Work Energy and Power

## (1) Introduction

In our everyday life we use terms like work and enegy.
Term work is generally used in context to any kind of activity requiring physical or mental effort.
But this is not the way how we define work done in physics.
When we push or pull a heavy load or lift it above the floor then we are doing work, but a man carrying heavy load and standing still is not doing any work according to scientific definiton of work.
Another term we often use is energy. Energy is usually associated with work done in the sence that a person feeling very energetic is capable of doing lot of work.
This way energy defined to be as capacity of doing work.
There are many forms of energy like chemical energy, mechanical energy, electrical energy, heat energy etc.
These forms of energies can be used in number of ways.
One form of energy can be converted into another form of energy.
In this chapter we will study about work, relation between work and energy, conservation of energy etc.

## (2) Work

We already know that work is said to be done when a force produces motion.
Work done is defined in such a way that it involves both force applied on the body and the displacement of the body.
Consider a block placed on a frictionless horizontal floor. This block is acted upon by a constant force F. Action of this force is to move the body through a distance $d$ in a straight line in the direction of force.
Now, work done by this force is equal to the product of the magnitude of applied force and the distance through which the body moves. Mathematically,
W=Fd


Figure 1. Force acting on a block at an angle $\theta$ to the displacement

In this case force acting on the block is constant but the direction of force and direction of displacement caused by this force is different. Here force $\mathbf{F}$ acts at an angle $\theta$ to the displacement $\mathbf{d}$
Effective component of force along the direction of displacement is Fcos $\theta$ and this component of force is responsible for the displacement of the block in the given direction.
Thus, work done by the force $\mathbf{F}$ in in displacing the body thruugh displacement $\mathbf{d}$ is
$\mathrm{W}=(|\mathbf{F}| \cos \theta)|\mathbf{d}|$
In equation 2 work done is defined as the product of magnitude of displacement $\mathbf{d}$ and the component of force in the direction of displacement.
We know that the scalar product of two vectors $\mathbf{A}$ and $\mathbf{B}$ where $\mathbf{A}$ makes an angle $\theta$ with $\mathbf{B}$ is given by


Figure 2

## $A . B=|A||B| \cos \theta$

Comparing equation 2 with definition of scalar products work done can be written as
W=F.d
Now consider two special cases:-
(i) When angle $\theta=0$ i.e., force is in the same direction as of displacement, then from equation (2)
$W=|\mathbf{F}||\mathbf{d}|$
this is the same result as of equation 1
(ii) when angle $\theta=90$ i.e., direction of force is perpandicular to that of displacement, then from equation (2)
$W=(|\mathbf{F}| \cos 90)|\mathbf{d}|=0$
that is force applies has no component along the displacement and hence force does not do any work on the body.
Work done by a force on a body can be positive, negative and zero i.e.,

## displacement of the body.

(c) Work done is zero :- Force is at rigt angle to the displacement for example work of a centripetal force on a body moving in a circle.
Unit of work done in any system of units is equal to the unit of force multiplied by the unit of distance. In SI system unit of work is 1 Nm and is given a name Joule(J). Thus,
$1 \mathrm{~J}=1 \mathrm{Nm}$
In CGS system unit is erg
1erg=1 dyne-cm
and $1 \mathrm{erg}=10^{-7} \mathrm{~J}$
When more then one forces acts on a body then work done by each force should be calculated separately and added togather.

## (3) Work done by variable force

So far we have defined work done by a force which is constant in both magnitude and direction.
However, work can be done by forces that varies in magnitude and direction during the displacement of the body on which it acts.
For simplicity consider the direction of force acting on the body to be along $x$-axis also consider the force $F(x)$ is some known function of position $x$
Now total displacement or path of the body can be decomposed into number of small intervals $\Delta x$ such that with in each interval force $F(x)$ can be considered to be approximately constant as shown below in the figure


Figure 3. Calculation of work done by variable force $F(x)$ in moving a body from $A$ to $B$

