Magnetic Field

(1) Introduction

We have allready studied about thermal effects of current and now in the present chapter we are studied about magnetic effect of current.
Earlier it was believe that there is no connection between electric and magnetic force and both of them are completely different.
But in 1820 Oersted showed that the electric current through a wire deflect the magnetic needle placed near the wire and the direction of deflection of needle is reversed if we reverse the direction of current in the wire.
So, Oersted's experiments establishes that a magnetic field is assoiated with current carrying wire.
Again if we a magnetic needlle near a bar magnet it gets deflectid and rests in some other direction.
This needle experiences the tourque which turn the needle to a definite direction.
Thus, the reagion near the bar magnet or current carrying where magnetic needle experience and suffer deflection is called magnetic field.

(2) The Magnetic Field

We all ready know that a stationery charges gets up a electric field E in the space surrounding it and this electric field exerts a force $\mathbf{F}=q_0\mathbf{E}$ on the test charge q_0 placed in magnetic field.
Similarly we can describe the intraction of moving charges that, a moving charge excert a magnetic field in the space surrounding it and this magnetic field exert a force on the moving charge.
Like electric field, magntic field is also a vector quantity and is represented by symbol **B**
Like electric field force which depend on the magnitude of charge and electric field, magnetic force is propotional to the magnitude of charge and the strength of magnetic field.
Apart from its dependence on magnitude of charge and magnetic field strength magnetic force also depends on velocity of the particle.

The magnitude magnetic force increase with increase in speed of charged particle.
Direction of magnetic force depends on direction of magnetc field B and velocity **v** of the chared particle.
The direction of magnetic force is not alonge the direction of magnetic field but direction of force is always perpendicular to direction of both magnetic field **B** and velocity **v**
Test charge of magnitude q_0 is moving with velocity **v** through a point P in magnetic field **B** experience a deflecting force **F** defined by a equation
F=q**v** X **B**
As mentioned earlier this force on charged particle is perpendicular to the plane formed by **v** and **B** and its direction is determined right hand thumb rule.

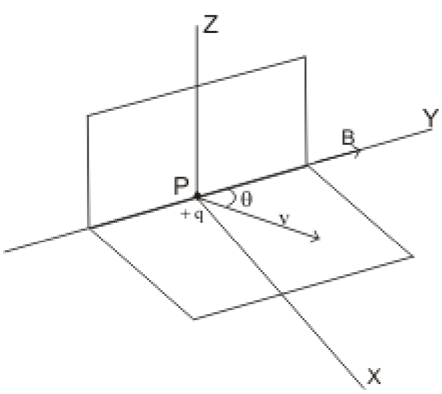

Figure 1. Particle having charge +q and velocity v through a point P in magnetic field

When moving charge is positive the direction of force **F** is the direction of advance of hand screw whose axis is perpendicular to the plane formed by **v** and **B**.

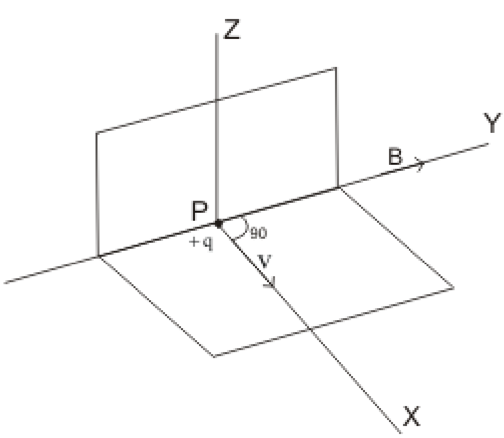

Figure 2. Particle having charge +q and velocity v through a point P in magnetic field with B perpendicular to v

Direction of force would be opposit to the direction of advance screw for negative charge moving in same direction.
Magnitude of force on charged particle is

$F=q_0vB\sin\theta$

where θ is the angle between v and B.

If **v** and **B** are at right angle to each other i.e. θ=90 then force acting on the particle would be maximum and is given by

$F_{max}=q_0vB$ ----(3)

When θ=180 or θ=0 i.e. v is parallel or antiparallel to B then froce acting on the particle would be zero.

