Motion in a plane

1. Introduction

In previous chapter we have learned about the motion of any particle along a straight line
Straight line motion or rectilinear motion is motion in one dimension.Now in this chapter ,we will consider both motion in two dimension and three dimension.
In two dimensional motion path of the particle is constrained to lie in a fixed plane.Example of such motion motion are projectile shot from a gun ,motion of moon around the earth,circular motion and many more.
To solve problems of motion in a plane,we need to generalize kinematic language of previous chapter to a more general using vector notations in two and three dimensions.

2.Average velocity

Consider a particle moving along a curved path in x-y plane shown below in the figue
Suppose at any time,particle is at the point P and after some time 't' is at point Q where points P and Q represents the position of particle at two different points.

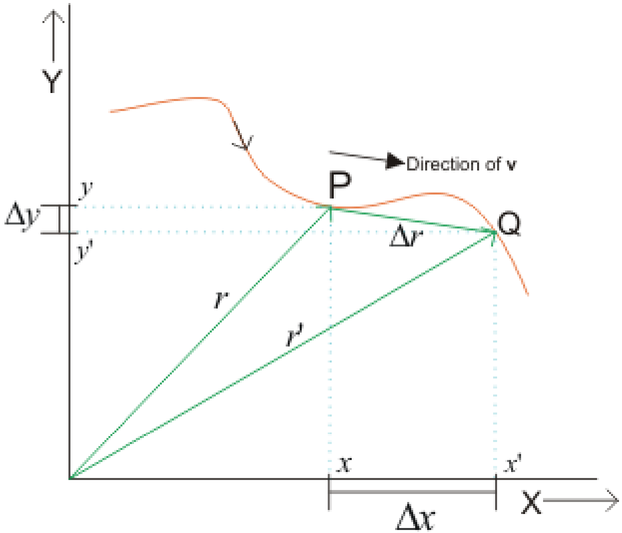


Figure 1. Displacement Δr between points P and Q

Position of particle at point P is described by the Position vector **r** from origin O to P given by

r=x**i**+y**j**

where x and y are components of **r** along x and y axis

As particle moves from P to Q,its displacement would be would be Δ**r** which is equal to the difference in position vectors **r** and **r'**.Thus

Δ**r** = **r'**-**r** = (x'**i**+y'**j**)-(x**i**+y**j**) = (x'-x)**i**+(y'-y)**j** = Δx**i**+Δy**j** (1)

where Δx=(x'-x) and Δy=(y'-y)

If Δt is the time interval during which the particle moves from point P to Q along the curved path then average velocity($\mathbf{v}_{avg}$) of particle is the ratio of displacement and corresponding time interval

$$v_{avg} = \frac{\Delta r}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = (v_x)_{avg}\hat{i} + (v_y)_{avg}\hat{j} \qquad (2)$$

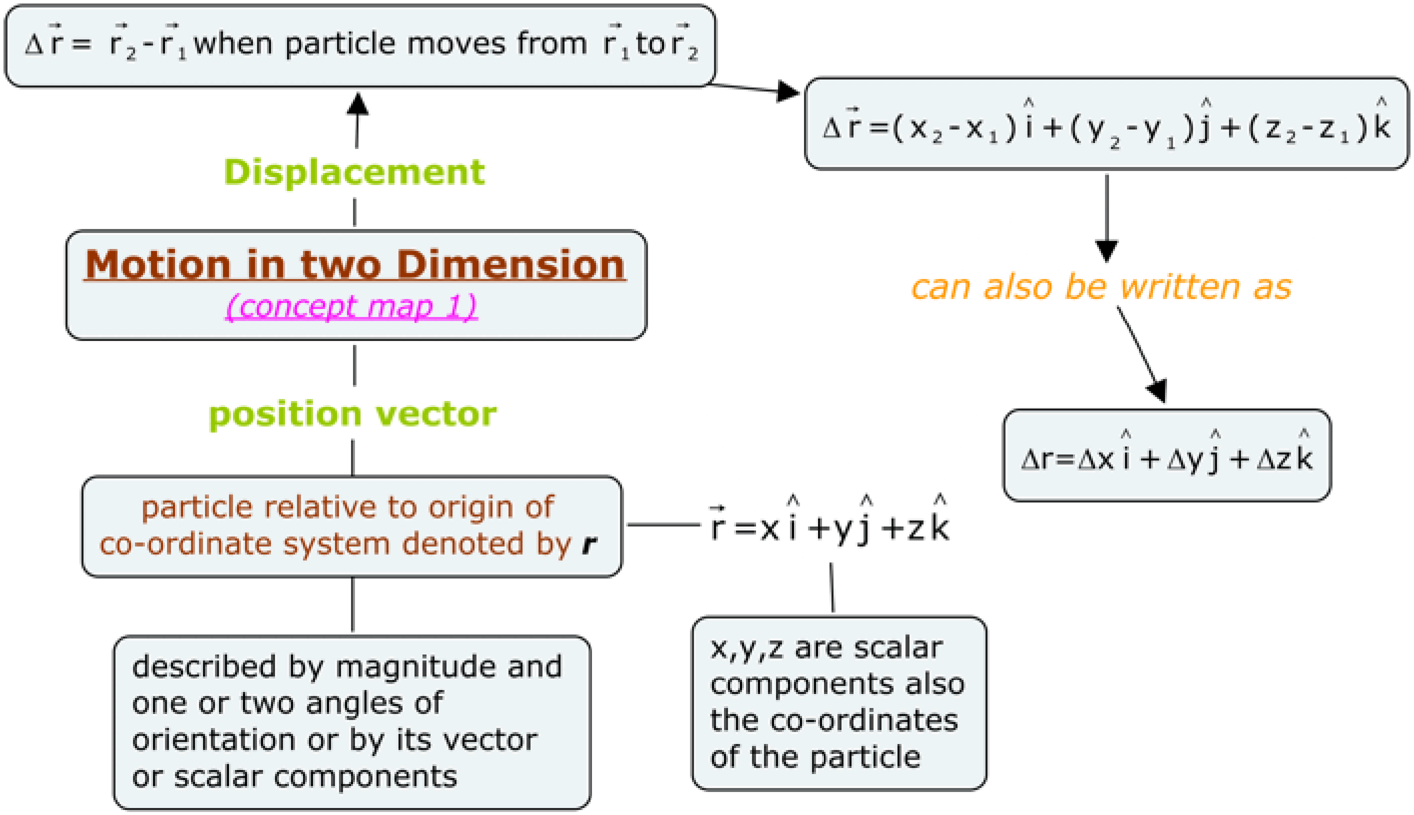
since $\mathbf{v}_{avg}$=Δ**r**/Δt , the direction of average velocity is same as that of Δ**r**

