Motion in straight Line

1. Introduction

In our daily life ,we see lots of things moving around for example car passing through from one place to other , person riding on a bicycle and many more like this.

In scientific terms an object is said to be in motion ,if it changes its position with the description of time and if it does not change it position with the passage of time then it is said to be at rest

Both the motion and rest are relative terms.

Let us explain above statement with the help of example where *mobile is kept on the table is resting at its position*.

Now what do you think about this mobile:-

(a)is it at rest or

(b) is it moving i.e., changing its position with the passage of time.

Now we have stated previously that:

Motion and rest are relative terms

- So for a person seeing mobile from earth it is at **rest**.
- Now consider a hypothetical scenario where we station our self on moon.
- Now from there earth seems to change its position with time (earth is rotating on its axis) and so mobile along with the table is changing its position.
- So, it is **moving** for a person tracking it from moon.

Simplest case of motion is rectilinear motion which is the motion of the object in a straight line.

In our description of object ,we will treat the object as an **point object**.

Object under consideration can be treated as point object if the size of the object is much smaller than the distance traveled by it in a reasonable time duration for example length of a motor car traveling a distance of 500 km can be neglected w.r.t distance traveled by it.

Here in **kinematics** ,we study ways to describe the motion without going into the cause of the motion.

2.Position and Displacement

(a) Position:

To locate the position in motion or at rest,we need a frame of refrence.

Simplest way to choose a frame of refrence is to choose three mutually perpendicular axis labelled as X-,Y- and Z- axis as shown in figure below

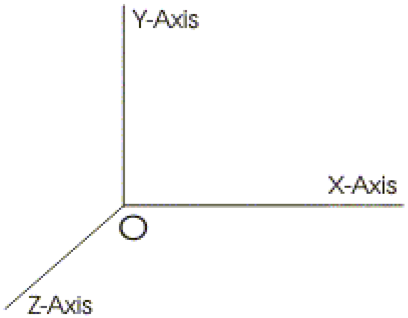


Figure 1. X , Y and Z axis

More about Frame of reference

We know that we need a frame of reference is needed to locate the position of any object. For that we choose rectangular co-ordinate system of three mutually perpendicular axis that are x, y and z axis. The point of intersection of these three axes is called *Origin 'O'* and is considered as the *Reference Point*.

The x, y and z co-ordinates describe the position of object with respect to the co-ordinate system. To measure time , we need a clock. The co-ordinate system along with a clock constitutes a ***Frame of reference***. So , the frame of reference is a co-ordinate system , with a clock w.r.t. which, an observer can describe the position, displacement etc. of an object.

Frames of reference are of two types:-

1. **Inertial frame of reference:-** These are the frame of reference in which Newton's first law of motion is applicable.

   Non-Inertial frame of reference:- These are the frame of reference in which , Newton's first law of motion is not applicable

Such system of labeling position of an object is known as rectangular coordinates system

If A(x,y,z) be the position of any point in rectangular co-ordinates system it can be labeled as follows

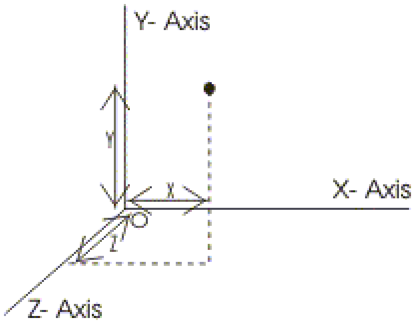


Figure 2.Figure shows X , Y and Z co-ordinates of point A

Point O is the point of intersection of these mutually perpendicular axis and is known as reference point or origin of frame of reference

To measure a time ,we can also attach a clock with this frame of reference

If any or all co-ordinates of the object under consideration changes with time in this frame of reference then the object is said to be in a motion w.r.t the frame of the reference otherwise it is at rest

For describing motion in one dimension we need one set of co-ordinates axis i.e only one of X,Y and Z axis

Similarly for two and three dimensions we need two or three set of axis respectively

Motion of an object along a straight line is an example of motion in one dimension

For such a motion, any one axis say X-axis may be choose so as to coincide with the path along which object is moving

Position of the object can be measured w.r.t origin O shown in the figure

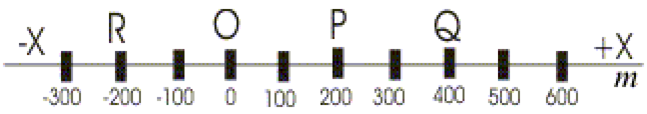


Figure 3.X-axis, origin and position of man walking at different times

Position to the right of the origin has positive values and those to the left of origin O has negative values.

(b) Distance and displacement:

In the graph shown below an object is at position P at time t_1 and at position R at time t_2.

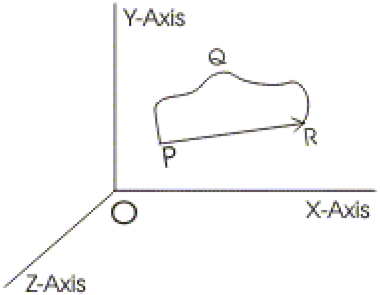


Figure 4

In the time interval from t_1 to t_2 particle has traveled path PQR and length of the path PQR is the distance traveled by the object in the time interval t_1 to t_2

Now connect the initial position of the object P with its final position R through a straight line and we get the displacement of the object.

Displacement of the object has both magnitude and direction i.e., displacement is a vector quantity.

Magnitude of displacement vector is equal to the length of straight line joining initial and final position and its direction points from the initial position of object towards its final position.

