Physics

NCERT Exemplar Problems

Chapter 10

Wave Optics

Answers

10.1 (c)

10.2 (a)

10.3 (a)

10.4 (c)

10.5 (d)

10.6 (a), (b), (d)

10.7 (b), (d)

10.8 (a), (b)

10.9 (a), (b)

10.10 Yes.

10.11 Spherical.

10.12 Spherical with huge radius as compared to the earth's radius so that it is almost a plane.

10.13 Sound wave have frequencies 20 Hz to 20 kHz. The corresponding wavelengths are 15m and 15mm, respectively. Diffraction effects are seen if there are slits of width a such that.

$\alpha : \lambda.$

For light waves, wavelengths ⊔ 10^{-7}m. Thus diffraction effects will show when

$a: \ 10^{-7}$ m.

whereas for sound they will show for

$15\text{mm} < a < 15\text{m}$.

10.14 The linear distance between two dots is $l = \frac{2.54}{300}$ cm ; 0.84×10^{-2}cm.

At a distance of Z cm this subtends an angle.

$$\phi: \ l/z \therefore z = \frac{l}{\phi} = \frac{0.84 \times 10^{-2}\text{cm}}{5.8 \times 10^{-4}} : \ 14.5\text{cm} \cdot$$

10.15 Only in the special cases when the pass axis of (III) is parollel to (I) or (II) there shall be no light emerging. In all other cases there shall be light emerging because the pass axis of (II) is no longer perpendicular to the pass axis of (III).

10.16 Polarisation by reflection occurs when the angle of incidence is the Brewster's angle i.e. $\tan\theta_B = \frac{n_2}{n_1}$ where $n_2 < n_1$.

When light travels in such a medium the critical angle is $\sin\theta_c = \frac{n_2}{n_1}$ where $n_2 < n_1$.

As $|\tan\theta_B| > |\sin\theta_c|$ for large angles, $\theta_B < \theta_C$.

Thus, polarisation by reflection shall definitely occur.

10.17 $$d_{\min} = \frac{1.22\lambda}{2\sin\beta}$$

where β is the angle subtended by the objective at the object.

For light of 5500 Å

$$d_{\min} = \frac{1.22 \times 5.5 \times 10^{-7}}{2\sin\beta}\text{m}$$

For electrons accelerated through 100V the deBroglie wavelength is

$$\lambda = \frac{h}{p} = \frac{1.227}{\sqrt{100}} = 0.13\text{nm} = 0.13 \times 10^{-9}\text{m}$$

$$\therefore d'_{\min} = \frac{1.22 \times 1.3 \times 10^{-10}}{2\sin\beta}$$

$$\therefore d'_{min} = \frac{1.22 \times 1.3 \times 10^{-10}}{2 \sin \beta}$$

$$\frac{d'_{min}}{d_{min}} = \frac{1.3 \times 10^{-10}}{5.5 \times 10^{-7}} : \; 0.2 \times 10^{-3}$$

10.18 $T_2P = D + x$, $T_1 \, P = D - x$

$$S_1P = \sqrt{(S_1T_1)^2 + (PT_1)^2}$$

$$= [D^2 + (D - x)^2]^{1/2}$$

$$S_2P = [D^2 + (D + x)^2]^{1/2}$$

Minima will occur when

$$[D^2 + (D + x)^2]^{1/2} - [D^2 + (D - x)^2]^{1/2} = \frac{\lambda}{2}$$

If $x = D$

$$(D^2 + 4D^2)^{1/2} = \frac{\lambda}{2}$$

$$(5D^2)^{1/2} = \frac{\lambda}{2}, \qquad \therefore D = \frac{\lambda}{2\sqrt{5}}.$$

10.19 Without P:

$$A = A_{\perp} + A_{11}$$

$$A_{\perp} = A_{\perp}^{1} + A_{\perp}^{2} = A_{\perp}^{0}\sin(kx - \omega t) + A_{\perp}^{0}\sin(kx - \omega t + \phi)$$

$$A_{11} = A_{11}^{(1)} + A_{11}^{(2)}$$

$$A_{11} = A_{11}^{0}[\sin(kx - wt) + \sin(kx - \omega t + \phi)]$$

where $A_{\perp}^{0}$, A_{11}^{0} are the amplitudes of either of the beam in $\perp$ and 11 polarizations.