Magnitude of Δ**r** is always the straight line distance from P to Q regardless of any shape of actual path taken by the particle.

Hence average velocity of particle from point P to Q in time interval Δt would be same for any path taken by the particle.

Concept Map's for motion in a plane

Concept Map's for Position and displacement in two dimensional motion



3.Instantaneous velocity

We already know that instantaneous velocity is the velocity of the particle at any instant of time or at any point of its path.

If we bring point Q more and more closer to point P and then calculate average velocity over such a short displacement and time interval then

$$v = \lim_{t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

where **v** is known as the instantaneous velocity of the particle.

Thus, instantaneous velocity is the limiting value of average velocity as the time interval aproaches zero.

As the point Q aproaches P, direction of vector **Δr** changes and aproaches to the direction of the tangent to the path at point P. So instantaneous vector at any point is tangent to the path at that point.

Figure below shows the direction of instantaneous velocity at point P.

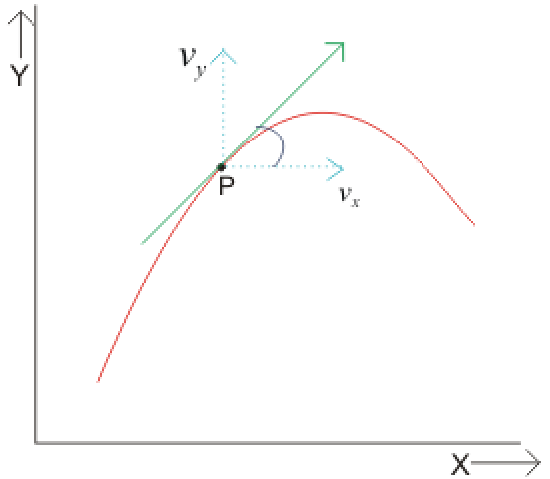


Figure 2. Direction of instantaneous velocity at any point P

Thus, direction of instantaneous velocity **v** at any point is always tangent to the path of particle at that point.

Like average velocity we can also express instantaneous velocity in component form

$$v = \frac{dr}{dt} = \frac{d(x\hat{i} + \Delta y\hat{j})}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j} \qquad (3)$$

where v_x and v_y are x and y components of instantaneous velocity.

Magnitude of instantaneous velocity is

$|\mathbf{v}|=\sqrt{[(v_x)^2+(v_y)^2]}$

and angle θ which velocity vector makes with x-axis is

$\tan\theta=v_x/v_y$

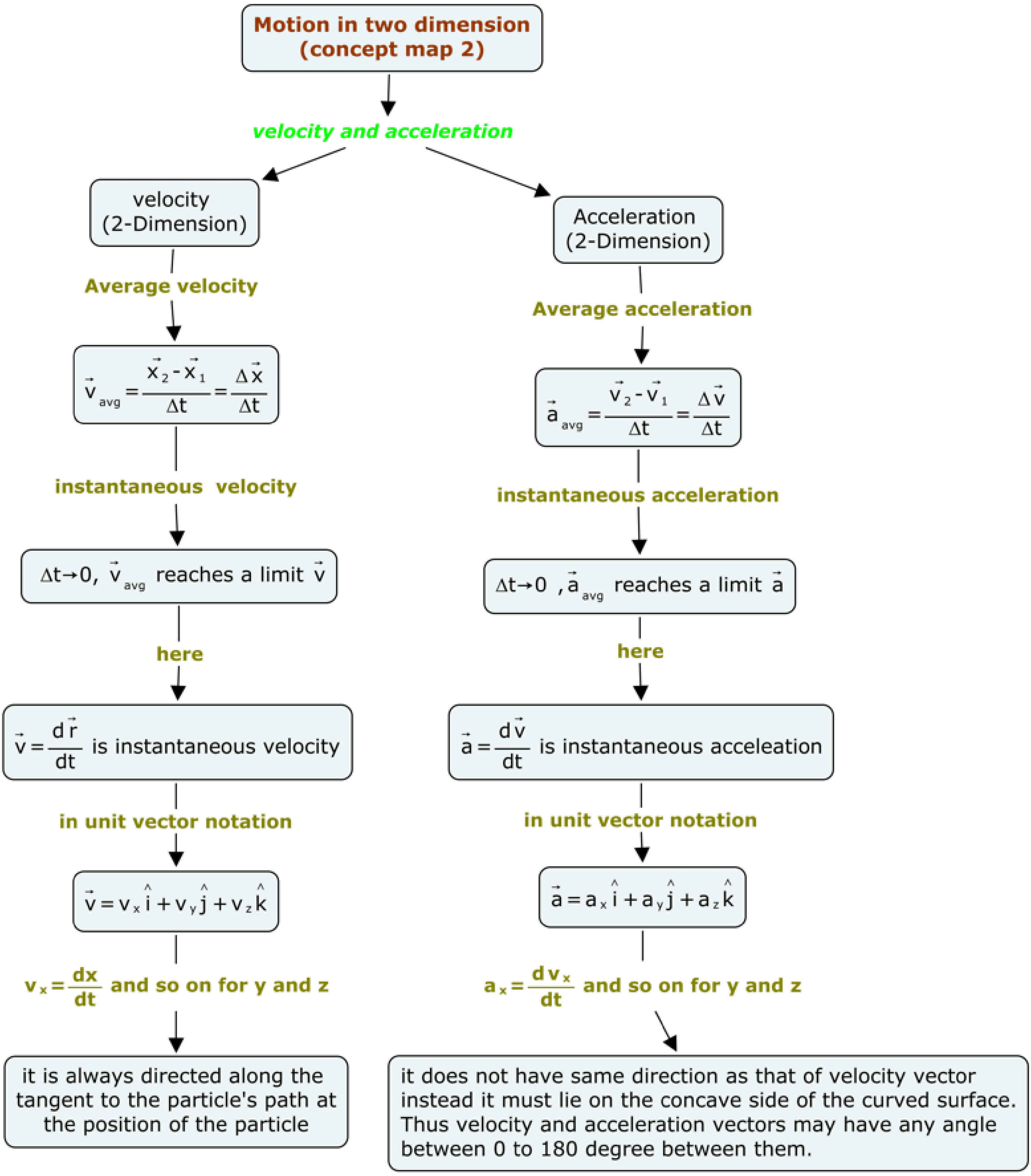
Expression for instantaneous velocity is

