemical process achieves this task of doing work in driving the positive charge from a low potential to a high potential. The effect of such a source is measured in terms of work done per unit charge in moving a charge once around the circuit. This is denoted by $\in$.
- **Significance of Ohm's Law:**
  Ohm's law is obeyed by many substances, but it is not a fundamental law of nature. It fails if
  a) V depends on I non- linearly. Example is when ρ increases with I (even if temperature is kept fixed).
  b) The relation between V and I depends on the sign of V for the same absolute value of V.
  c) The relation between V and I is non- unique. For e.g., GaAs
  An example of (a) & (b) is of a rectifier
- When a source of emf ((ε)) is connected to an external resistance R, the voltage V_{ext} across R is given by

$$V_{ext} = IR = \frac{\varepsilon}{R+r} R$$

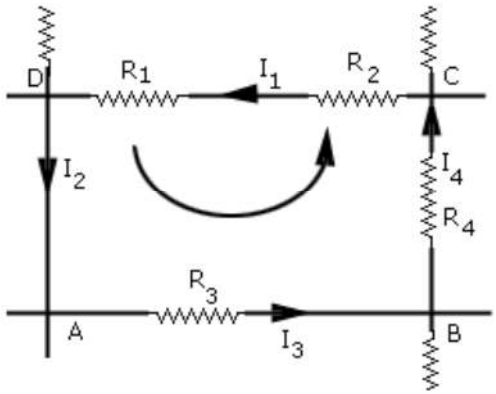
  Where r is the internal resistance of the source.
- **Kirchhoff's First Rule**:
  At any junction of several circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
  In the above junction, current I enters it and currents I_1 and I_2 leave it. Then,

$$I = I_1 + I_2$$



  This is a consequence of charge conservation and assumption that currents are steady, that is no charge piles up at the junction.

- **Kirchhoff's Second Rule**:
  The algebraic sum of changes in potential around any closed resistor loop must be zero. This is based on the principle that electrostatic forces alone cannot do any work in a closed loop, since this work is equal to potential difference, which is zero, if we start at one point of the loop and come back to it.

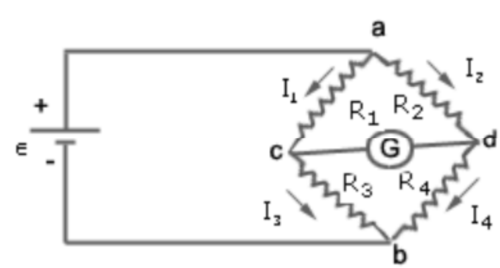


  This gives: $(R_1 + R_2)\, I_1 + R_3\, I_3 + R_4\, I_4 = 0$

- **In case of current loops:**
  i) Choose any closed loop in the network and designate a direction (in this example counter clockwise) to traverse the loop.
  ii) Go around the loop in the designated direction, adding emf's and potential differences. An emf is counted as positive when it is traversed (-) to (+) and negative in the opposite case i.e., from (+) to (-). An IR term is counted negative if the resistor is traversed in the same direction of the assumed current, and positive if in the opposite direction.
  iii) Equate the total sum to zero.

- **Wheatstone Bridge:**
  Wheatstone bridge is an arrangement of four resistances R_1, R_2, R_3, R_4. The null point condition is given by,

$$\therefore \frac{R_1}{R_2} = \frac{R_3}{R_4}$$



  This is also known as the balanced condition. If R_1, R_2, R_3 are known, R_4 can be determined.

$$R_4 = \left(\frac{R_2}{R_1}\right) R_3$$

- In a balanced condition of the meter bridge,

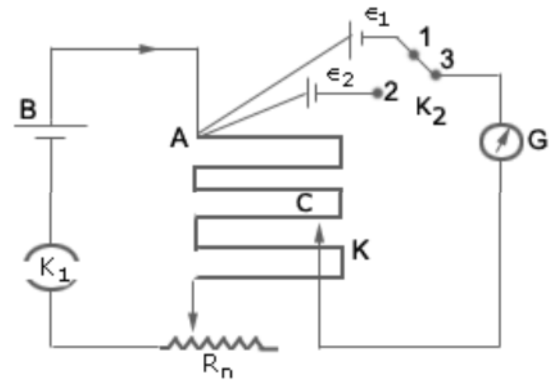
$$\frac{R}{S} = \frac{P}{Q} = \frac{\sigma l_1}{100 - l_1}$$

$$\therefore R = \frac{S l_1}{(100 - l_1)}$$

Where σ is the resistance per unit length of wire and l_1 is the length of wire from one end where null point is obtained.

- **Potentiometer:**

  The potentiometer is a device to compare potential differences. Since the method involves a condition of no current flow, the device can be used to measure potential differences; internal resistance of a cell and compare emf's of two sources.



- **Potential Gradient:**

  The potential gradient of the wire in a potentiometer depends on the current in the wire.

- If an emf $\in_1$ is balanced against length l_1, then

  $\in_1 = \rho l_1$

  Similarly, if $\in_2$ is balanced against l_2, then

  $\in_2 = \rho l_2$

  The comparison of emf's of the two cells is given by,

$$\therefore \frac{\in_1}{\in_2} = \frac{l_1}{l_2}$$