Workdone in moving the body from $x_{1}$ to $x_{2}$ is given by
$\Delta \mathrm{W}=\mathrm{F}\left(\mathrm{x}_{1}\right) \Delta \mathrm{x}_{1}$ where $\Delta \mathrm{x}_{1}=\mathrm{x}_{2}-\mathrm{x}_{1}$
Total workdone in moving the body from point $A$ to point $B$
$\mathrm{W}=\mathrm{F}\left(\mathrm{x}_{1}\right) \Delta \mathrm{x}_{1}+\mathrm{F}\left(\mathrm{x}_{2}\right) \Delta \mathrm{x}_{2}+\mathrm{F}\left(\mathrm{x}_{3}\right) \Delta \mathrm{x}_{3} \ldots . .+\mathrm{F}\left(\mathrm{x}_{\mathrm{n}}\right) \Delta \mathrm{x}_{\mathrm{n}}$
$W=\Sigma F\left(x_{i}\right) \Delta x_{i}$ where $i=1$ to $i=n$
Where $\Sigma$ is the symbol of summation
Summation in equation 4 is equal to shaded area in figure 3(a). More accuracy of results can be obtained by making these interval infintesimally smaller
We get the exact value of workdone by making each interval so much small such that \#916;x-> 0 which means curved path being decomposed into infinte number of line segment i.e
$W=\operatorname{Lim}_{\Delta x->}{ }^{\Sigma} \mathrm{F}\left(\mathrm{x}_{\mathrm{i}}\right) \Delta \mathrm{x}_{\mathrm{i}}$
$W=\int F(r) d r$ with in the limits $r_{A}$ and $r_{B}$
Where $F(\mathbf{r})=F(x) \mathbf{i}+F(y) \mathbf{j}+F(z) \mathbf{k}$
and $d r=d x i+d y j+d z k$
Here $F(x), F(y)$ and $F(z)$ are rectangular components of the force along $x, y$ and $z$ axis. Similary $d x, d y$ and $d z$ are rectangular components of displacement along
$x, y$ and $z$ axis

## (4) Mechanical Energy

we already have an idea that energy is associated closely with work and we have defined energy of a body as the capacity of the body to do work
In dynamics body can do work either due to its motion, due to its position or both due to its motion and position Ability of a body to do work due to its motion is called kinetic energy for example piston of a locomotive is capable of doing of work
Ability of a body to do work due to its position or shape is called potential energy For example workdone by a body due to gravity above surface of earth
Sum of kinetic energy and Potential energy of body is known as its mechanical energy
Thus
M.E=K.E+P.E
(5) Kinetic energy

Kinetic energy is the energy possesed by the body by virtue of its motion
newton's second law of motion

## $\mathrm{F}=\mathrm{ma}$

Where a is the acceleration of the body
If due to this acceleration a, velocity of the body increases from $\mathbf{v}_{\mathbf{1}}$ to $\mathbf{v}_{\mathbf{2}}$ during the displacement $d$ then from equation of motion with constant acceleration we have
$\mathrm{v}_{2}{ }^{2}-\mathrm{v}_{1}{ }^{2}=2 \mathrm{ad}$ or
$a=v_{2}{ }^{2}-v_{1}{ }^{2} / 2 d$ Using this acceleration in Newton's second law of motion we have
$F=m\left(v_{2}{ }^{2}-v_{1}{ }^{2}\right) / 2 d$
or
$\mathrm{Fd}=\mathrm{m}\left(\mathrm{v}_{2}{ }^{2}-\mathrm{v}_{1}{ }^{2}\right) / 2$
or
$\mathrm{Fd}=\mathrm{mv}_{2}{ }^{2} / 2-\mathrm{mv}_{1}{ }^{2} / 2$
We know that Fd is the workdone by the force $F$ in moving body through distance $d$ In equation(7), quantity on the right hand side $\mathrm{mv}^{2} / 2$ is called the kinetic energy of the body
Thus
$K=m v^{2} / 2$
Finally we can define KE of the body as one half of the product of mass of the body and the square of its speed
Thus we see that quantity ( $\mathrm{mv}^{2} / 2$ ) arises purely becuase of the motion of the body In equation 7 quantity
$K_{2}=\mathrm{mv}_{2}{ }^{2} / 2$
is the final KE of the body and
$K_{1}=\mathrm{mv}_{1}{ }^{2} / 2$
is the initial KE of the body .Thus equation 7 becomes
$W=K_{2}-K_{1}=\Delta K$