$$v = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

Thus, if expression for the co-ordinates x and y are known as function of time then we can use equations derived above to find x and y components of velocity.

Concept Map's for motion in a plane

Concept Map's for Velocity and acceleration in two dimensional motion



Motion in two dimension (concept map 2)
velocity and acceleration
velocity (2-Dimension)
Average velocity
$\vec{v}_{avg} = \frac{\vec{x}_2 - \vec{x}_1}{\Delta t} = \frac{\Delta \vec{x}}{\Delta t}$
instantaneous velocity
$\Delta t \to 0$, $\vec{v}_{avg}$ reaches a limit $\vec{v}$
here
$\vec{v} = \frac{d\vec{r}}{dt}$ is instantaneous velocity
in unit vector notation
$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$
$v_x = \frac{dx}{dt}$ and so on for y and z
it is always directed along the tangent to the particle's path at the position of the particle
Acceleration (2-Dimension)
Average acceleration
$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$
instantaneous acceleration
$\Delta t \to 0$, $\vec{a}_{avg}$ reaches a limit $\vec{a}$
here
$\vec{a} = \frac{d\vec{v}}{dt}$ is instantaneous acceleation
in unit vector notation
$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$
$a_x = \frac{dv_x}{dt}$ and so on for y and z
it does not have same direction as that of velocity vector instead it must lie on the concave side of the curved surface. Thus velocity and acceleration vectors may have any angle between 0 to 180 degree between them.

4. Average and instantaneous acceleration

Suppose a particle moves from point P to point Q in x-y plane as shown below in the figure

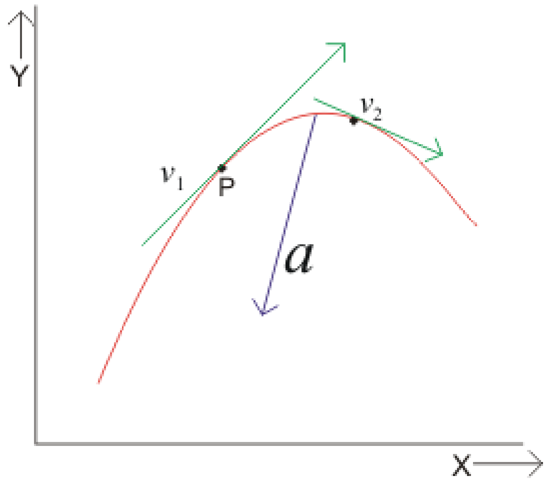


Figure 3(a).Vector representation for average acceleration between P and Q

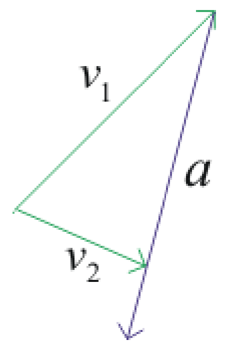


Figure 3(b).Digrametic vector representation for obtaining Δv

Suppose $\mathbf{v}_1$ is the velocity of the particle at point P and $\mathbf{v}_2$ is the velocity of particle at point Q

Average acceleration is the change in velocity of particle from $\mathbf{v}_1$ to $\mathbf{v}_2$ in time interval Δt as particle moves from point P to Q. Thus average acceleration is

$$a_{avg} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{\Delta t} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad (6)$$

Average accelaration is the vector quantity having direction same as that of Δ**v**.

Again if point Q aproaches point P, then limiting value of average acceleration as time aproaches zero defines instantaneous acceleration or simply the acceleration of particle at that point. Ths, instantaneous acceleration

$$\boldsymbol{a}=\lim_{t\to 0}\frac{\Delta\boldsymbol{v}}{\Delta t}=\frac{d\boldsymbol{v}}{dt}$$

or,

$$\boldsymbol{a}=\frac{d\boldsymbol{v}}{dt}=\frac{d(v_x\hat{\boldsymbol{i}}+v_y\hat{\boldsymbol{j}})}{dt}=\hat{\boldsymbol{i}}\frac{dv_x}{dt}+\hat{\boldsymbol{j}}\frac{dv_y}{dt}$$

or,

$$\boldsymbol{a}=\hat{\boldsymbol{i}}a_x+\hat{\boldsymbol{j}}a_y \qquad (7)$$

where,

$$a_x=\frac{dv_x}{dt},\ a_y=\frac{dv_y}{dt}$$

Figure below shows instantaneous acceleration a at point P.