Again from equation 2 if the velocity of the palticle in the magnetic field is zero i.e., particle is stationery in magnetic field then it does not experience any force.

SI unit of strength of magnetic field is tesla (T). It can be defined as follows

$B=F/qv\sin\theta$

for F=1N,q=1C and v=1m/s and θ=90

$1T=1NA^{-1}m^{-1}$

Thus if a charge of 1C when moving with velocity of 1m/s along the direction perpendicular to the magnetic field experiences a force of 1N then magnitude of field at that point is equal to 1 tesla (1T).

Another SI unit of magnetic field is weber/m^2 Thus

$1\ Wb\text{-}m^{-2}=1T=1NA^{-1}m^{-1}$

In CGS system, the magnetic field is expressed in 'gauss'. And 1T= 10^4 gauss. Dimention formula of magnetic field (B) is $[MT^{-2}A^{-1}]$

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(3) Lorentz Force

We know that force acting on any charge of magnitude q moving with velocity **v** inside the magnetic field B is given by

F=q(**v** X **B**)

and this is the magnetic force on charge q due to its motion inside magnetic field.

If both electric field E and magnetic field B are present i.e., when a charged particle moves through a reagion of space where both electric field and magnetic field are present both field exert a force on the particle and the total force on the particle is equal to the vector sum of the electric field and magnetic field force.

F=q**E**+q(**v** X **B**) (4)

This force in equation(4) is known as Lorentz Force.

Where important point to note is that magnetic field is not doing any work on the charged particle as it always act in perpendicular direction to te motion of the charge.

(4)Motion of Charged Particle in The Magnetic Field

As we have mentioned earlier magnetic force F=(vXB) does not do any work on the particle as it is perpendicular to the velocity.

Hence magnetic force does not cause any change in kinetic energy or speed of the particle.

Let us consider there is a uniform magnetic field **B** perpendicular to the plane of paper and directed in downward direction and is indicated by the symbol C in figure shown below.

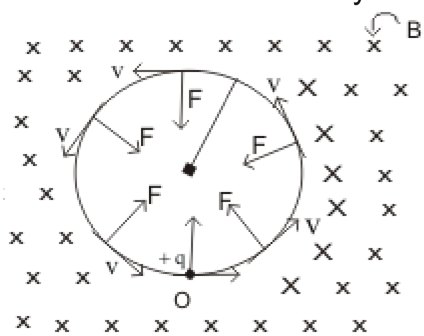

Figure 3. Motion of charged particle in uniform magnetic field

Now a charge particle +q is projected with a velocity v to the magnetic field at point O with velocity v directed perpendicular to the magnetic field.

Magnetic force acting on the particle is

F=q(**v** X **B**) = qvBsinθ

Since v is perpendicular to B i.e., angle between v and B is θ=90 Thus charged particle at point O is acted upon by the force of magnitude

F=qvB

and the direction of force would be perpendicular to both **v** and **B**

Since the force f is perpendicular to the velocity, it would not change the magnitude of the velocity and the peffect of this force is only to change the direction of the velocity.

Thus under the action of the magnetic force of the particle will more along the circle perpendicular to the field.

Therefore the charged particle describe an anticlockwise circular path with constant speed v and here magnetic force work as centripetal force. Thus

$F=qvB=mv^2/r$

where radius of the circular path traversed by the particle in the magnetic in field B is given as

r=mv/qB ---(5)

thus radius of the path is proportional to the momentum mv pof the charged particle.

2πr is the distance traveled by the particle in one revolution and the period T of the complete revolution is

T=2 πr /v

From equation(5)

r/v=m/qB

time period T is

T=2πm/qB (6)

and the frequency of the particle is f=1/T=qB/2πm (7)

From equation (6) and (7) we see that both time period and frequency does not dependent on the velocity of the moving charged particle.

Increasing the speed of the charged particle would result in the increace in the radius of the circle. So that time taken to complete one revolution would remains same.

