Physics

NCERT Exemplar Problems

Chapter 8

Electromagnetic Waves

Answers

8.1 (c)

8.2 (b)

8.3 (b)

8.4 (d)

8.5 (d)

8.6 (c)

8.7 (c)

8.8 (a), (d)

8.9 (a), (b), (c)

8.10 (b), (d)

8.11 (a), (c), (d)

8.12 (b), (d)

8.13 (a), (c), (d)

8.14 As electromagnetic waves are plane polarised, so the receiving antenna should be parallel to electric/magnetic part of the wave.

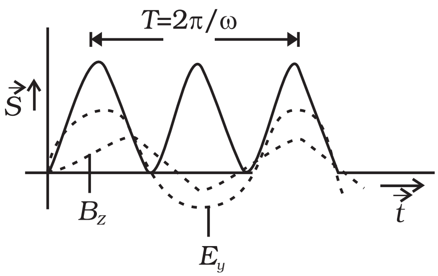
8.15 Frequency of the microwave matches the resonant frequency of water molecules.

8.16 $i_C = i_D = \dfrac{dq}{dt} = -2\pi q_0 \nu \sin 2\pi\nu t$.

8.17 On decreasing the frequency, reactance $X_c = \dfrac{1}{\omega C}$ will increase which will lead to decrease in conduction current. In this case $i_D = i_C$; hence displacement current will decrease.

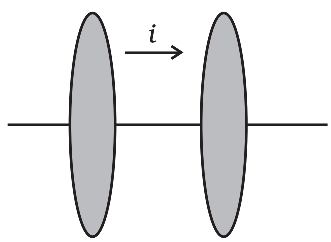
8.18 $I_{av} = \dfrac{1}{2} c \dfrac{B_0^2}{\mu_0} = \dfrac{1}{2} \times \dfrac{3\times10^8 \times (12\times10^{-8})^2}{1.26\times10^{-6}} = 1.71 W/m^2$.

8.19



8.20 EM waves exert radiation pressure. Tails of comets are due to solar solar radiation.

8.21 $B = \dfrac{\mu_0 2I_D}{4\pi r} = \dfrac{\mu_0 1}{4\pi r} = \dfrac{\mu_0}{2\pi r}\varepsilon_0 \dfrac{d\phi_E}{dt}$



$$= \frac{\mu_0\varepsilon_0}{2\pi r}\frac{d}{dt}(E\pi r^2)$$

$$= \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}.$$

8.22 (a) $\lambda_1 \rightarrow$ Microwave, $\lambda_2 \rightarrow$ UV

$\lambda_3 \rightarrow$ X rays, $\lambda_4 \rightarrow$ Infrared

(b) $\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$

(c) Microwave - Radar
UV - LASIK eye surgery
X-ray - Bone fracture identification (bone scanning)
Infrared - Optical communication.

8.23 $S_{av} = c^2\varepsilon_0 |\mathbf{E}_0 \times \mathbf{B}_0| \frac{1}{T}\int_0^T \cos^2(kx - \omega t)dt$ as $\mathbf{S} = c^2\varepsilon_0(\mathbf{E}\times\mathbf{B})$

$$= c^2\varepsilon_0 E_0 B_0 \frac{1}{\not{T}} \times \frac{\not{T}}{2}$$

$$= c^2\varepsilon_0 E_0 \left(\frac{E_0}{c}\right) \times \frac{1}{2} \quad \left(as\, c = \frac{E_0}{B_0}\right)$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$= \frac{E_0^2}{2\mu_0 c} \quad as\left(c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

8.24 $i_D = C\frac{dV}{dt}$

$$1\times10^{-3} = 2\times10^{-6}\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{1}{2}\times10^3 = 5\times10V/s$$

Hence, applying a varying potential difference of 5 × 10² V/s would produce a displacement current of desired value.

8.25 Pressure

$$P = \frac{Force}{Area} = \frac{F}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t} \quad \left(F = \frac{\Delta p}{\Delta t} = \text{rate of change of momentum}\right)$$

$$= \frac{1}{A}\cdot\frac{U}{\Delta tc} \quad (\Delta pc = \Delta U = \text{energy imparted by wave in time}\,\Delta t)$$

$$= \frac{I}{c}\left(\text{intensity}\; I = \frac{U}{A\Delta t}\right)$$

8.26 Intensity is reduced to one fourth. Tis is beacause the light beam spreads, as it propogates into a spherical region of area $4\pi r^2$, but LASER does not spread and hence its intensity remains constant.