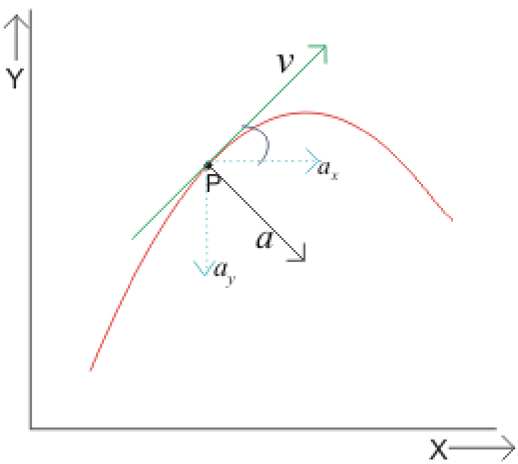


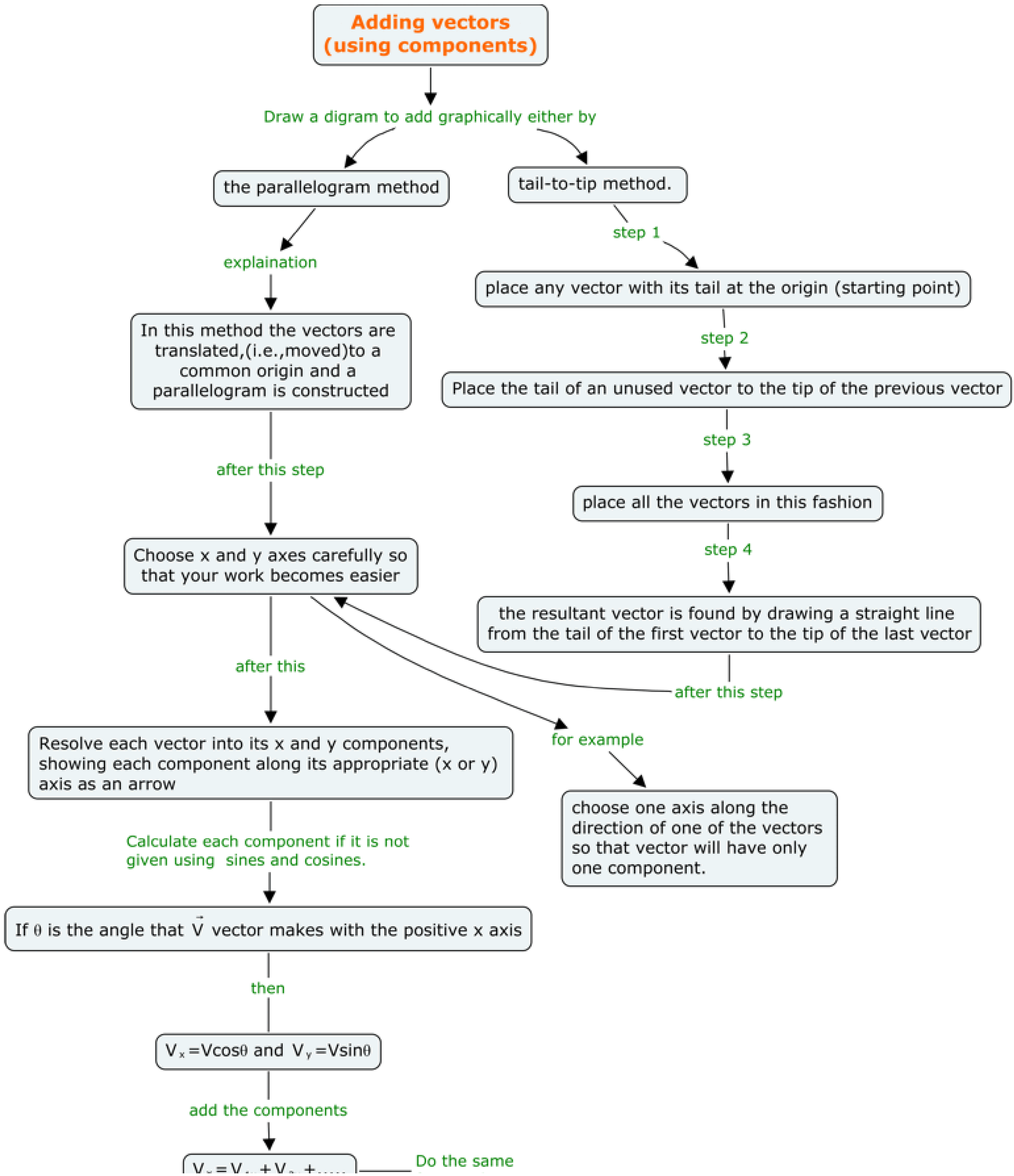
Figure 4. Acceleration at point P

Instantaneous acceleration does not have same direction as that of velocity vector instead it must lie on the concave side of the curved surface.

Thus velocity and acceleration vectors may have any angle between 0 to 180 degree between them.

Concept Map's for motion in a plane

Concept Map's for Vector addition



5. Motion with constant acceleration

Motion in two dimension with constant acceleration we we know is the motion in which velocity changes at a constant rate i.e, acceleration remains constant throughout the motion

We should set up the kinematic equation of motion for particle moving with constant acceleration in two dimensions.