If the moving charged particle exerts the magnetic field in such a that velocity **v** of particle makes an angle θ with the magnetic field then we can resolve the velocity in two components

$v_{parallel}$: Compenents of the velocity parallel to field

$v_{perpendicular}$:component of velocity perpendicular to magnetic field B

The component v_{par} would remain unchanged as magnetic force is perpendicular to it.

In the plane perpendicular to the field the particle travels in a helical path. Radius of the circular path of the helex is

$$r=mv_{perpendicular}/qB=mv\sin\theta/qB \qquad (8)$$

Magnetic Field

(5) Cyclotron

Cyclotron is a machine for producing high energy particles ,first developed by E.O.Lawrence and M.S.Livingston in 1931.Figure below shows the path of a charged particle in a cyclotron

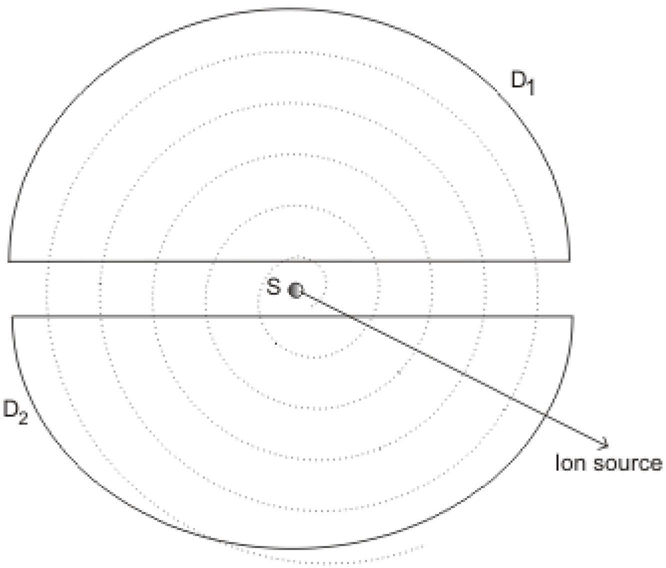

Figure 4. Path of a charged particle in a cyclotron

Construction

cyclotron consists of two horizontal D-shaped hollow metal segments D_1 and D_1 with a small gap between them

These D'ees are placed in between the poles of a large electromagnet so that that magnetic field is Perpendicular to the plane of the D'ees

The whole space inside the D'ees is evacuated to pressure of about 10^{-6} mm of Hg

An ion source S is kept at the center between the D'ees

The two D'ees are connected to the terminals of high freqency oscillating A.C circuit.This changes the charge of each D'ees several million time per sec

Theory and working

Suppose that any instant ,alternating potential is in the direction which makes D_1 positive and D_2 negative

A positive ion starting from source S will be attracted by the Dee D_2

Since Uniform magnetic field B acts at right angles to the plane of the Dees ,the positively charged ion of the charge q and mass m will move in a circular motion of radius

$r=mv/qB$

where v is the speed of the particle and it is constant

After traversing half a cycle the ion comes to the edge of D_2.If we adjust the frequency of the oscillator in such a way that by the time ,ion comes to the edge of D_2,potential difference changes direction so as to make D_1 negative and D_2 positive.

The ion will then get then attracted to D_1 and its speed will increasae due to acceleration

Once inside D_1 ,the ion is now in electric field free zone and again it will move in a circular path with constant speed which is higher then the previous constant speed in D_2.Radius of the path in D_1 will be larger then D_2

After traversing the semi -circular path in D_1 ,the ion will come to the egde of D_1 where if the direction of electric field changes ,it will receive additional energy

This way the ion will continue travelleing in semi circles of increasing radii every time it goes from D_2 to D_1 and from D_1 to D_2

Time taken by the ion to traverse the semi-circular path in the Dee is given by

$t=\pi r/v$

Thus by adjusting the magnetic field B,t can be made the same as that required to change the potential of the D_1 and D_2,so that positive charge ion always crosses the alternating electric field across the gap in correct phase