## Unit of KE is same as that of work i.e Joule

If there are number of forces acting on a body then we can find the resultant force ,which is the vector sum of all the forces and then find the workdone on the body Again equation (9) is a generalized result relating change in KE of the object and the net workdone on it.This equation can be summerized as $\mathrm{K}_{\mathrm{f}}=\mathrm{K}_{\mathrm{i}}+\mathrm{W}$
which says that kinetic energy after net workdone is equal to the KE before net work plus network done.Above statement is also known as work-kinetic energy theorem of particles
Work energy theorem holds for both positive and negative workdone.if the workdone is positive then final KE increases by amount of the work and if workdone is negative then final KE decreases by the amount of workdone

## (6)Potential energy

Potential energy is the energy stored in the body or a system by virtue of its position in field of force or by its configuration
Force acting on a body or system can change its PE
Few examples of bodies possesing PE are given below
i) Stretched or compressed coiled spring
ii) Water stored up at a height in the Dam possess PE
iii) Any object placed above the height H from the surface of the earth posses PE

Potential energy is denoted by letter $\cup$
In next two topics we would discuss following two example of PE
i)PE of a body due to gravity above the surface of earth
ii) PE of the spring when it is compressed orr elongated by the application of some external force

## (7) Gravitational PE near the surface of the earth

All bodies fall towards the earth with a constant acceleration known as acceleration due to gravity Consider a body of mass $m$ placed at height $h$ above the surface of the earth
Now the body begins to fall towards the surface of the earth and at any time $t$, it is at height $h^{\prime}\left(h^{\prime}<h\right)$ above the surface of the earth
During the fall of the body towards the earth a constant force $\mathrm{F}=\mathrm{mg}$ acts on the body where direction of force is towards the earth
Workdone by the constant force of gravity is
W=F.d
$\mathrm{W}=\mathrm{mg}(\mathrm{h}-\mathrm{h}$ )
W=mgh-mgh' -(12)
From equation (12), we can clearly see that workdone depends on the difference in height or position
So a potential energy or more accurately gravitational potential energy can be associated with the body such that
U=mgh
Where h is the height of the body from the refrence point
if
$\mathrm{U}_{\mathrm{i}}=\mathrm{mgh} \mathrm{U}_{\mathrm{f}}=\mathrm{mgh}$ ' or
$W=U_{i}-U_{f}=-\Delta U$
The potential energy is greater at height h and smaller at lower height h
We can choose any position of the object and fix it zero gravitational PE level.PE at height above this level would he mah

The position of zero PE is choosen according to the convenience of the problem and generally earth surface is choosen as position of zero PE
One important point to note is that equation (13) is valid even for the case when object object of mass $m$ is thrown vertically upwards from height $h$ to $h^{\prime}\left(h^{\prime}>h\right)$
Points to keep in mind
i) If the body of mass $m$ is thrown upwards a height $h$ above the zero refrence level then its PE increase by an amount mgh
ii) If the body of mass falls vertically downwards through a height $h$,the PE of body decreases by the amount mgh

## (8) Conversion of Gravitational PE to KE

Consider a object of mass placed at height H above the surface of the earth
By virtue of its position the object possess PE equal to mgH
When this objects falls it begins to accelerate towards earth surface with acceleration equal to acceleration due to gravity g
When object accelerates it gathers speed and hence gains kinetic energy at the expense of gravitational PE To Analyze this conversion of PE in KE consider the figure given below

$$
\mathrm{h}=\mathrm{H}: \mathrm{v}=0
$$



## Figure 4. Conversion of PE into KE when object of mass m falls from height H

Now at any given height h,PE of the object is given by the $\mathrm{U}=\mathrm{mgh}$ and at this height speed of the object is given by v
Change in KE between two height $h_{1}$ and $h_{2}$ would be equal to the workdone by the downward force
$m v_{1}{ }^{2} / 2-m v_{2}{ }^{2} / 2=m g d$
Where $d$ is the distance between two heights .here one important thing to note is that both force and are in the same downwards direction
Corresponding change in Potential energy
$\mathrm{U}_{1}-\mathrm{U}_{2}=\mathrm{mg}\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)=-\mathrm{mgd}$
Here the negative sign indicates the decrease in PE whch is exacty equal to the gain in KE
From equation (14) and (15)
$\mathrm{mv}_{1}{ }^{2} / 2-\mathrm{mv}_{2}{ }^{2} / 2=\mathrm{U}_{2}-\mathrm{U}_{1}$
or $K E_{1}+P E_{1}=K E_{2}+P E_{2}$
Thus totat energy of the system KE and gravitational PE is conserved. There is a conversion of one form of energy into another during the motion but sum remains same
This can be proved as follows
At height H ,velocity of the object is zero
ie
at $\mathrm{H}=\mathrm{H}, \mathrm{v}=0=>\mathrm{KE}=0$
So KE+PE=mgH
at $\mathrm{H}=\mathrm{h}, \mathrm{v}=\mathrm{v}=>\mathrm{KE}=\mathrm{mv}^{2} / 2=\mathrm{mg}(\mathrm{H}-\mathrm{h})$ and $\mathrm{U}=\mathrm{mgh}$
So KE+PE==mgH
at $\mathrm{H}=0, \mathrm{v}=\mathrm{v}_{\text {max }}=>\mathrm{KE}=\mathrm{mv}_{\max ^{2}}{ }^{2} / 2=\mathrm{mgH}$