$\therefore$ Intensity =

$$= \left\{\left|A_{\perp}^{0}\right|^2 + \left|A_{11}^{0}\right|^2\right\}[\sin^2(kx - wt)(1 + \cos^2\phi + 2\sin\phi) + \sin^2(kx - \omega t)\ \sin^2\phi]_{average}$$

$$= \left\{\left|A_{\perp}^{0}\right|^2 + \left|A_{11}^{0}\right|^2\right\}\left(\frac{1}{2}\right).2(1 + \cos\phi)$$

$$= 2\left|A_{\perp}^{0}\right|^{2}.(1+\cos\phi)\,\text{since}\left|A_{\perp}^{0}\right|_{\text{average}} = \left|A_{11}^{0}\right|_{\text{average}}$$

With P:

Assume $A_{\perp}^{2}$ is blocked:

Intensity $= (A_{11}^{1} + A_{11}^{2})^{2} + (A_{\perp}^{1})^{2}$

$$= \left|A_{\perp}^{0}\right|^{2}(1+\cos\phi) + \left|A_{\perp}^{0}\right|^{2}.\frac{1}{2}$$

Given: $I_o = 4\left|A_{\perp}^{0}\right|^{2}$ = Intensity without polariser at principal maxima.

Intensity at principal maxima with polariser

$$= \left|A_{\perp}^{0}\right|^{2}\left(2+\frac{1}{2}\right)$$

$$= \frac{5}{8}I_0$$

Intensity at first minima with polariser

$$= \left|A_{\perp}^{0}\right|^{2}(1-1) + \frac{\left|A_{\perp}^{0}\right|^{2}}{2}$$

$$= \frac{I_0}{8}.$$

10.20 Path difference $= 2d\sin\theta + (\mu - 1)l$

$\therefore$ For principal maxima,

$$2d\sin\theta + 0.5l = 0$$

$$\sin\theta_0 = \frac{-l}{4d} = \frac{-1}{16} \qquad \left(Q\; l = \frac{d}{4}\right)$$

$$\therefore\ OP = D\tan\theta_0 \approx -\frac{D}{16}$$

For the first minima:

$$\therefore\ 2d\sin\theta_1 + 0.5l = \pm\frac{\lambda}{2}$$

$$\sin\theta_1 = \frac{\pm\lambda/2 - 0.5l}{2d} = \frac{\pm\lambda/2 - \lambda/8}{2\lambda} = \pm\frac{1}{4} - \frac{1}{16}$$

On the positive side: $\sin\theta^+ = \frac{3}{16}$

On the negative side: $\sin\theta^- = -\frac{5}{16}$

The first principal maxima on the positive side is at distance

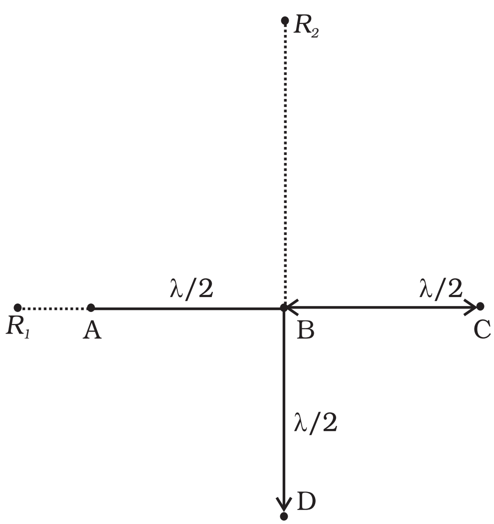
$$D\tan\theta^+ = D\frac{\sin\theta^+}{\sqrt{1-\sin^2\theta}} = D\frac{3}{\sqrt{16^2 - 3^2}} \text{ above O.}$$

In the –ve side, the distance will be $D\tan\theta^- = \frac{5}{\sqrt{16^2 - 5^2}}$ below O.