Ions gain tremendous amount of energy after traversing through reversal rotation.when they come near the circumference of the Dees,an auxillary electric field is used to deflect them from the circular path to eventually reach a target

Frequency F of charged particle moving in a cyclotron is

$f=\omega/2\pi =\upsilon/2\pi r=B\upsilon/2\pi m$ -- (10)

where $\upsilon=1/2t$

If f and B are adjusted to keep charged ion always in phase each time ,the ion crosses the gap .it receive additional energy and at the same time it describes a flat spiral of increasing radius

KE of ion emerging from the cyclotron if R is radius of the D'e is

$$KE = \frac{1}{2}M\left(\frac{BqR}{M}\right)^2$$

$$= \frac{q^2B^2R^2}{2M} = 2\pi^2R^2f^2M$$

Above relation shows that the maximum energy attained by the ion is limited by the radius R of the Dees ,magnetic field B or the frequency f

Maximum energy acquired by the charged particle in a particular cyclotron is independent of the alternating potential i.e when the voltage is small the ion makes a large number of the turns before reaching the periphery and for the large voltage number of turns is small.Total energy remians the same in both the cases,provided both B and R are unchanged

These days cyclotron are not in wide use but others based on principle of cylotron are used

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(6) Magnetic force on a current carrying wire

We know that current flowing in a conductor is nothing but the drift of free electron's from lower potential end of the conductor to the higher potential end
when a current carrying conductor is placed in a magnetic field ,magnetic forces are exerted on the moving charges with in the conductor
Equation -1 which gives force on a moving charge in a magnetic field can also be used for calculating the magnetic force exerted by magnetic field on a current carrying conductor (or wire)
Let us consider a straight conducting wire carrying current **I** is placed in a magnetic field B(x).Consider a small element dl of the qire as shown below in the figure

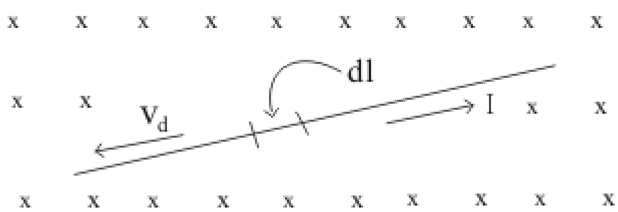

Figure 5.

Drift velocity of electrons in a conductor and current **I** flowing in the conductor is given by **I**=neAv_d
Where A is the area of cross-section of the wire and n is the number of free electrons per unit volume
Magnetic force experienced by each electron in presence of magnetic field is
$\mathbf{F}=e(\mathbf{v_d} \times \mathbf{B})$
where e is the amount of charge on an electron
Total number of electron in length dl of the wire
N=nAdl
Thus magnetic force on wire of length dl is
$d\mathbf{F}=(nAdl)(e\mathbf{v_d} \times \mathbf{B})$
if we denote length dl along the direction of current by the vector **dl** the above equation becomes
$d\mathbf{F}=(nAev_d)(\mathbf{dl} \times \mathbf{B})$
or $d\mathbf{F}=I(\mathbf{dl} \times \mathbf{B})$ -- (12)
where the quantity **IdL** is known as current element
If a straight wire of length l carrying current **I** is placed in a uniform magnetic field then force on wire would be equal to
$d\mathbf{F}=I(\mathbf{L} \times \mathbf{B})$ -- (13)

Direction of force

Direction of force is always perpendicular to the plane containing the current element **IdL** and magnetic field **B**

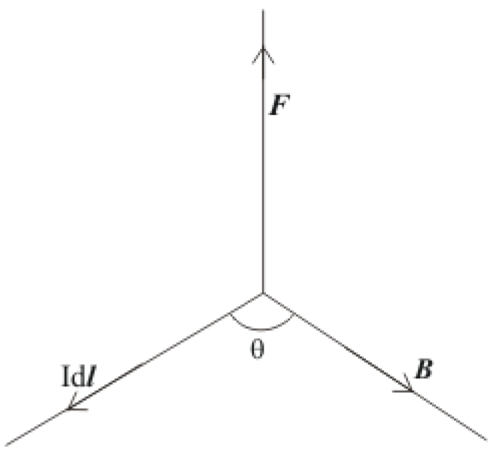

Figure 6.