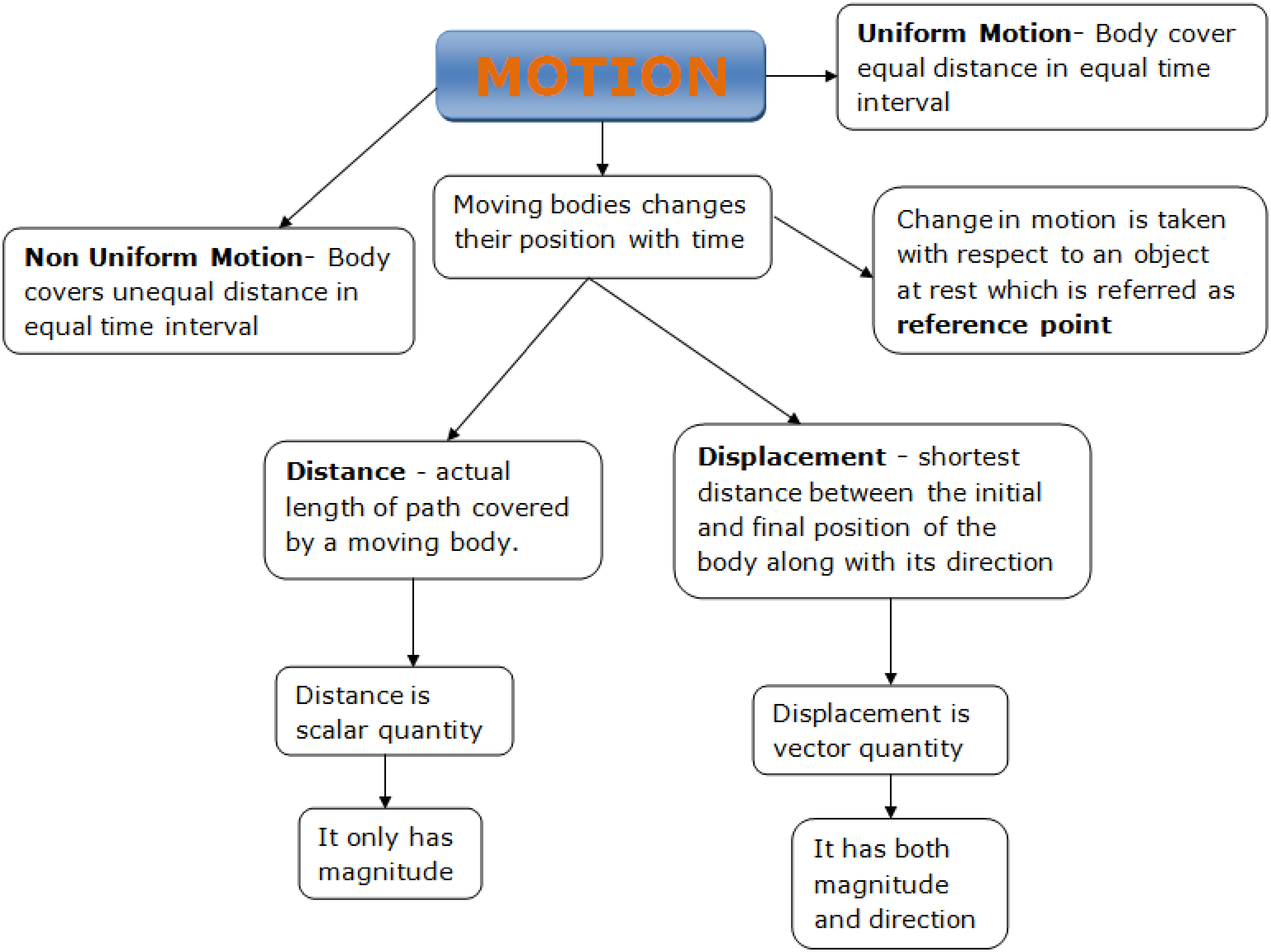
In contrast to displacement distance is scalar quantity.

Comparison between distance and displacement

1. For a moving particle in a given time interval distance can be many valued function, but displacement would always be single valued function
2. Displacement could be positive, negative or zero, but distance would always be positive.
3. Displacement can decrease with time, but distance can never decrease with time.
4. Distance is always greater than or equal to the magnitude of displacement.
5. Distance would be equal to displacement if and only is particle is moving along straight line without any change in direction.

Kinematics Concept Maps

Concept Map of Motion in General



Motion in straight Line

3. Average velocity and speed

What is velocity?

Velocity is defined as the rate of change of displacement.

1. It is a vector quantity, both magnitude and direction are required to define it.
2. It's direction is same as that of displacement.
3. SI unit of velocity is m/s

What is speed?

Speed is defined as the rate of change in distance with respect to time.

1. It is a scalar quantity. Only magnitude is required to define the speed.
2. Speed and velocity both have same unit.

Velocity and speed are very important concepts in Kinematics.