10.21 (i) Consider the disturbances at R_1 which is a distance d from A. Let the wave at R_1 because of A be $Y_A = a\cos\omega t$. The path difference of the signal from A with that from B is $\lambda/2$ and hence the phase difference is π.

Thus the wave at R_1 because of B is

$$y_B = a\cos(\omega t - \pi) = -a\cos\omega t.$$



The path difference of the signal from C with that from A is λ and hence the phase difference is 2π.

Thus the wave at R_1 because of C is $y_c = a\cos\omega t$.

The path difference between the signal from D with that of A is

$$\sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - (d - \lambda/2)$$

$$= d\left(1 + \frac{\lambda}{4d^2}\right)^{1/2} - d + \frac{\lambda}{2}$$

$$= d\left(1 + \frac{\lambda^2}{8d^2}\right)^{1/2} - d + \frac{\lambda}{2}$$

If $d >> \lambda$ the path difference $: \frac{\lambda}{2}$ and hence the phase difference is π.

$\therefore y_D = -a\cos\omega t$.

Thus, the signal picked up at R_1 is

$y_A + y_B + y_C + y_D = 0$

Let the signal picked up at R_2 from B be $y_B = a_1 \cos \omega t$.
The path difference between signal at D and that at B is $\lambda/2$.

$\therefore y_D = -a_1 \cos\omega t$

The path difference between signal at A and that at B is

$$\sqrt{(d)^2 + \left(\frac{\lambda}{2}\right)^2} - d = d\left(1 + \frac{\lambda^2}{4d^2}\right)^{1/2} - d : \frac{1}{8}\frac{\lambda^2}{d^2}$$

$\therefore$ The phase difference is $\dfrac{2\pi}{8\lambda}\cdot\dfrac{\lambda^2}{d^2} = \dfrac{\pi\lambda}{4d} = \phi :\ 0.$

Hence, $y_A = a_1 \cos(\omega t - \phi)$
Similarly, $y_C = a_1 \cos(\omega t - \phi)$
$\therefore$ Signal picked up by R_2 is
$y_A + y_B + y_C + y_D = y = 2a_1 \cos(\omega t - \phi)$

$\therefore |y|^2 = 4a_1^2 \cos^2(\omega t - \phi)$

$\therefore \langle I \rangle = 2a_1^2$

Thus R_1 picks up the larger signal.

(ii) If B is switched off,
R_1 picks up $y = a \cos \omega t$

$\therefore \langle I_{R_1} \rangle = \dfrac{1}{2} a^2$

R_2 picks up $y = a \cos \omega t$

$\therefore \langle I_{R_2} \rangle = \dfrac{1}{2} a_1^2$

Thus R_1 and R_2 pick up the same signal.

(c) If D is switched off.
R_1 picks up $y = a \cos \omega t$

$\therefore \langle I_{R_1} \rangle = \dfrac{1}{2} a^2$

R_2 picks up $y = 3a \cos \omega t$

$\therefore \langle I_{R_2} \rangle = \dfrac{1}{2} 9a^2$

Thus R_2 picks up larger signal compared to R_1.

(iv) Thus a signal at R_1 indicates B has been switched off and an enhanced signal at R_2 indicates *D* has been switched off.

10.22 (i) Suppose the postulate is true, then two parallel rays would proceed as shown in Fig. 1. Assuming ED shows a wave front then all points on this must have the same phase. All points with the same optical path length must have the same phase.