## So $\mathrm{KE}+\mathrm{PE}=\mathrm{mgH}$

This shows that total energy of the system is always conserved becuase total energy $\mathrm{KE}+\mathrm{PE}=\mathrm{mgH}$, which remains same as plotted in the figure given below


Hight h $\longrightarrow$
Figure 5. Plots of KE and PE during the free fall of an object of mass $m$

We know conclude that energy can neither be produced nor be destroyed .It can only be transformed from one form to another

## (9) PE of the spring

To study this ,consider an electric spring of negligibly small mass .One end of the spring is attached to the rigid wall and another end of spring is attached to a block of mass $m$ which can move on smooth frictionless horizozntal surface
Consider the figure given below


Figure 6. PE of a spring
Here unstrectched or un compressed position of the spring is taken at $x=0$
We now take the block from its unstrecthed position to a point $P$ by stretching the spring
At this point P restorinng force is exerted by the spring on the block trying it bring it back to the equlibrium position.
Similar restoring force developes in the spring when we try to compress it
For an ideal spring, this restoring force F is proportional to displacement x and direction of restoring force is opposite to that displacement
Thus force and displacement are related as
Fax
or $F=-k x$
where K is called the spring constant and this equation (16) is known as Hook's law.negative sign indicates that force oppose the motion of the block along $x$
To stetch a spring we need to apply the external force which should be equal in magnitude and opposite to the direction of the restoring force mentioned above i.e for stretching the spring

Workdone in both elongation and compression of spring is stored in the spring as its PE which can be easily calculated
If the spring is stretched through a distance $x$ from its equilibrium position $x=0$ then
$W=\int F_{e x t} d x$
Since both $\mathrm{F}_{\text {ext }}$ and dx have same direction Now
$W=\int K x d x$
On integrating with in the limits $x=0$ to $x=x$
We have
$W=K x^{2} / 2$
This workdone is positive as force is towards the right and spring also moves towards the right Same amount of external is done on the spring when it is compressed through a distance $x$ Workdone as calculated in equation (17) is stored as PE of the spring.Therefore
$\mathrm{U}=\mathrm{Kx} \mathrm{K}^{2} / 2$

## (10)Conservation of energy of spring mass system

Again consider the figure (6) where we have stretched the spring to a distance $\mathrm{x}_{0}$ from its equlibrium position $\mathrm{x}=0$
If we release the block then speed of the block begin to increase from zero and reach its maxmimum value at $x=0$ (equilbrium position)
Assuming that there is no dissaption of energy due to air resistance and frictional forces , whole of the PE is coverted in KE
This means gain in KE is exactly equal to the loss in PE
After reaching the equilbrium position at $x=0$,the block then crosses the equilibrium position ,it speed begin to decreass until it reaches the point $x=-x_{0}$

This would happen if whole the conversion between KE and PE is perfect without any dissipaton effect Now at $x=-x_{0}$ PE of the block is maximum ,KE is zero and the restoring force again pulls the block towards its equilibrium position.This way block would keep on oscillating

From all discussion ,we see that total mechanical energy (KE+PE) of the system always remain constant which is equal to $\mathrm{Kx}_{0}{ }^{2} / 2$
We can analyze this as follows

$$
\begin{aligned}
& x=x_{0}: v=0: K E=0: P E=K x_{0}^{2} / 2: T E=K x_{0}^{2} / 2 \\
& x=x: v=v: K E=m v^{2} / 2=k\left(x_{0}^{2}-x^{2}\right) / 2: P E=K x^{2} / 2: T E=K x_{0}^{2} / 2 \\
& x=0: v=v_{\max }: K E=m v_{\max }^{2} / 2=K x_{0}^{2} / 2: P E=0: T E=K x_{0}^{2} / 2 \\
& x=-x_{0}: v=0: K E=0: P E=K x_{0}^{2} / 2: T E=K x_{0}^{2} / 2 \\
& \text { Graph of } K E \text { and } P E \text { are shown below in figure }
\end{aligned}
$$



