Physics

NCERT Exemplar Problems

Chapter 13

Nuclei

Answers

13.1 (c)

13.2 (b)

13.3 (b)

13.4 (a)

13.5 (a)

13.6 (b)

13.7 (b)

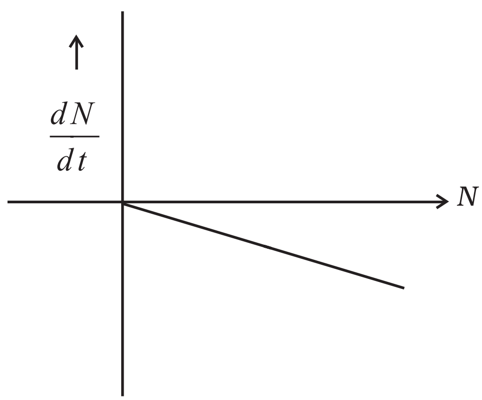
13.8 (a), (b)

13.9 (b), (d)

13.10 (c), (d)

13.11 No, the binding energy of H_1^3 is greater.

13.12



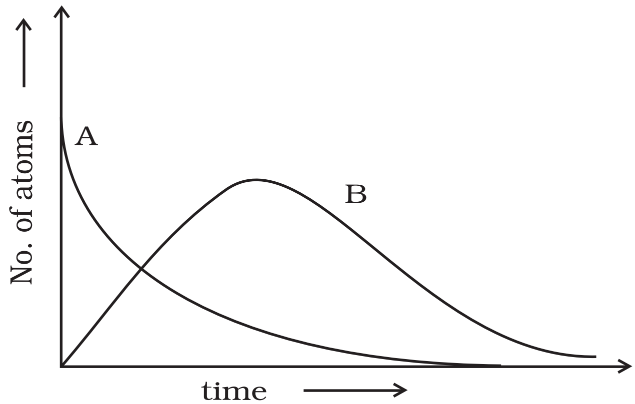
13.13 B has shorter mean life as λ is greater for B.

13.14 Excited electron because energy of electronic energy levels is in the range of eV, only not in MeV. as γ-radiation has energy in MeV.

13.15 2γ photons are produced which move in opposite directions to conserve momentum.

13.16 Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so, that an excess of neutrons which produce only attractive forces, is required for stability.

13.17



At t = 0, $N_A = N_O$ while N_B = 0. As time increases, N_A falls off exponentially, the number of atoms of B increases, becomes maximum and finally decays to zero at ∞ (following exponential decay law).

13.18 $t = \dfrac{1}{\lambda}\ln\dfrac{R_0}{R}$

$$= \frac{5760}{0.693}\ln\frac{16}{12} = \frac{5760}{0.693}\ln\frac{4}{3}$$

$$= \frac{5760}{0.693}\times 2.303\log\frac{4}{3} = \quad 2391.12 \text{ years.}$$

13.19 To resolve two objects separated by distance d, the wavelength λ of the proving signal must be less than d. Therefore, to detect separate parts inside a nucleon, the electron must have a wavelength less than 10^{-15} m.

$$\lambda = \frac{h}{p} \text{ and } K \approx pc \Rightarrow K \approx pc = \frac{hc}{\lambda}$$

$$= \frac{6.63\times10^{34}\times3\times10^{8}}{1.6\times10^{-19}\times10^{-15}}\text{eV}$$

$= 10^9$ eV. = 1 GeV.

13.20 (a) ${}^{23}_{11}\text{Na} : Z_1 = 11,\ N_1 = 12$

$\therefore$ Mirror isobar of ${}^{23}_{11}\text{Na} = {}^{23}_{12}\text{Mg}$.

(b) Since $Z_2 > Z_1$, Mg has greater binding energy than Na.

13.21 $^{38}\text{S} \xrightarrow[2.48\,\text{h}]{} {}^{38}\text{Cl} \xrightarrow[0.62\,\text{h}]{} {}^{38}\text{Ar}$

At time t, Let ^{38}S have $N_1(t)$ active nuclei and ^{38}Cl have $N_2(t)$ active nuclei.

$$\frac{dN_1}{dt} = -\lambda_1 N_1 = \text{rate of formation of Cl}^{38}\text{. Also}$$

$$\frac{dN_2}{dt} = -\lambda_1 N_2 + \lambda_1 N_1$$

But $N_1 = N_0 e^{-\lambda_1 t}$

$$\frac{dN_2}{dt} = -\lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2$$

Multiplying by $e^{\lambda_2 t} dt$ and rearranging

$$e^{\lambda_2 t} dN_2 + \lambda_2 N_2 e^{\lambda_2 t} dt = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$

Integrating both sides.

$$N_2 e^{\lambda_2 t} = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} e^{(\lambda_2 - \lambda_1)t} + C$$

Since at $t = 0$, $N_2 = 0$, $C = -\dfrac{N_0 \lambda_1}{\lambda_2 - \lambda_1}$

$$\therefore N_2 e^{\lambda_2 t} = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} \left(e^{(\lambda_2 - \lambda_1)t} - 1\right)$$

$$N_2 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

For maximum count, $\dfrac{dN_2}{dt} = 0$

On solving, $t = \left(\ln \dfrac{\lambda_1}{\lambda_2}\right) / (\lambda_1 - \lambda_2)$

$$= \ln \frac{2.48}{0.62} / (2.48 - 0.62)$$

$$= \frac{\ln 4}{1.86} = \frac{2.303 \log 4}{1.86}$$

$$= 0.745 \text{ s.}$$

13.22 From conservation of energy

$$E - B = K_n + K_p = \frac{p_n^{\,2}}{2m} + \frac{p_p^{\,2}}{2m} \qquad (1)$$

From conservation of momentum

$$p_n + p_p = \frac{E}{c} \tag{2}$$

If $E = B$, the first equation gives $p_n = p_p = 0$ and hence the second equation cannot be satisfied, and the process cannot take place.