Direction of force when current element **IdL** and **B** are perpendicular to each other can also be find using either of the following rules

i) Fleming'e left hand rule:-

If fore finger ,the middle finger and thumb of the left hand are stretched in such a way that the all are mutually perpendicular to each other then,if the fore finger points in the direction of the field (B) and middle finger points in the direction of current I ,the thumb will point in the direction of the force

ii) Right hand palm Rule:

Stretch the finger and thumb of the right hand so that they are perpendicular to each other .If the fingers point in the direction of current I and the palm in the direction of field B then the thumb will point in the direction of force

Magnetic Field

(7) Torque on a current carrying rectangular loop in a magnetic field

Consider a rectangular loop ABCD being suspended in a uniform magnetic field B and direction of B is paralle to the plane of the coil as shown below in the figure

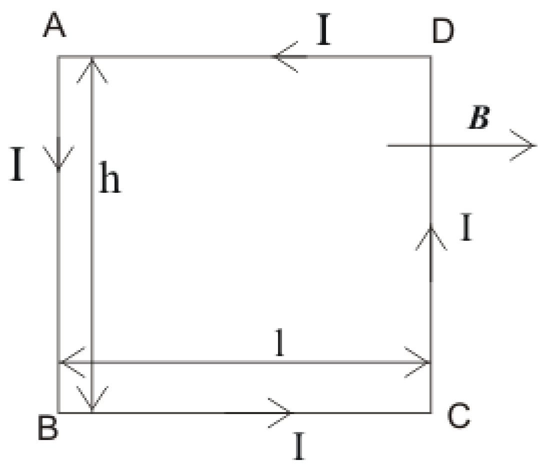

Figure 7.

Magnitude of force on side AM according to the equation(13) is

F_{AB}=IhB (angle between I and B is 90^0)

And direction of force as calculated from the right hand palm rule would be normal to the paper in the upwards direction

Similarly magnitude of force on CD is

F_{CD}=ihB

and direction of F_{CD} is normal to the page but in the downwards direction going into the page

The forces F_{AB} and F_{CD} are equal in magnitude and opposite in direction and hence they constitute a couple

Torque τexerted by this couple on rectangular loop is

$\tau = IhlB$

Since torque = one of the force * perpendicular distance between them

No force acts on the side BC since current element makes an angle θ=0 with B due to which the product (**IL**X**B**) becomes equal to zero

Similary on the side DA ,no magnetic force acts since current element makes an angle $\theta=180^0$ with **B**

Thus total torque on rectangular current loop is

$\tau = IhlB$

$=IAB$ --(15)

Where A=hl is the area of the loop

If the coil having N rectangular loop is placed in magnetic field then torque is given by

$\tau = NIAB$ ----(16)

Again if the normal to the plane of coil makes an angle θ with the uniform magnetic field as shown below in the figure then

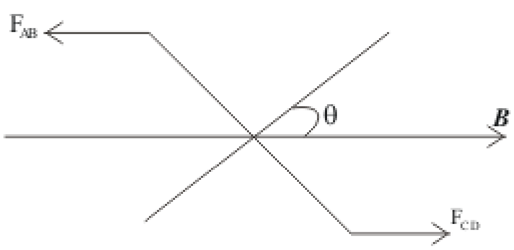

Figure 8.