Equation's for position and velocity vector can be found generalizing the equation for position and velocity derived earliar while studying motion in one dimension

Thus velocity is given by equation

$\mathbf{v}=\mathbf{v_0}+\mathbf{a}t$ (8)

where

$\mathbf{v}$ is velocity vector

$\mathbf{v_0}$ is Intial velocity vector

$\mathbf{a}$ is Instantanous acceleration vector

Similary position is given by the equation

$\mathbf{r}-\mathbf{r_0}=\mathbf{v_0}t+(1/2)\mathbf{a}t^2$ (9)

where $\mathbf{r_0}$ is Intial position vector

i,e

$\mathbf{r_0}=x_0\mathbf{i}+y_0\mathbf{j}$

and average velocity is given by the equation

$\mathbf{v_{av}}=(1/2)(\mathbf{v}+\mathbf{v_0})$ (10)

Since we have assumed particle to be moving in x-y plane,the x and y components of equation (8) and (9) are

$v_x=v_{x0}+a_xt$ (11a)

$x-x_0=v_{0x}t+(1/2)a_xt^2$ (11b)

and

$v_y=v_{y0}+a_yt$ (12a)

$y-y_0=v_{0y}t+(1/2)a_yt^2$ (12b)

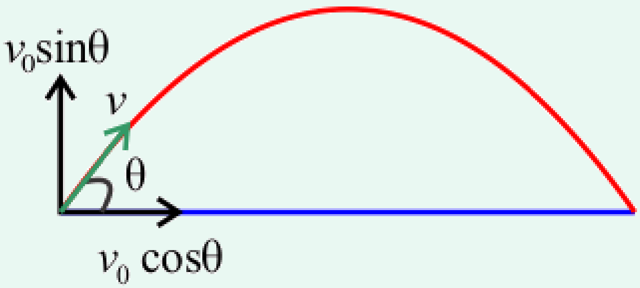
from above equation 11 and 12 ,we can see that for particle moving in (x-y) plane although plane of motion can be treated as two seperate and simultanous 1-D motion with constant acceleration

Similar result also hold true for motion in a three dimension plane (x-y-z)

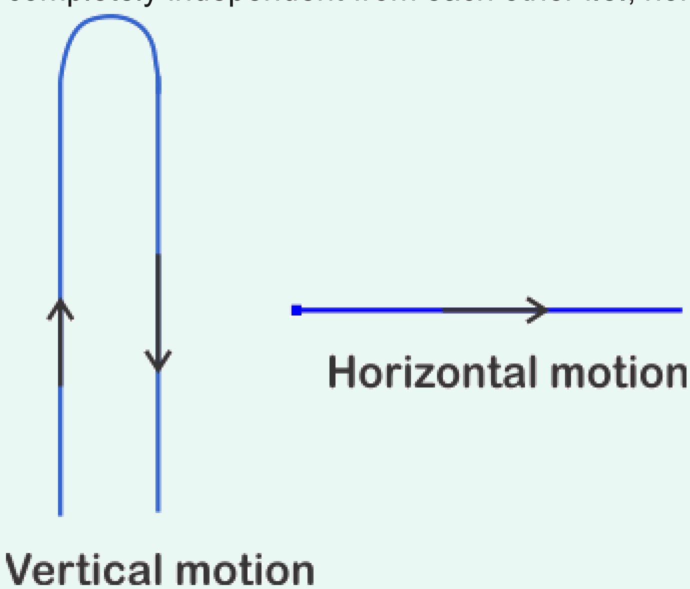
6. Projectile Motion

Basic Concept

1. Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown with some initial velocity near the earth's surface, and it moves along a curved path under the action of gravity alone.
2. It is an example of two dimensional motion with constant acceleration.
3. If F is constant then a is constant and when force is oblique direction with initial velocity , the result is parabolic path.



4. Projectile motion is considered as two simultaneous motion in mutually perpendicular directions which are completely independent from each other i.e., horizontal motion and vertical motion



Projectile thrown at an angle with horizontal

So we have seen our basics let us now learn the projectile motion in little more detail about Projectile thrown at an angle with horizontal. There are certain approximate assumptions we make while studying projectile motion and they are

1. We ignore the frictional resistance due to air.
2. The effect due to rotation of earth and the curvature of earth is negligible.
3. The acceleration due to gravity is constant in magnitude and direction at all points during the motion of projectile.

Projectile motion is a special case of motion in two dimension when acceleration of particle is constant in both magnitude and direction.

An object is referred as projectile when it is given an initial velocity which subsequently follows a path determined by gravitational forces acting on it. For example bullet fired from the rifle javelin thrown by the athlete etc.

Path followed by a projectile is called its trajectory.

In this section we will study the motion of projectile near the earth surface. We would be neglecting the air resistance.

Acceleration acting on a projectile is constant which is acceleration due to gravity (g=9.81 m/s) directed along vertically downward direction.

We shall treat the projectile motion in a Cartesian co-ordinates system taking y axis in vertically upwards direction and x axis along horizontal directions

Now x and y components of acceleration of projectile is

${a_x} = 0$ and ${a_x} = 0$

Since acceleration in horizontal direction is zero, this shows that horizontal component of velocity is constant and vertical motion is simply a case of motion with constant acceleration.

Suppose at time $t=0$ object is at origin of co-ordinate system and velocity components ${v_{0x}}$ and ${v_{0y}}$. From above components of acceleration are ${a_x} = 0$ and ${a_y} = - g$. From equation (11) and (12) in the previous section components of position and velocity are

$x = {v_{0x}}t$ (14a)

${v_x} = {v_{0x}}$ (14b)

and ${v_y} = {v_{0y}} - gt$ (15b)

$y = {v_{0y}} - \frac{1}{2}g{t^2}$ (15a)

Figure below shows motion of an object projected with velocity ${v_0}$ at an angle ${\theta _0}$.

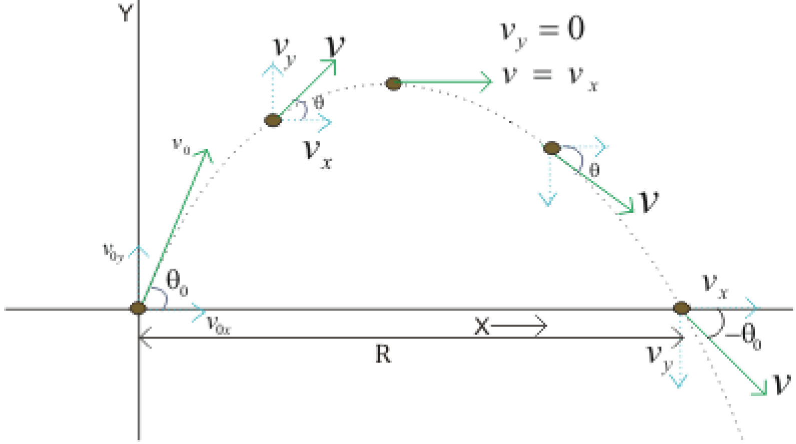


Figure 5. Trajectory of a body projected with an initial velocity v_0

In terms of initial velocity ${v_0}$ and angle ${\theta _0}$ components of initial velocity are