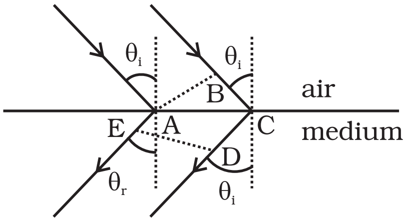


Fig.1

Thus $-\sqrt{\varepsilon_r\mu_r}\ AE = \mathrm{BC} - \sqrt{\varepsilon_r\mu_r}\ \mathrm{CD}$

or $BC = \sqrt{\varepsilon_r\mu_r}\,(CD - AE)$

BC > 0, CD > AE

As showing that the postulate is reasonable. If however, the light proceeded in the sense it does for ordinary material (viz. in the fourth quadrant, Fig. 2)

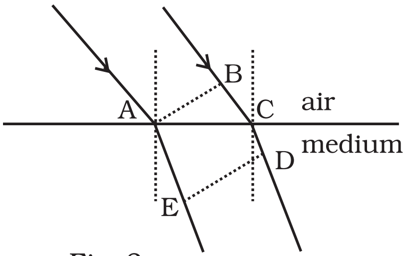


Fig. 2

Then $-\sqrt{\varepsilon_r\mu_r}\ AE = \mathrm{BC} - \sqrt{\varepsilon_r\mu_r}\ \mathrm{CD}$

or, $BC = \sqrt{\varepsilon_r\mu_r}\,(CD - AE)$

As AE > CD, BC < O

showing that this is not possible. Hence the postalate is correct.

(ii) From Fig. 1.

BC = AC sin θ_i and CD-AE = AC sin θ_r:

Since $-\sqrt{\varepsilon_r\mu_r}\,(AE - CD) = BC$

$-n \sin \theta_r = \sin \theta_i$.

10.23 Consider a ray incident at an angle *i*. A part of this ray is reflected from the air-film interface and a part refracted inside. This is partly reflected at the film-glass interface and a part transmitted. A part of the reflected ray is reflected at the film-air interface and a part transmitted as r_2 parallel to r_1. Of course succesive reflections and transmissions will keep on decreasing the amplitude of the wave. Hence rays r_1 and r_2 shall dominate the behavior. If incident light is to be transmitted through the lens, r_1 and r_2 should interfere destructively. Both the reflections at A and D are from lower to higher refractive index and hence there is no phase change on reflection. The optical path difference between r_2 and r_1 is
n (AD + CD) – AB.

If d is the thickness of the film, then

$$AD = CD = \frac{d}{\cos r}$$

$$AB = AC \sin i$$

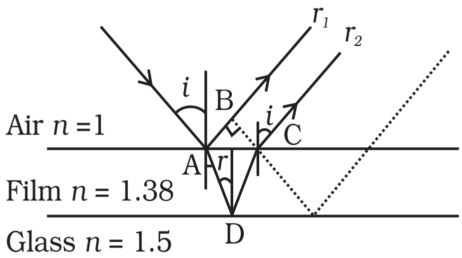
$$\frac{AC}{2} = d \tan r$$

$$\therefore AC = 2d \tan r$$

Hence, AB = 2d tan$r$ sin$i$

Thus the optical path difference is

$$2n\frac{d}{\cos r} - 2d\tan r\sin i$$



$$= 2.\frac{\sin i}{\sin r}\frac{d}{\cos r} - 2d\frac{\sin r}{\cos r}\sin i$$

$$= 2d\sin\left[\frac{1-\sin^2 r}{\sin r\cos r}\right]$$

$= 2nd \cos r$

For these waves to interefere destructively this must be $\lambda/2$.

$$\Rightarrow 2nd\cos r = \frac{\lambda}{2}$$

or $nd \cos r = \lambda/4$

For a camera lens, the sources are in the vertical plane and hence $i \approx r \approx 0$

$$\therefore nd \approx \frac{\lambda}{4}.$$

$$\Rightarrow d = \frac{5500\,\mathring{A}}{1.38\times 4} \approx 1000\,\mathring{A}$$