Figure 7. Graphs for KE and PE of a spring

From above graph we can easily conclude that when KE increases PE decreases and when PE increase KE decrease but total energy of the system remains constant
Above discussed case is the ideal case when other dissipated forces like frictional forces or air resistance are assumed to be absent
However in realty some part of the energy of the system is dissipated due to these resistive forces and after some time system looses all its energy and come to rest

## (11) Conservative Forces

Consider the gravitational force acting on a body .If we try to move this body upwards by applying a force on it then work is done against gravitation
Consider a block of mass $m$ being raised to height $h$ vertically upwards as shown in fig 8(a). Workdone in this case is mgh
Now we make the block travel the path given in figure 8(b) to raise its height h above the ground.In this path workdone during the horizontal motion is zero because there is no change in height of the body due to which there would be no change in gravtitaional PE of the body and if there is no change in the speed KE would also remains same

Thus for fig 8(b) if we add up the workdone in two vertical paths the result we get is equal to mgh


Figure 8. Conservative force:- Work done in raising a block of mass $m$ to height $h$ is independent of the path taken

Again if we move the the block to height h above the floor through an arbitary path as shown in fig 8(c) ,the workdone can be caluctlated by breaking the path into elementary horizontal and vertical portions Now workdone along the horizontal path would be zero and along the vertical paths its add up to mgh Thus we can say that workdone in raising on object against gravity is independent of the path taken and depends only on the intial and final position of the object Now we are in position to define the conservative forces
" If the workdone on particle by a force is independent of how particle moves and depends only on initial and final position of the objects then such a force is called conservative force" Gravitatinal force,electrostatic force ,elastic force and magnetic forces are conservative forces Total workdone by the conservative force is zero when particle moves around any closed path returning to its

Concept of PE is associated with conservative forces only .No such PE is associated with non-conservative forces like frictional forces

## (12) Power

Power is defined as rate of doing the work
if $\Delta \mathrm{W}$ amount of work is done in time interval $\Delta \mathrm{t}$, the instantanous power delivered will be
$\mathrm{P}=\Delta \mathrm{W} / \Delta \mathrm{t}$ or $\mathrm{P}=\mathrm{dW} / \mathrm{dt}$
For total workdone W in total time t,then average power
$P_{\mathrm{avg}}=\mathrm{W} / \mathrm{t}$
If P does not vary with time ,then $\mathrm{P}=\mathrm{P}_{\text {avg }}$
SI unit of power is joule/sec also called watt
Another wunit of power is horsepower(hp)
$1 \mathrm{Hp}=746 \mathrm{watt}$

## (13) Principle of conservation of energy

We have already learned that KE+PE remains constant or conserved in absence of dissipative power As discussed in case of spring mass system in reality dissipative forces like friction and air resistance are always present and some of the energy of the system gets dissipated in the force of heat energy by increasing the internal energy of the spring This continues until system finally comes to rest If one can somehow measure this energy ,the sum of KE,PE and this internal (or energy dissipated) would remain constant .This can be extended to all types of energy
Energy can not be created or destroyed .It can only be transferred from one form to another form.Total energy in a closed system always remains constant
This is the law of conservation of energy although emprical one but it has never been found violated

## WORK ENERGY AND POWER

## WORK

## PHYSICAL DEFINITION

When the point of application of force moves in the direction of the applied force under its effect then work is said to be done.