${v_{0x}} = {v_0}\cos{\theta _0}$ (16a)

${v_{0y}} = {v_0}\sin{\theta _0}$ (16b)

Using these relations in equation 14 and 15 we find

$x=(v_0\cos\theta_0)t$ (17a)

$y=(v_0\sin\theta_0)t-(1/2)gt^2$ (17b)

$v_x=v_0\cos\theta_0$ (17c)

$v_y=v_0\sin\theta_0-gt$ (17d)

Above equations describe the position and velocity of projectile as shown in fig 5 at any time t.

(A)Equation of Path of projectile(Trajectory)

From equation 17a

$t=x/v_0\cos\theta_0$

now putting this value of t in equation 17b,we find

$y=(\tan\theta_0)x-[g/2(v_0\cos\theta_0)^2]x^2$ (18)

In equation (18),quantities θ_0,g and v_0 are all constants and equation (18) can be compared with the equation

$y=ax-bx^2$

where a and b are constants

This equation $y=ax-bx^2$ is the equation of the parabola.From this we conclude that path of the projectile is a parabola as shown in figure 5

B) Time of Maximum height

At point of maximum height $v_y=0$.Thus from equation (17d)

$v_y=v_0\sin\theta_0-gt$

$0=v_0\sin\theta_0-gt$ -

or $t_m=v_0\sin\theta_0/g$ (19)

y=0 because when projectile reaches ground ,verical distance travelled is zero.This from equation (17b)

$t_f=2(v_0\sin\theta_0)/g$ (20)

or

$t_f=2t_m$

Maximum height reached by the projectile can be calculated by substituting $t=t_m$ in equation 17b

$y=H_m=(v_0\sin\theta_0)(v_0\sin\theta_0/g)-(g/2)(v_0\sin\theta_0/g)^2$

or

$H_m=v_0^2\sin^2\theta_0/2g$ (21)

(C) Horizontal Range of Projectile

Since acceleration g acting on the projectile is acting vertically ,so it has no component in horizontal direction. So, projectile moves in horizontal direction with a constant velocity $v_0\cos\theta_0$. So range R is

R=OA=velocity x time of flight

$$R = v_0 \cos\theta_0 t_f = \frac{v_0 \cos\theta_0 \times 2v_0 \sin\theta_0}{g}$$