8.27 Electric field of an EM wave is an oscillating field and so is the electric force caused by it on a charged particle. This electric force averaged over an integral number of cycles is zero since its direction changes every half cycle. Hence, electric field is not responsible for radiation pressure.

8.28 $$\mathbf{E} = \frac{\lambda \hat{\mathbf{e}}_s}{2\pi\varepsilon_o a}\hat{\mathbf{j}}$$

$$\mathbf{B} = \frac{\mu_o i}{2\pi a}\hat{\mathbf{i}}$$

$$= \frac{\mu_o \lambda v}{2\pi a}\hat{i}$$

$$\mathbf{S} = \frac{1}{\mu_o}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_o}\left(\frac{\lambda \hat{\mathbf{j}}_s}{2\pi\varepsilon_o a}\hat{\mathbf{j}}\times\frac{\mu_o \lambda v}{2\pi a}\hat{\mathbf{i}}\right)$$

$$= \frac{-\lambda^2 v}{4\pi^2\varepsilon_0 a^2}\hat{\mathbf{k}}$$

8.29 Let the distance between the plates be d. Then the electric field $E = \frac{V_o}{d}\sin(2\pi\nu t)$. The conduction current density is given by the Ohm's law = E.

$$\Rightarrow J^c = \frac{1}{\rho}\frac{V_o}{d}\sin(2\pi\nu t) = \frac{V_0}{\rho d}\sin(2\pi\nu t)$$

$$= J_0^c \sin 2\pi\nu t$$

where $J_0^c = \frac{V_0}{\rho d}$.

The displacement current density is given as

$$J^d = \varepsilon\frac{\partial E}{dt} = \varepsilon\frac{\partial}{dt}\left\{\frac{V_o}{d}\sin(2\pi\nu t)\right\}$$

$$= \frac{\varepsilon 2\pi\nu V_o}{d}\cos(2\pi\nu t)$$

$= J_0^d \cos(2\pi\nu t)$, where $J_0^d = \frac{2\pi\nu\varepsilon V_0}{d}$

$$J_o^d \big/ J_o^c = \frac{2\pi\nu\varepsilon V_o}{d} \cdot \frac{\rho d}{V_o} = 2\pi\nu\varepsilon\rho = 2\pi\times 80\,\varepsilon_o\nu\times 0.25 = 4\pi\varepsilon_o\nu\times 10$$

$$= \frac{10\,\text{v}}{9\times 10^9} = \frac{4}{9}$$

8.30 (i) Displacement curing density can be found from the relation be $\mathbf{J}_D = \varepsilon_0 \frac{d\mathbf{E}}{dt}$

$$= \varepsilon_o\,\mu_o\,I_o\,\frac{\partial}{\partial t}\cos\,(2\pi\nu t).\,\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

$$= \frac{1}{c^2} I_0 2\pi\nu^2 (-\sin(2\pi\nu t))\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

$$= \left(\frac{\nu}{c}\right)^2 2\pi\,I_0\sin(2\pi\nu t)\ln\left(\frac{a}{s}\right)\hat{k}$$

$$= \frac{2\pi}{\lambda^2} I_0 \ln\left(\frac{a}{s}\right)\sin(2\pi\nu t)\,\hat{\mathbf{k}}$$

(ii) $I^d = \int J_D s\,ds\,d\theta$

$$= \frac{2\pi}{\lambda^2} I_0 2\pi \int_{s=0}^{a} \ln\left(\frac{a}{s}\right).s\,ds\,\sin(2\pi\nu t)$$

$$= \left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_{s=0}^{a} \frac{1}{2} ds^2\, l\,n\left(\frac{a}{s}\right).\sin(2\pi\nu t)$$

$$= \frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_{s=0}^{a} d\left(\frac{s}{a}\right)^2 ln\left(\frac{a}{s}\right)^2.\sin(2\pi\nu t)$$

$$= -\frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_0^1 \ln\xi\,d\,\xi.\sin(2\pi\nu t)$$

$$= +\left(\frac{a}{2}\right)^2\left(\frac{2\pi}{\lambda}\right)^2 I_0\sin 2\pi\nu t \quad (\therefore \text{ The integral has value } -1)$$