Consider a particle undergoing motion along a straight line i.e. particle is moving along X-axis.
Here in this case X co-ordinate describing motion of the particle from origin O varies with time or we can say that X co-ordinate depends on time.
If at time $t=t_1$ particle is at point P , at a distance x_1 from origin and at time $t=t_2$ it is at point Q at a distance x_2 from the origin then displacement during this time is a vector from point P to Q and is

$\Delta x = {x_2} - {x_1}$ (1)

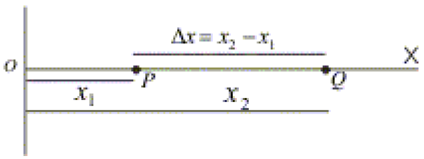


Figure 5a. Particle moving on X-axis

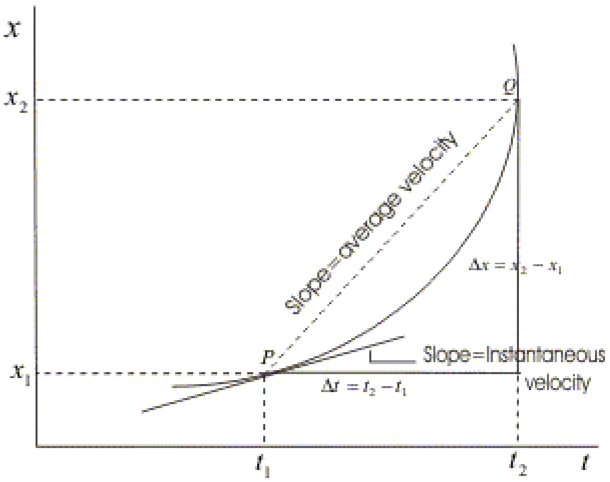


Figure 5b. Co-ordinate time graph of the motion

The average velocity of the particle is defined as the ratio of the displacement Δx of the particle in the time interval $\Delta t = t_2 - t_1$. If v_{avg} represents average velocity then,

$$v_{avg} = \frac{\Delta x}{\Delta t} \qquad (2)$$

Figure (5b) represents the co-ordinate time graph of the motion of the particle i.e., it shows how the value of x-coordinate of moving particle changes with the passage of time.

In figure (5b) average velocity of the particle is represented by the slope of chord PQ which is equal to the ratio of the displacement Δx occurring in the particular time interval Δt.

Like displacement average velocity $\mathbf{v}_{\mathbf{avg}}$ also has magnitude as well as direction i.e., average velocity is a vector quantity.

Average velocity of the particle can be *positive as well as negative* and its positive and negative value depends on the sign of displacement.

If displacement of particle is zero its average velocity is also zero.

Graphs below shows the x-t graphs of particle moving with positive, negative average velocity and the particle at rest.

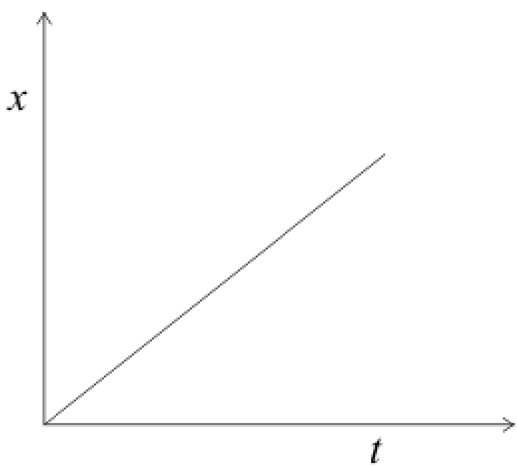


Figure 5c.Positioin time graph of particle moving with positive average velocity

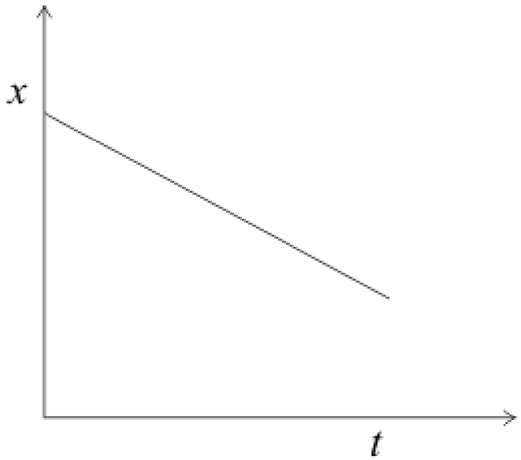


Figure 5d.Positioin time graph of particle moving with negative average velocity

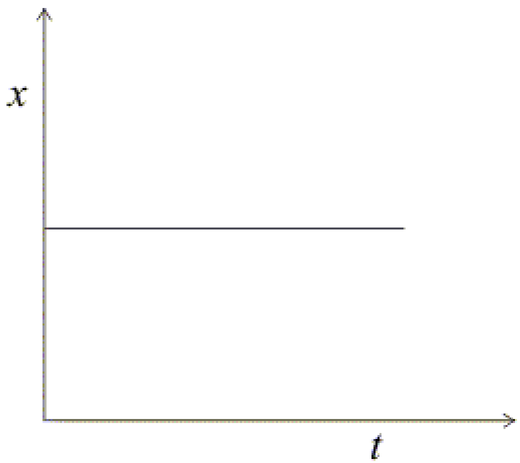


Figure 5e.Positioin time graph of particle moving with zero average velocity

From graph 5c it is clear that for positive average velocity slope of line slants upwards right or we can say that it has *positive slope*.

For negative average velocity slope line slants upwards down to the right i.e. it has *negative slope*.

For particles at rest slope is *zero*.

So far we have learned that Average speed is defined as total distance traveled divided by time taken.

Displacement of the object is different from the actual distance traveled by the particle.

For actual distance traveled by the particle its average speed is defined as the total distance traveled by the particle in the time interval during which the motion takes place.

Mathematically,

$$Average\ speed\ = \frac{Total\ distance\ travelled}{Total\ time\ interval} \qquad (3)$$

Since distance traveled by an particle does not involve direction so speed of the particle depending on distance traveled does not involve direction and hence is a scalar quantity and is always positive.