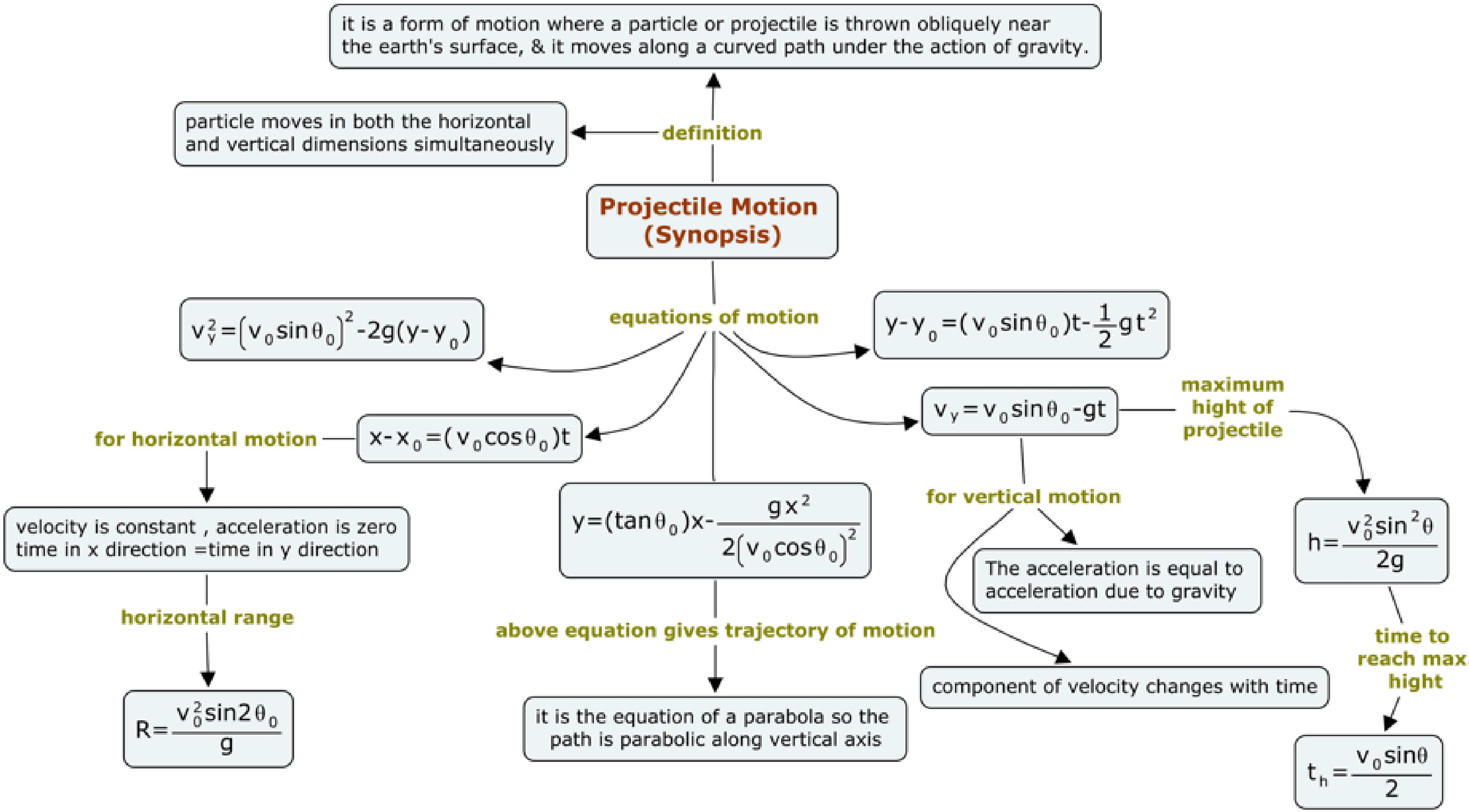
or,

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \qquad (22)$$

Maximum range is obtained when $\sin2\theta_0=1$ or $\theta_0=45^0$. Thus when $\theta_0=45^0$ maximum range achieved for a given initial velocity is $(v_0)^2/g$.

Concept Map's for motion in a plane

Concept Map's for projectile motion



7. Uniform circular motion

When an object moves in a circular path at a constant speed then motion of the object is called uniform circular motion.

In our every day life ,we came across many examples of circular motion for example cars going round the circular track and many more .Also earth and other planets revolve around the sun in a roughly circular orbits

Here in this section we will mainly consider the circular motion with constant speed

if the speed of motion is constant for a particle moving in a circular motion still the particles accelerates becuase of costantly changing direction of the velocity.

Here in circular motion ,we use angular velocity in place of velocity we used while studying linear motion

(A) Angular velocity

Consider an object moving in a circle with uniform velocity v as shown below in the figure

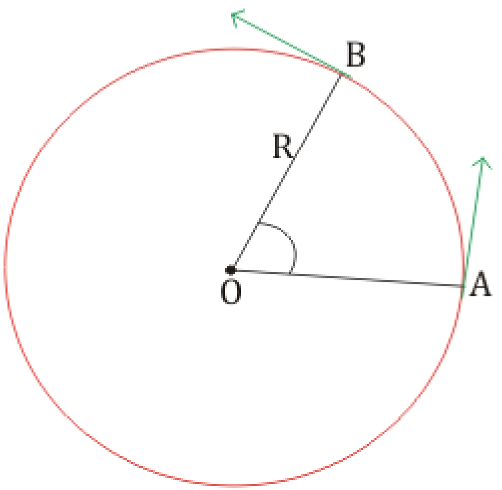


Figure 6

The velocity v at any point of the motion is tangential to the circle at that point.Let the particle moves from point A to point Balong the circumference of the circle .The distance along the circumference from A to B is

$s=R\theta$ (23)

Where R is the radius of the circle and θ is the angle moved in radian's

Magnitude of velocity is

$v=ds/dt=Rd\theta/dt$ (24)

Since radius of the circle remains constant quantity,

$\omega=d\theta/dt$ (25)

is called the angular velocity defined as the rate of change of angle swept by radius with time.

Angular velolcity is expressed in radians per second ($rads^{-1}$)

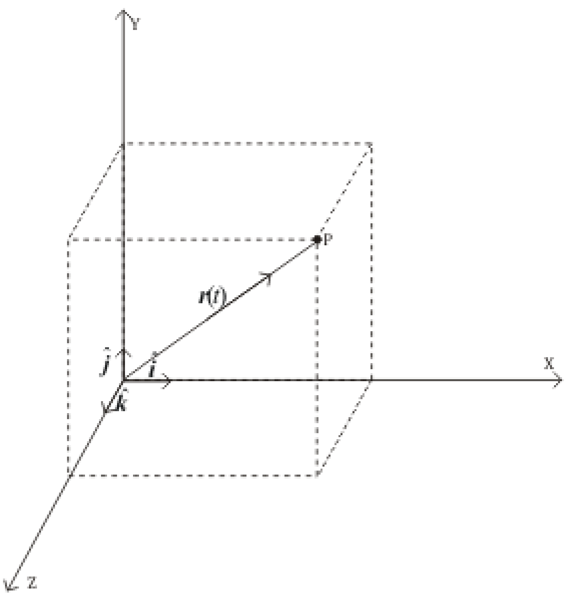
8. Motion in three dimensions

We have already studied physical quantities like displacement ,velocity,acceleration etc in one and two dimension

In this topic ,we will generalize our previous knowlegde of motion in 1 and 2 -dimension to three dimension's

As we have used vectors to represent motion in a plane,we can freely use vectors and its properties in 3-dimension as we have done in case of motion in a plane

In three dimensions ,we have three units vectors **i** ,**j** and **k** associated with each co-ordinate axis of cartesian co-ordinates system shown below in the figure



Consider a particle moving in 3-D space .Let P be its position at any point t.Position vector of this particle at point P would be

r=x**i**+y**j**+z**k**

Where x,y and z are co-ordinates of point P

Similarly velocity and acceleration vectors of particle moving in 3-D space are

v=v_x**i**+v_y**j**+v_z**k** where v_x=dx/dt,v_y=dy/dt and v_z=dz/dt

and

a=v_x**i**+a_y**j**+a_z**k**

where a_x=dv_x/dt,a_y=dv_y/dt and a_z=dv_z/dt

All the relations we have derived incase of motion in plane are valid for 3-D motion with one added co-ordinate

KINEMATICS

*rest and Motion are relative terms, nobody can exist in a state of absolute rest or of absolute motion.

*One dimensional motion:- The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.

*Two dimensional motion:- The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.

*Three dimensional motion:- The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

*The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

Displacement ≤ Actual distance

*Speed:- It is rate of change of distance covered by the body with respect to time.