$\tau = NIAB\sin\theta$

We know that when an electric dipole is placed in external electric field then torque experienced by the dipole is

$\tau = \mathbf{P} \times \mathbf{E} = PE\sin\theta$

Where **P** is the electric dipole moment

comparing expression for torque experienced by electric dipole with the expression for torque on a current loop i.e ,

$\tau = (NIA)B\sin\theta$

if we take NIA as magnetic dipole moment (m) analogus to electric dipole moment (p),we have

$\mathbf{m} = NIA$ -- (18)

then

$\tau = \mathbf{m} \times \mathbf{B}$ -- (19)

The coil thus behaves as a magnetic dipole

The direction of magnetic dipole moment lies along the axis of the loop

Magnetic Effect of current

(1) Introduction

In the previous chapter we have defined concept of magnetic field represented by vector **B**
We defined magnetic field B in terms of force it exerts on moving charges and on current carrying conductors
We also know that magnetic field is produced by the motion of the electric charges or electric current
In this chapter we would study the magnetic field produced by the steady current
we would study about how various factors affect the magnitude and direction of the magnetic field
We will also learn to calculate the equation for magnetic field B if the current configuration is known using Biot-savart's law and ampere circuital law

(2)Biot Savart Law

We know that electric current or moving charges are source of magnetic field
A Small current carrying conductor of length **dl** (length element) carrying current **I** is a elementary source of magnetic field .The force on another similar conductor can be expressed conveniently in terms of magnetic field **dB** due to the first
The dependence of magnetic field **dB** on current **I** ,on size and orientation of the length element **dl** and on distance **r** was first guessed by Biot and savart
The magnitude of the magnetic field **dB** at a distance **r** from a current element **dl** carrying current **I** is found to be proportional to **I** ,to the length dl and inversely proportional to the square of the distance |**r**|

The direction of the magnetic Field is perpendicular to the line element **dl** as well as radius **r**
Mathematically, Field **dB** is written as

$$d\boldsymbol{B} = \left(\frac{\mu_0}{4\pi}\right) I \frac{d\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2}$$

or,

$$d\boldsymbol{B} = \left(\frac{\mu_0}{4\pi}\right) I \frac{d\boldsymbol{l} \times \boldsymbol{r}^3}{r^2} \qquad (1)$$

Here ($\mu_0/4\pi$) is the proportionality constant such that

$\mu_0/4\pi = 10^{-7}$ Tesla Meter/Ampere(Tm/A)

Figure below illustrates the relation between magnetic field and current element

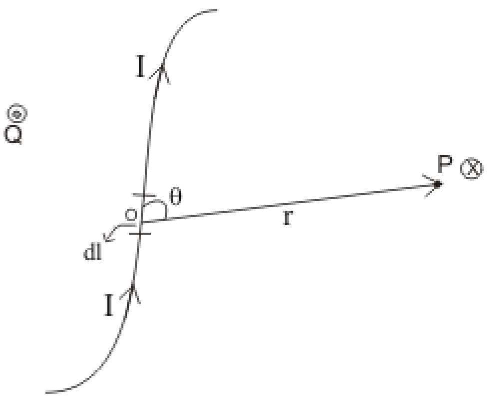

Figure 1. Field at point P is perpendicular to the plane of paper pointing into it

if in figure, Consider that line element **dl** and radius vector **r** connecting line element mid point to the field point P at which field is to be found are in the plane of the paper

From equation (1) ,we expect magnetic field to be perpendicular to both **dl** and **r**.Thus direction of **dB** is the direction of advance of right hand screw whose axis is perpendicular to the plane formed by **dl** and **r** and which is rotated from **dl** to **r** (right hand screw rule of vector product)

Thus in figure ,**dB** at point P is perpendicular directed downwards represented by the symbol (x) and point Q field is directed in upward direction represented by the symbol (•)

The magnitude of magnetic field is

$$dB = \left(\frac{\mu_0}{4\pi}\right) \frac{I\,|\,\mathbf{dl}\,|\sin\theta}{r^2} \qquad (2)$$

where θ is the angle between the line element dl and radius vector **r**

The resultant field at point P due to whole conductor can be found by integrating equation (1) over the length of the conductor i.e.