Magnitude of average speed may differ from average velocity because motion in case of average speed involve distance which may be greater than magnitude of displacement. Let us consider an example

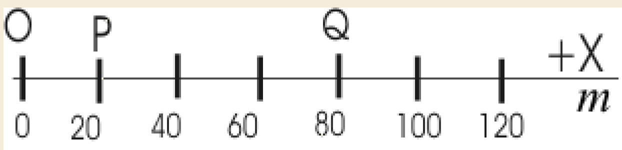


Figure 6

here a man starts traveling from origin till point Q and return to point P then in this case displacement of man is

Displacement from O to Q is OQ=80m

Displacement from Q to P is =20m-80m=-60m

total displacement of particle in moving from O to Q and then moving Q to P is = 80m + (-60m) = 20 m

Now total distance traveled by man is $OQ+OP= 80m +60m = 140m$

Hence during same course of motion distance traveled is greater then displacement.

From this we can say that average speed depending on distance is in general greater than magnitude of velocity.

Let us now consider some of the solved examples for the concepts Average speed and Average velocity. Questions are important for understanding the concept. Try to understand these solved examples and then try to solve few questions on your own.

Examples based on Average Speed

Question 1 A car travels first half distance between two places with a speed of 40 Km/hr and the rest half with a speed of 60 Km/hr. Find the average speed of the car.

Solution Let x be the total distance traveled by the car.

Time taken to travel first half distance ${t_1} = \frac{{x/2}}{{40}} = \frac{x}{{80}}hr$

Time taken to travel rest half distance ${t_2} = \frac{{x/2}}{{60}} = \frac{x}{{120}}hr$

Therefore Average speed = (Total distance)/(Total time) = $\frac{x}{{(x/80) + (x/120)}} = 48Km/hr$

Question 2 A point traveling along a straight line traverse one third the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

Solution Let s be the total distance. Let $\frac{s}{3}$ be the distance covered in time t_1 while the remaining distance $\frac{2s}{3}$ in time t_2 second

$\frac{s}{3} = {v_0}{t_1}$

${t_1} = \frac{s}{{3{v_0}}}$ (1)

and,

$\frac{{2s}}{3} = {v_1}\left({\frac{{{t_2}}}{2}} \right) + {v_2}\left({\frac{{{t_2}}}{2}} \right)$

or,

${t_2} = \frac{{4s}}{{3({v_1} + {v_2})}}$ (2)

Average Velocity $ = \frac{s}{{{t_2} + {t_1}}}$

or, Average Velocity $ = \frac{s}{{\left({\frac{s}{{3{v_0}}}} \right) + \left({\frac{{4s}}{{3({v_1} + {v_2})}}} \right)}} = \frac{{3{v_0}({v_1} + {v_2})}}{{{v_1} + {v_2} + 4{v_0}}}$

Examples based on Average Velocity

Question 1 Usually "average speed" means the ratio of total distance covered to total the total time elapsed. However sometimes the phrase "average speed" can mean magnitude of average velocity. Are the two same?

Solution No, usually they have different meanings, as according to their definitions

Average speed is defined as total distance traveled divided by time taken and Average velocity is defined as change of displacement divided by the time taken. Now since distance traveled by any particle is either grater then the displacement or it is equal to the displacement , the velocity would be given as

${v_{av}} \geq |{{\vec v}_{av}}|$

that is usually average speed is greater than the magnitude of average velocity.

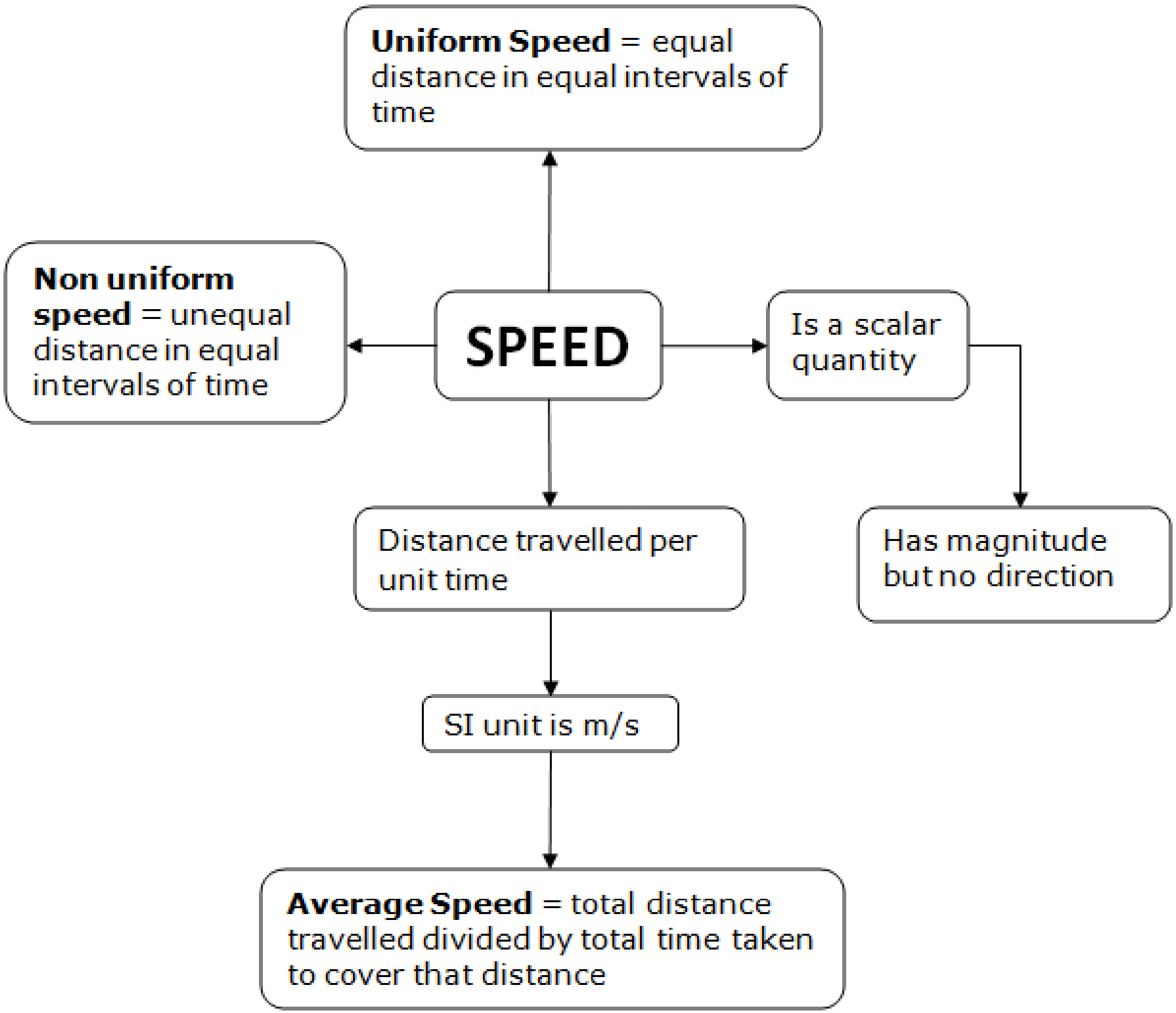
For example if a body returns to its starting point after some motion, then as distance travel would be finite while displacement would be zero. So in this case average speed would be greater then zero but magnitude of average velocity would be equal to zero.