Speed = Distance travelled /time taken

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \rightarrow 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

Velocity =Displacement /Time

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

Acceleration = Change in velocity /time taken

It is a vector quantity, Its SI unit is meter/ $sec.^2$ and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

For accelerated motion,	For deceleration motion
$V = u + at$	$v = u - at$
$S = ut + {}^1/_2 at^2$	$S = ut - {}^1/_2 at^2$
$V^2 = u^2 + 2as$	$V^2 = u^2 - 2as$
$Sn = u + {}^a/_2(2n - 1)$	$Sn = u - {}^a/_2(2n - 1)$

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec^2.

*Relative Motion:- The rate of change of distance of one object with respect to the other is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A.

*Relative velocity of object A with respect to object B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

When both objects are move in same direction, then the relative velocity of object B with respect to the object A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

When the object B moves in opposite direction of object A .

$$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$$

When V_A and V_B are incident to each other at angle Θ

$$V_{AB} = (V^2_A + V^2_B - 2V_AV_B\text{Cos }\Theta)^{1/2}$$

*Scalars :- The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

*Vectors :- The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc.

For examples : Displacement, velocity, acceleration, force , momentum etc.

- **Addition of Vectors :-**

  (i) Only vectors of same nature can be added.

  (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan \beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

  (iii) Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

  (iv) Vector addition is associative,

  $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

  (v) R is maximum if Θ = 0 and minimum if Θ = 180^0.

Subtraction of two vectors :-

(i) Only vector of same nature can be subtracted.

(ii) Subtraction of B from A = vector addition of A and (-B),

$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (\overrightarrow{-B})$

Where $R = [A^2 + B^2 + 2AB \cos(180 - \Theta)]^{1/2}$ and

$\tan\beta = B \sin(180 - \Theta) / [A + B \cos(180 - \Theta)]$, Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

(iii) Vector subtraction is not commutative $\overrightarrow{(A} - \vec{B}) \neq (\vec{B} - \vec{A})$

(iv) Vector subtraction is not associative,

$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$

Rectangular components of a vector in a plane :- If A makes an angle Θ with x-axis and A_x and B_y be the rectangular components of A along X-axis and Y- axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{B}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

Here $A_x = A \cos\Theta$ and $A_y = A \sin\Theta$

And $A = (A_x^2 + A_y^2)^{1/2}$

And $\tan\Theta = A_y/A_x$

Dot product or scalar product : - The dot product of two vectors A and B, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \,.\, \vec{B} = AB \cos\Theta$$

(i) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

(ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Cross or Vector product :-

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad \hat{\jmath} \times \hat{k} = \hat{I}$ $\quad \hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t ,

$$V = [(u^2 - 2ugt\sin\theta + g^2t^2)]^{1/2}$$

- Horizontal range

$$R = u^2 \sin 2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

- Flight time

$$T = 2u \sin\Theta/g$$

- Height

$$H = u^2 \sin^2\Theta/2g$$

For maximum height $\Theta = 90^0$

$$H_{max.} = u^2/2g$$

Very Short answer type questions (1 marks)

Q1. What does the slope of v-t graph indicate ?

Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?

Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.
Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : 90^{0}.

Q8. What is the angular velocity of the hour hand of a clock ?

Ans : $W = 2\pi/12 = \pi/6$ rad h^{-1},

Q9. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .

Ans : Null vector .

Short Answer type question (2 marks)

Q1. Derive an equation for the distance travelled by an uniform acceleration body in n^{th} second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n - 1)$

Q2. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s

Ans. Differentiation of the given equation eq. w.r.t. time

We get $a = 18 + 18t$

At $t = 2$ sec.

$a = 54\ m/sec^2$.

Q3.what is relative velocity in one dimension, if V_A and V_B are the velocities of the body A and B respectively then prove that $V_{AB}=V_A-V_B$?

Ans. Relative Motion:- The rate of change of separation between the two object is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A .

*Relative velocity of object A with respect to object B

$V_{AB} = V_A - V_B$

When both objects are moving in same direction , then the relative velocity of object B with respect to the object A

$V_{BA} = V_B - V_A$

Q4. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$R = u^2 \text{Sin}2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

and Height

$$H = u^2 \sin^2\Theta/2g$$

For $\Theta = 45^0$

$$H = u^2/4g = 1/4 \text{ of the } R_{max.}$$

Q6. State the parallelogram law of vector addition. Derive an expression for magnitude and direction of resultant of the two vectors.

Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$$

And $R = (A^2 + B^2 + 2AB \text{ Cos}\Theta)^{1/2}$

And $\tan\beta = B \text{ Sin}\Theta / (A + B \text{ Cos}\Theta)$,

Where Θ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q7. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,

Ans. Because bullet follow parabolic trajectory under constant downward acceleration.

Q8. Derive the relation between linear velocity and angular velocity.

Ans : Derive the expression

$$V = r\,\omega$$

Q9. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .

Q10. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?

Ans: we know that

$$H_{max.} = R_{max} /2$$

$$\text{So} \quad h = R/2$$

$$\text{Or} \quad R = 2h$$

Short answer questions (3 marks)

Q1. If 'R' is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q2. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0}$.

Q3. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.

Q4: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

(i) Zero

Q5. Two vectors **A** and **B** are inclined to each other at an angle Θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Q6. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Q7. When the angle between two vectors of equal magnitudes is $2\pi/3$, prove that the magnitude of the resultant is equal to either.

Q8. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. ($g = 9.8\ m/sec^2$)

Q9. Find the value of λ so that the vector $\vec{A} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{B} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.

Q10. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.

Long answer question (5 marks)

Q1. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.

Q2. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q3. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q4. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q5. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

HOTS

Q1. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : 60^0

Q2. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q3. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $u\text{Cos}45^0$ is present on highest point.)

Q4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1 t_2 = 2R/g$

ans : for equal range the particle should either be projected at an angle Θ and ($90 - \Theta$) ,

$$\text{then} \qquad t_1 = 2u\,\text{Sin}\Theta/g$$

$$t_2 = 2u\,\text{Sin}(90 - \Theta)/g = 2u\,\text{Cos}\Theta/g$$

$$t_1 t_2 = 2R/g \,.$$