(iii) The displacement current

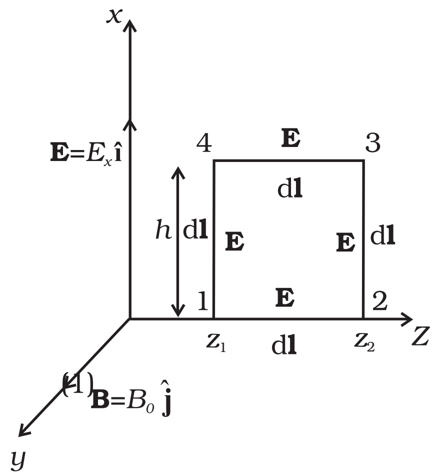
$$I^d = \left(\frac{a}{2}\cdot\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi\nu t = I_0^d sin 2\pi\nu t$$

$$\frac{I_0^d}{I_0} = \left(\frac{a\pi}{\lambda}\right)^2.$$

8.31 (i) $$\oint E.dl = \int_1^2 E.dl + \int_2^3 E.dl + \int_3^4 E.dl + \int_4^1 E.dl$$

$$= \int_1^2 E.dl \cos 90° + \int_2^3 E.dl \cos 0 + \int_3^4 E.dl \cos 90° + \int_4^1 E.dl \cos 180°$$

$$= \mathbf{E}_0 h[sin(kz_2 - \omega t) - sin(kz_1 - \omega t)] \quad (1)$$

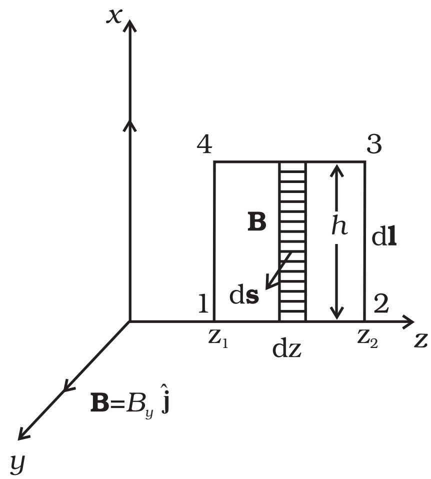


(ii) For evaluating $\int \mathbf{B.ds}$ let us consider the rectangle 1234 to be made of strips of area ds = $h\ dz$ each.

$$\int \boldsymbol{B.ds} = \int B ds \cos 0 = \int B ds = \int_{z_1}^{z_2} B_0 sin(kz - \omega t) h dz$$

$$= \frac{-B_o h}{k}[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)] \quad (2)$$

(iii) $$\oint \mathbf{E.dl} = \frac{-d\phi_B}{dt}$$



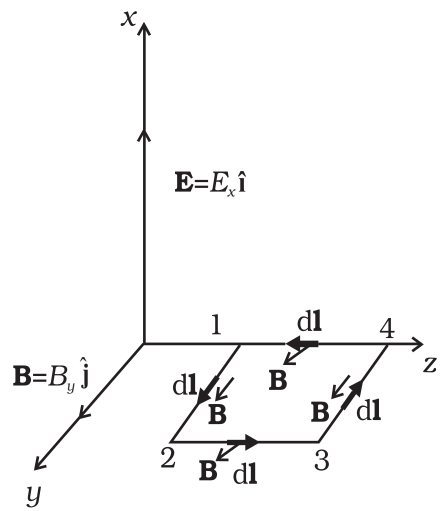
Using the relations obtained in Equations (1) and (2) and simplifiying, we get

$$E_0 h[sin(kz_2 - \omega t) - sin(kz_1 - \omega t)] = \frac{B_o h}{k}\omega[sin(kz_2 - \omega t) - sin(kz_1 - \omega t)]$$

$$E_0 = B_0 \frac{\omega}{k}$$

$$\frac{E_0}{B_0} = c$$

(iv) For evaluating $\oint \mathbf{B.dl}$, let us consider the loop 1234 in yz plane as shown in Fig.