However in case of motion along the straight line without change in direction, magnitude of displacement would be equal to distance and two definitions would mean the same.

Question 2 A student argues that the mean velocity during an interval of time can also be expressed as $\vec v = \frac{{{{\vec v}_f} + {{\vec v}_i}}}{2}$ and this should always be equal to $\vec v =

Kinematics Concept Maps

Concept Map of Speed



4. Instantaneous velocity and speed

Velocity of particle at any instant of time or at any point of its path is called instantaneous velocity.
Again consider the graph 5b and imagine second point Q being taken more and more closer to point P then calculate the average velocity over such short displacement and time interval.
Instantaneous velocity can be defined as limiting value of average velocity when second point comes closer and closer to the first point.
Limiting value of $\Delta x / \Delta t$ as Δt approaches zero is written as dx/dt, and is known as instantaneous velocity.Thus instantaneous velocity is

$$v = \lim_{\Delta t \to 0} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (4)$$

As point Q approaches point P in figure 5a in this limit slope of the chord PQ becomes equal to the slope of tangent to the curve at point P.
Thus we can say that instantaneous velocity at any point of a coordinate time graph is equal to the slope of the tangent to the graph at that point.
Instantaneous speed or speed is the magnitude of the instantaneous velocity unlike the case of average velocity and average speed where average speed over an finite interval of time may be greater than or equal to average velocity.
Unit of average velocity , average speed, instantaneous velocity and instantaneous speed is ms_{-1} in SI system of units.
Some other units of velocity are $ft.s_{-1}$, $cm.s_{-1}$.

5. Acceleration

Acceleration is the rate of change of velocity with time.
For describing average acceleration we first consider the motion of an object along X-axis.
Suppose at time t_1 object is at point P moving with velocity v_1 and at time t_2 it is at point Q and has velocity v_2.
Now average acceleration of object in moving from P to Q is

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad (5)$$

which is the change in velocity of object with the passage of time.
Instantaneous acceleration can be defined in the same way as instantaneous velocity

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (6)$$

The instantaneous acceleration at any instant is the slope of v-t graph at that instant.

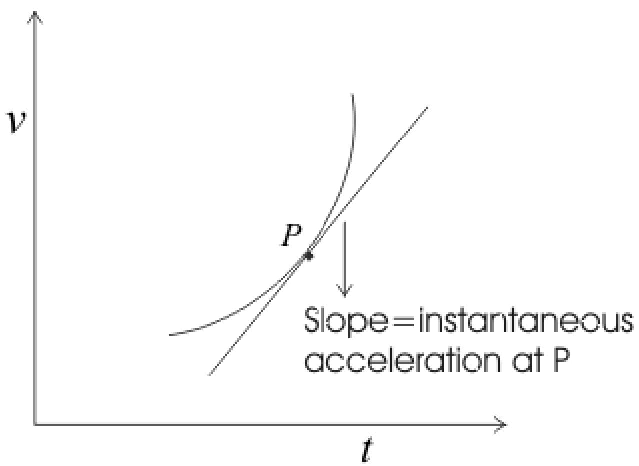


Figure 7. Velocity time graph of particle

In figure 7 instantaneous acceleration at point P is equal to the slope of tangent at this point P.
Since velocity of a moving object has both magnitude and direction likewise acceleration depending on velocity has both magnitude and direction and hence acceleration is a vector quantity.
Acceleration can also be positive, negative and zero.
SI unit of acceleration is ms^{-2}

Question:

A car moves in a straight road. The velocity at point A is 50 km/hr, It presses the accelerator paddle and reach point B in 10 sec. The velocity ar point B is 100Km/hr

Find the average acceleration of the car from point A to point B? For solution click here

Question:

True and False Statement

a) Acceleration is a scalar quantity?