For the process to take place, Let $E = B + \lambda$, where λ would be $<<B$.

Then : substituting for p_n from Equation (2) into Equation (1),

$$\lambda = \frac{1}{2m}(p_p^2 + p_n^2) = \frac{1}{2m}\left(p_p^2 + (p_p - E/c)^2\right)$$

$$\therefore 2p_p^2 - \frac{2E}{c}p_p + \left(\frac{E^2}{c^2} - 2m\lambda\right) = 0$$

$$\therefore p_p = \frac{2E/c \pm \sqrt{4E^2/c^2 - 8\left(\frac{E^2}{c^2} - 2m\lambda\right)}}{4}$$

Since the determinant must be positive for p_p to be real :

$$\frac{4E^2}{c^2} - 8\left(\frac{E^2}{c^2} - 2m\lambda\right) = 0$$

Or, $16m\lambda = \dfrac{4E^2}{c^2}$, $\therefore \lambda = \dfrac{E^2}{4mc^2} \approx \dfrac{B^2}{4mc^2}$.

13.23 The binding energy in H atom $E = \dfrac{me^4}{8\varepsilon_0^2 h^2} = 13.6\,\text{eV}$. (1)

If proton and neutron had charge e' each and were governed by the same electrostatic force, then in the above equation we would need to replace electronic mass m by the reduced mass m' of proton-neutron and the electronic charge e by e'.

$$m' = \frac{M}{2} = \frac{1836m}{2} = 918m.$$

$\therefore$ Binding energy $= \dfrac{918me'}{8\varepsilon_0^2 h^2} = 2.2\text{MeV}$ (given) (2)

Diving (2) by (1)

$$918\left(\frac{e'}{e}\right)^4 = \frac{2.2\text{MeV}}{13.6\text{eV}}$$

$$\Rightarrow \frac{e'}{e} \approx 11.$$

13.24 Before β decay, neutron is at rest. Hence $E_n = m_n c^2$, $p_n = 0$

After β decay, from conservation of momentum:

$$\mathbf{p}_n = \mathbf{p}_p + \mathbf{p}_e$$

Or $\mathbf{p}_p + \mathbf{p}_e = 0 \Rightarrow |\mathbf{p}_p| = |\mathbf{p}_e| = p$

Also, $E_p = (m_p{}^2c^4 + p_p{}^2c^2)^{\frac{1}{2}}$,

$$E_e = (m_e{}^2c^4 + p_e{}^2c^2)^{\frac{1}{2}} = (m_e{}^2c^4 + p_p c^2)^{\frac{1}{2}}$$

From conservation of energy:

$$(m_p{}^2c^4 + p^2c^2)^{\frac{1}{2}} + (m_e{}^2c^4 + p^2c^2)^{\frac{1}{2}} = m_n c^2$$

$$m_p c^2 \approx 936\text{MeV}, m_n c^2 \approx 938\text{MeV}, m_e c^2 = 0.51\text{MeV}$$

Since the energy difference between n and p is small, pc will be small, $pc << m_p c^2$, while pc may be greater than $m_e c^2$.

$$\Rightarrow m_p c^2 + \frac{p^2c^2}{2m_p{}^2c^4} ; \quad m_n c^2 - pc$$

To first order $pc ; \quad m_n c^2 - m_p c^2 = 938\text{MeV} - 936\text{MeV} = 2\text{MeV}$

This gives the momentum.

Then,

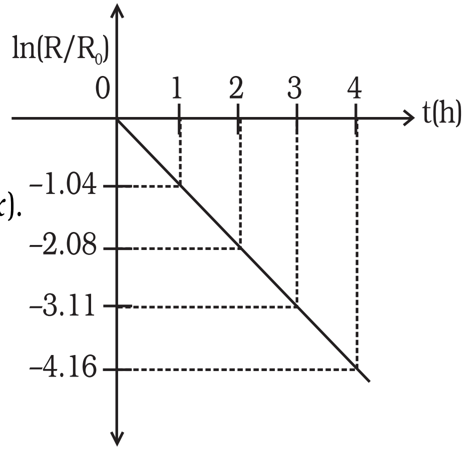
$$E_p = (m_p{}^2c^4 + p^2c^2)^{\frac{1}{2}} = \sqrt{936^2 + 2^2} ; \quad 936\text{MeV}$$

$$E_e = (m_e{}^2c^4 + p^2c^2)^{\frac{1}{2}} = \sqrt{(0.51)^2 + 2^2} ; \quad 2.06\text{MeV}$$

13.25 (i) $t_{1/2}$ = 40 min (approx).

(ii) Slope of graph = $-\lambda$

So $\lambda = -\left(\dfrac{-4.16+3.11}{1}\right) = 1.05\text{h}$



So $t_{1/2} = \dfrac{0.693}{1.05} = 0.66\text{h} = 39.6\,\text{min}\, or\, 40\,\text{min}\,(approx).$

13.26 (i) $S_{pSn} = (M_{119,70} + M_H - M_{120,70})c^2$

$= (118.9058 + 1.0078252 - 119.902199)c^2$

$= 0.0114362\ c^2$

$S_{pSb} = (M_{120,70} + M_H - M_{121,70})c^2$

$= (119.902199 + 1.0078252 - 120.903822)c_2$

$= 0.0059912\ c^2$

Since $S_{pSn} > S_{pSb}$, Sn nucleus is more stable than Sb nucleus.

(ii) It indicates shell structure of nucleus similar to the shell structure of an atom. This also explains the peaks in BE/nucleon curve.