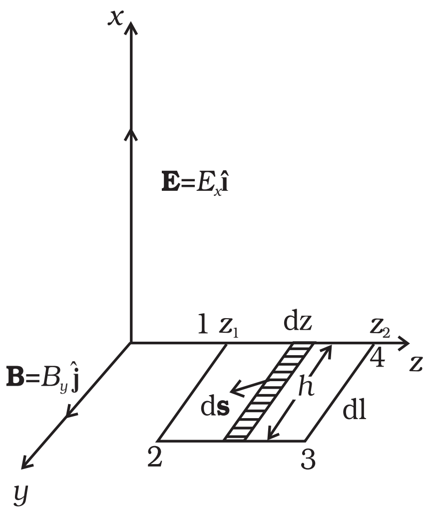


$$\oint \mathbf{B}.\mathbf{dl} = \int_1^2 \mathbf{B}.\mathbf{dl} + \int_2^3 \mathbf{B}.\mathbf{dl} + \int_3^4 \mathbf{B}.\mathbf{dl} + \int_4^1 \mathbf{B}.\mathbf{dl}$$

$$= \int_1^2 B\,dl\cos 0 + \int_2^3 B\,dl\cos 90° + \int_3^4 B\,dl\cos 180° + \int_4^1 B\,dl\cos 90°$$

$$= B_0 h[sin(kz_1 - \omega t) - sin(kz_2 - \omega t)] \qquad (3)$$

Now to evaluate $\phi_E = \int \mathbf{E}.\mathbf{ds}$, let us consider the rectangle 1234 to be made of strips of area *hdz* each.



$$\phi_E = \int \mathbf{E}.\mathbf{ds} = \int E\,ds\cos 0 = \int E\,ds = \int_{z_1}^{z_2} E_0 sin(kz_1 - \omega t)hdz$$

$$= \frac{-E_0 h}{k}[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)]$$

$$\therefore \frac{d\phi_E}{dt} = \frac{E_0 h\omega}{k}[sin(kz_1 - \omega t) - sin(kz_2 - \omega t)] \qquad (4)$$

In $\oint \mathbf{B}.\mathbf{dl} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$, I = conduction current

= 0 in vacuum.

$$\therefore \oint \mathbf{B}.\mathbf{dl} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

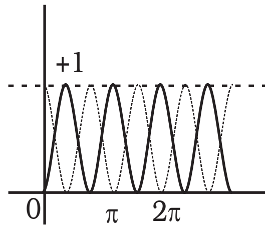
Using relations obtained in Equations (3) and (4) and ssimplifying, we get

$$B_0 = E_0 \frac{\omega}{k} \cdot \mu_0 \varepsilon_0$$

$$\frac{E_0}{B_0}\frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0}$$ But $E_0/B_0 = c$, and $\omega = ck$

or $c.c = \dfrac{1}{\mu_0 \varepsilon_0}$ Therefore, $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

8.32 (a) E - field contribution is $u_E = \frac{1}{2}\varepsilon_0 E^2$



B - field contribution is $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

Total energy density $u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$ (1)

The values of E^2 and B^2 vary from point to point and from moment to moment. Hence, the effective values of E^2 and B^2 are their time averages.

$$(E^2)_{av} = E_0^2[\sin^2(kz-\omega t)]_{av}$$

$$(B^2)_{av} = (B^2)_{av} = B_0^2[\sin^2(kz-\omega t)]_{av}$$

The graph of $\sin^2\theta$ and $\cos^2\theta$ are identical in shape but shifted by $\pi/2$, so the average values of $\sin^2\theta$ and $\text{Cos}^2\theta$ are also equal over any integral multiple of π.

and also $\sin^2\theta + \cos^2\theta = 1$

So by symmetry the average of $\sin^2\theta$ = average of $\cos^2\theta = \frac{1}{2}$

$$\therefore (E^2)_{av} = \frac{1}{2}E_0^2 \text{ and } (B^2)_{av} = \frac{1}{2}B_0^2$$

Substuting in Equation (1),

$$u = \frac{1}{4}\varepsilon_0 E^2 + \frac{1}{4}\frac{B_0^2}{\mu} \quad (2)$$

(b) We know $\frac{E_0}{B_0} = c$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\therefore \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{E_0^2/c^2}{4\mu_0} = \frac{E_0^2}{4\mu_0}\mu_0\varepsilon_0 = \frac{1}{4}\varepsilon_0 E_0^2$.

Therefore, $u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2$, and $I_{av} = u_{av} c = \frac{1}{2}\varepsilon_0 E_0^2$.