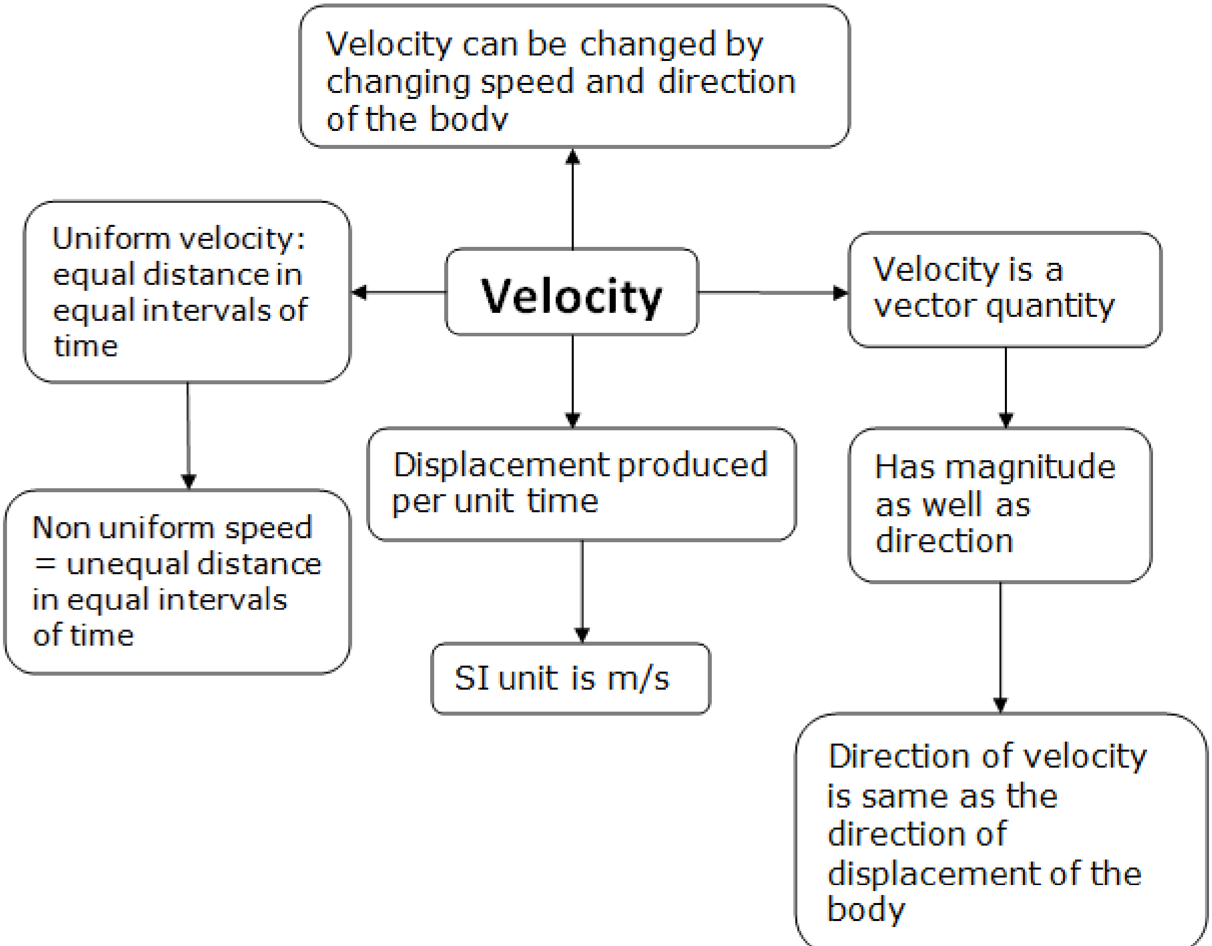
b) if the velocity of the object is zero,then acceleration will always be zero?

c) Instantaneous speed is always equal to magnitude of instantaous velocity

For solution click here

Kinematics Concept Maps

Velocity Concept Map



7. Free fall acceleration

Freely falling motion of any body under the effect of gravity is an example of uniformly accelerated motion. Kinematic equation of motion under gravity can be obtained by replacing acceleration **'a'** in equations of motion by acceleration due to gravity **'g'**.

Value of g is equal to 9.8 $m.s^{-2}$.

Thus kinematic equations of motion under gravity are

$v = v_0 + gt$ (16a)

$x = v_0t + ½ (gt^2)$ (16b)

$v^2 = (v_0)^2 + 2gx$ (16c)

The value of g is taken positive when the body falls vertically downwards and negative when the body is projected up against gravity.

8. Relative velocity

Consider two objects A and B moving with uniform velocities v_A and v_B along two straight and parallel tracks. Let x_{OA} and x_{OB} be their distances from origin at time t=0 and x_A and x_B be their distances from origin at time t.

For object A

$x_A = x_{OA} + v_At$ (18)

and for object B

$x_B = x_{OB} + v_Bt$ (19)

subtracting equation 18 from 19

$x_B - x_A = (x_{OB} - x_{OA}) + (v_B - v_A) t$ (20)

Above equation 20 tells that as seen from object A , object B seems to have velocity ($v_B - v_A$) .