B=∫d**B**

Relation between permeability (μ_0 and permittivity (ε_0) of the free space

We know that

$\mu_0/4\pi=10^{-7}$ N/A^2 ----(a)

and

$1/4\pi\varepsilon_0=9*10^9$ N-m^2/C^2 ----(b)

Dividing equation a by b we get

$\mu_0\varepsilon_0 =1/(9*10^{16})\ (C/Am)^2$

we know that

1C=1A-s

So $\mu_0\varepsilon_0 =1/(3*10^8\ m/s)^2$

And $3*10^8$ m/s is the speed of the light in free space

So $\mu_0\varepsilon_0 =1/c^2$

or $c=1/\sqrt{(\mu_0\varepsilon_0)}$

SUMMARY

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- **Force on a Straight Conductor:**
  Force F on a straight conductor of length l and carrying a steady current I placed in a uniform external magnetic field B,
  $$\vec{F} = I\vec{l}x\vec{B}$$
- **Lorentz Force:**
  Force on a charge q moving with velocity v in the presence of magnetic and electric fields B and E.
  $$\vec{F} = q(\vec{v}x\vec{B} + \vec{E})$$
- **Magnetic Force:**
  The magnetic force $\vec{F_B} = q(\vec{v}x\vec{B})$ is normal to $\vec{V}$ and work done by it is zero.
- **Cyclotron:**
  A charge q executes a circular orbit in a plane normal with frequency called the cyclotron frequency given by,
  $$v_c = \frac{qB}{2\pi m}$$
  This cyclotron frequency is independent of the particle's speed and radius.
- **Biot – Savart Law:**
  It asserts that the magnetic field $d\vec{B}$ due to an element $d\vec{l}$ carrying a steady current I at a point P at a distance r from the current element is,
  $$d\vec{B} = \frac{\mu_0}{4\Pi} I \frac{d\vec{l}x\vec{r}}{r^3}$$
- **Magnetic Field due to Circular Coil:**
  Magnetic field due to circular coil of radius R carrying a current I at an axial distance X from the centre is
  $$B = \frac{\mu_0 I R^2}{2(X^2 + R^2)^{3/2}}$$
  At the centre of the coil,
  $$B = \frac{\mu_0 I}{2R}$$
- **Ampere's Circuital Law:**
  For an open surface S bounded by a loop C, then the Ampere's law states that
  $$\oint_C \vec{B}.d\vec{l} = \mu_0 I$$
  where I refers to the current passing through S.
- If B is directed along the tangent to every point on the perimeter them

$$BL = \mu_0 I_e$$

Where I_e is the net current enclosed by the closed circuit.

- **Magnetic Field:**

  Magnetic field at a distance R from a long, straight wire carrying a current I is given by,

  $$B = \frac{\mu_0 I}{2R}$$

  The field lines are circles concentric with the wire.

- **Magnetic field B inside a long Solenoid carrying a current I:**

  $$B = \mu_0 nI$$

  Where n is the number of turns per unit length.

- For a toriod,

  $$B = \frac{\mu_0 NI}{2\Pi r}$$

  Where N is the total numbers of turns and r is the average radius.

- **Magnetic Moment of a Planar Loop:**

  Magnetic moment m of a planar loop carrying a current I, having N closely wound turns, and an area A, is

  $$\vec{m} = NI\vec{A}$$

- **Direction of $\vec{m}$ is given by the Right – Hand Thumb Rule:**

  Curl and palm of your right hand along the loop with the fingers pointing in the direction of the current, the thumb sticking out gives the direction of

  $$\vec{m}(and\,\vec{A})$$

- **Loop placed in a Uniform Magnetic Field:**

  a) When this loop is placed in a uniform magnetic field B,

  Then, the force F on it is, F = 0

  And the torque on it is, $\vec{\tau} = \vec{m} x \vec{B}$

  In a moving coil galvanometer, this torque is balanced by a counter torque due to a spring, yielding.

  $$k\phi = NI \text{ AB}$$

  where ϕ is the equilibrium deflection and k the torsion constant of the spring.

- **Magnetic Moment in an Electron:**

  An electron moving around the central nucleus has a magnetic moment μ_l , given by

  $$\mu_l = \frac{e}{2m} l$$

  where l is the magnitude of the angular momentum of the circulating electron about the central nucleus.

- **Bohr Magneton:**

  The smallest value of μ_l is called the Bohr magneton μ_B

  $\mu_B = 9.27 \times 10^{-24}$ J/T