Thus ($v_B - v_A$) is the velocity of object B relative to object A. Thus,

$v_{BA} = (v_B - v_A)$ (21)

Similarly velocity of object A relative to object B is

$v_{AB} = (v_A - v_B)$ (22)

If $v_B = v_A$ then from equation 20

$x_B - x_A = (x_{OB} - x_{OA})$

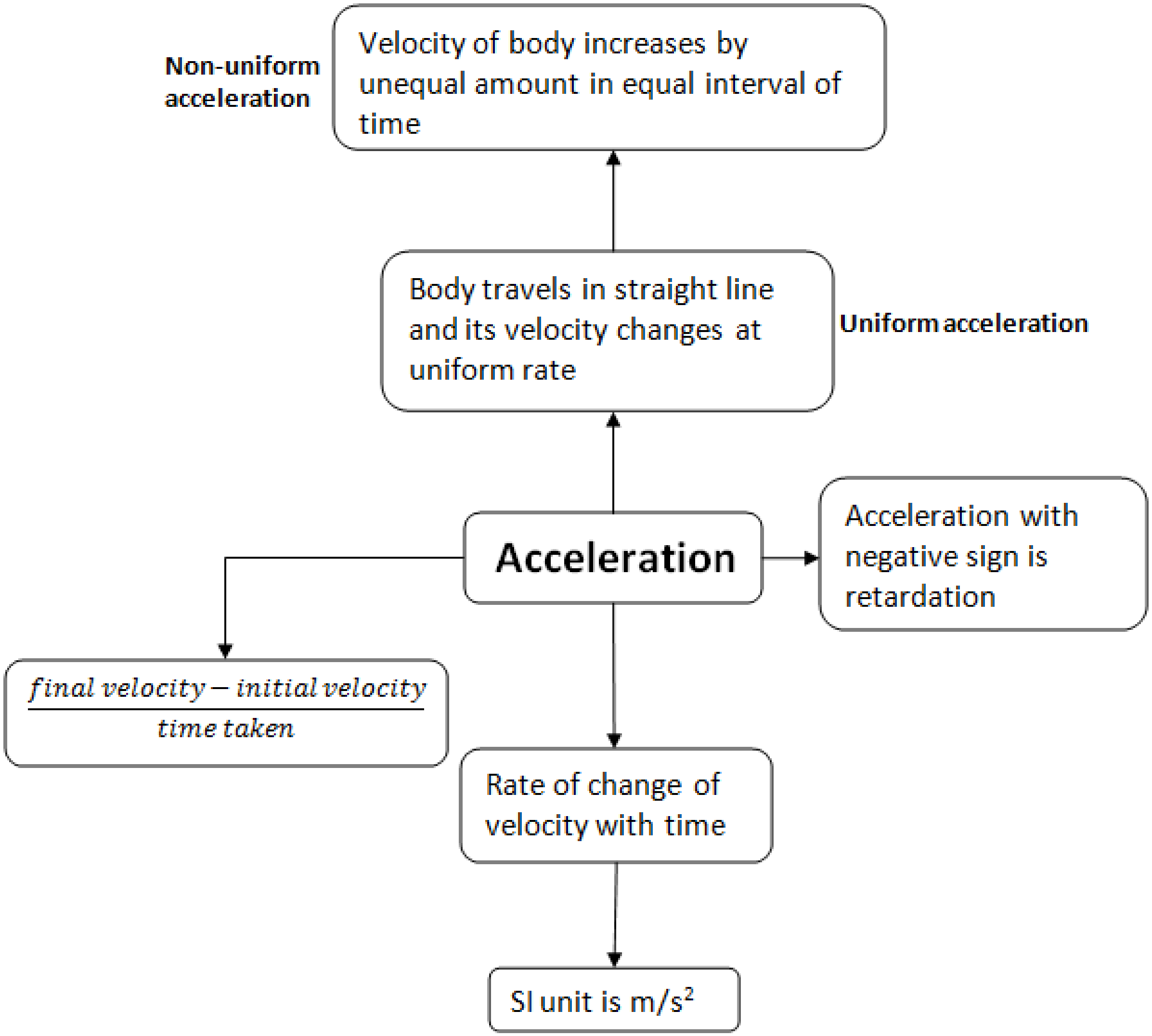
i.e., two objects A and B stays apart at constant distance.

$v_A > v_B$ then ($v_B - v_A$) would be negative and the distance between two objects will go on decreasing by an amount ($v_A - v_B$) after each unit of time. After some time they will meet and then object A will overtake object B.

If v_A and v_B have opposite signs then magnitude of v_{BA} or v_{AB} would be greater then the magnitude of velocity of A or that of B and objects seems to move very fast.

Kinematics Concept Maps

Concept Map Acceleration



KINEMATICS

*rest and Motion are relative terms, nobody can exist in a state of absolute rest or of absolute motion.

*One dimensional motion:- The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.

*Two dimensional motion:- The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.

*Three dimensional motion:- The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

*The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

Displacement ≤ Actual distance

*Speed:- It is rate of change of distance covered by the body with respect to time.

Speed = Distance travelled /time taken

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \rightarrow 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

Velocity =Displacement /Time

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

Acceleration = Change in velocity /time taken

It is a vector quantity, Its SI unit is meter/ sec.^{2} and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

For accelerated motion,	For deceleration motion
$V = u + at$	$v = u - at$
$S = ut + {}^1/_2\, at^2$	$S = ut - {}^1/_2\, at^2$
$V^2 = u^2 + 2as$	$V^2 = u^2 - 2as$
$Sn = u + {}^a/_2(2n - 1)$	$Sn = u - {}^a/_2(2n - 1)$

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec^2.

*Relative Motion:- The rate of change of distance of one object with respect to the other is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A.

*Relative velocity of object A with respect to object B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

When both objects are move in same direction, then the relative velocity of object B with respect to the object A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

When the object B moves in opposite direction of object A .

$$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$$

When V_A and V_B are incident to each other at angle Θ

$$V_{AB} = (V^2_A + V^2_B - 2V_AV_B \text{Cos } \Theta)^{1/2}$$

*Scalars :- The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

*Vectors :- The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc.

For examples : Displacement, velocity, acceleration, force , momentum etc.

- **Addition of Vectors :-**

  (i) Only vectors of same nature can be added.

  (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan \beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

  (iii) Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

  (iv) Vector addition is associative,

  $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

  (v) R is maximum if Θ = 0 and minimum if Θ = 180 ^{0}.

Subtraction of two vectors :-

(i) Only vector of same nature can be subtracted.

(ii) Subtraction of B from A = vector addition of A and (-B),

$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (\overrightarrow{-B})$

Where $R = [A^2 + B^2 + 2AB \cos(180 - \Theta)]^{1/2}$ and

$\tan\beta = B \sin(180 - \Theta) / [A + B \cos(180 - \Theta)]$, Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

(iii) Vector subtraction is not commutative $\overrightarrow{(A - B)} \neq (\vec{B} - \vec{A})$

(iv) Vector subtraction is not associative,

$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$

Rectangular components of a vector in a plane :- If A makes an angle Θ with x-axis and A_x and B_y be the rectangular components of A along X-axis and Y- axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{B}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

Here $A_x = A \cos\Theta$ and $A_y = A \sin\Theta$

And $A = (A_x^2 + A_y^2)^{1/2}$

And $\tan\Theta = A_y/A_x$

Dot product or scalar product : - The dot product of two vectors A and B, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \,.\, \vec{B} = AB \cos\Theta$$

(i) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

(ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Cross or Vector product :-

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad \hat{\jmath} \times \hat{k} = \hat{I}$ $\quad \hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t ,

$$V = [(u^2 - 2ugt\sin\theta + g^2t^2)]^{1/2}$$

- Horizontal range

$$R = u^2 \, \mathrm{Sin}2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

- Flight time

$$T = 2u \, \mathrm{Sin}\Theta/g$$

- Height

$$H = u^2 \sin^2\Theta/2g$$

For maximum height $\Theta = 90^0$

$$H_{max.} = u^2/2g$$

Very Short answer type questions (1 marks)

Q1. What does the slope of v-t graph indicate ?

Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?

Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.
Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : 90^0.

Q8. What is the angular velocity of the hour hand of a clock ?

Ans : W = 2π/12 = π/6 rad h^{-1},

Q9. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .

Ans : Null vector .

Short Answer type question (2 marks)

Q1. Derive an equation for the distance travelled by an uniform acceleration body in n^{th} second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n-1)$

Q2. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s

Ans. Differentiation of the given equation eq. w.r.t. time

We get $a = 18 + 18t$

At $t = 2$ sec.

$a = 54\ m/sec^2$.

Q3.what is relative velocity in one dimension, if V_A and V_B are the velocities of the body A and B respectively then prove that $V_{AB}=V_A-V_B$?

Ans. Relative Motion:- The rate of change of separation between the two object is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A .

*Relative velocity of object A with respect to object B

$V_{AB} = V_A - V_B$

When both objects are moving in same direction , then the relative velocity of object B with respect to the object A

$V_{BA} = V_B - V_A$

Q4. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$R = u^2 \, Sin2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

and Height

$$H = u^2 \sin^2\Theta/2g$$

For $\Theta = 45^0$

$$H = u^2/4g = 1/4 \text{ of the } R_{max.}$$

Q6. State the parallelogram law of vector addition. Derive an expression for magnitude and direction of resultant of the two vectors.

Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$$

And $R = (A^2 + B^2 + 2AB\,Cos\Theta)^{1/2}$

And $\tan\beta = B\,Sin\Theta/(A + B\,Cos\Theta)$,

Where Θ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q7. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,

Ans. Because bullet follow parabolic trajectory under constant downward acceleration.

Q8. Derive the relation between linear velocity and angular velocity.

Ans : Derive the expression

$$V = r\,\omega$$

Q9. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .

Q10. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?

Ans: we know that

$$H_{max.} = R_{max} /2$$

So $\quad h = R/2$

Or $\quad R = 2h$

Short answer questions (3 marks)

Q1. If 'R' is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q2. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0}$.

Q3. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.

Q4: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

(i) Zero

Q5. Two vectors **A** and **B** are inclined to each other at an angle Θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Q6. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Q7. When the angle between two vectors of equal magnitudes is 2π/3, prove that the magnitude of the resultant is equal to either.

Q8. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. (g = 9.8 m/sec^2)

Q9. Find the value of λ so that the vector $\vec{A} = 2\hat{i} + \lambda\,\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular to each.

Q10. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.

Long answer question (5 marks)

Q1. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.

Q2. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q3. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q4. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q5. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

HOTS

Q1. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : 60^0

Q2. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q3. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $uCos45^0$ is present on highest point.)

Q4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1t_2 = 2R/g$

ans : for equal range the particle should either be projected at an angle Θ and ($90 - \Theta$) ,

then $t_1 = 2u \, \text{Sin}\Theta/g$

$t_2 = 2u \, \text{Sin}(90 - \Theta)/g = 2u \, \text{Cos}\Theta/g$

$t_1 t_2 = 2R/g$.