## MATHEMATICAL DEFINITION OF WORK



Work is defined as the product of force and displacement in the direction of force

$$
\mathbf{W}=\mathrm{F} \times \mathrm{s}
$$



If force and displacement are not parallel to each other rather they are inclined at an angle, then in the evaluation of work component of force (F) in the direction of displacement (s) will be considered.

$$
\begin{aligned}
& W=(F \cos \theta) \times s \\
& W=F s \cos \theta
\end{aligned}
$$

## VECTOR DEFINITION OF WORK



Force and displacement both are vector quantities but their product, work is a scalar quantity, hence work must be scalar product or dot product of force and displacement vector.

$$
w=\vec{F} \cdot \vec{s}
$$

## WORK DONE BY VARIABLE FORCE

## Force varying with displacement

In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.

$$
\begin{aligned}
\mathrm{dW} & =\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{ds}} \\
\mathbf{W} & =\int_{\mathrm{s}_{1}}^{\mathrm{s}_{2}} \overrightarrow{\mathrm{~F}} \cdot \overrightarrow{\mathrm{ds}}
\end{aligned}
$$

Force varying with time
In this condition we consider the force to be constant for any elementary displacement and work done in that elementary displacement is evaluated.

$$
\mathrm{dW}=\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{ds}}
$$

Multiplying and dividing by dt,

$$
\begin{aligned}
& \mathrm{dW}=\frac{\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{ds}} \mathrm{dt}}{\mathrm{dt}} \\
& \mathrm{dW}=\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{v}} \mathrm{dt} \quad(\mathrm{v}=\mathrm{ds} / \mathrm{dt})
\end{aligned}
$$

Total work is obtained by integrating the elementary work from initial to final limits.

$$
\mathbf{W}=\int_{t_{1}}^{t_{2}} \vec{F} \cdot \vec{v} d t
$$

## WORK DONE BY VARIABLE FORCE FROM GRAPH

Let force be the function of displacement \& its graph be as shown.


To find work done from $s_{1}$ to $s_{2}$ we consider two points $M \& N$ very close on the graph such that magnitude of force ( $F$ ) is almost same at both the points. If elementary displacement from M to N is ds, then elementary work done from M to N is.

$$
\begin{aligned}
& \mathrm{dW}=\mathrm{F} . \mathrm{ds} \\
& \mathrm{dW}=\text { (length } x \text { breadth)of strip MNds } \\
& \mathrm{dW}=\text { Area of strip MNds }
\end{aligned}
$$

Thus work done in any part of the graph is equal to area under that part. Hence total work done from $s_{1}$ to $s_{2}$ will be given by the area enclosed under the graph from $\mathrm{s}_{1}$ to $\mathrm{s}_{2}$.

$$
\mathrm{W}=\operatorname{Area}\left(\mathrm{ABS}_{2} \mathrm{~S}_{1} \mathrm{~A}\right)
$$

## DIFFERENT CASES OF WORK DONE BY CONSTANT FORCE

Case i) Force and displacement are in same direction

$$
\theta=0
$$

Since,
$W=F s \operatorname{Cos} \theta$
Therefore
$\mathrm{W}=\mathrm{Fs} \operatorname{Cos} 0$
or,
$\mathbf{W}=\mathrm{Fs}$
Ex-Coolie pushing a load horizontally


Case ii) Force and displacement are mutually perpendicular to each other
$\theta=90$
Since,
$W=F s \operatorname{Cos} \theta$
Therefore
W = Fs Cos 90
or,
$\mathbf{W}=\mathbf{0}$

Ex - coolie carrying a load on his head \& moving horizontally with constant velocity. Then he applies force vertically to balance weight of body \& its displacement is horizontal.


|  | $\theta=180$ |
| :--- | :--- |
| Since, | $W=F s \operatorname{Cos} \theta$ |
| Therefore | $W=F s \operatorname{Cos} 180$ |
| or, | $\mathbf{W}=-$ Fs |



Ex - Coolie carrying a load on his head \& moving vertically down with constant velocity. Then he applies force in vertically upward direction to balance the weight of body \& its displacement is in vertically downward direction.

## ENERGY

Capacity of doing work by a body is known as energy.

Note - Energy possessed by the body by virtue of any cause is equal to the total work done by the body when the cause responsible for energy becomes completely extinct.

## TYPES OF ENERGIES

There are many types of energies like mechanical energy, electrical, magnetic, nuclear, solar, chemical etc.

## MECHANICAL ENERGY

Energy possessed by the body by virtue of which it performs some mechanical work is known as mechanical energy. It is of basically two types-
(i) Kinetic energy
(ii) Potential energy

## KINETIC ENERGY

Energy possessed by body due to virtue of its motion is known as the kinetic energy of the body. Kinetic energy possessed by moving body is equal to total work done by the body just before coming out to rest.


Consider a body of man ( m ) moving with velocity $\left(\mathrm{v}_{\mathrm{o}}\right)$.After travelling through distance (s) it comes to rest.

Applying,

$$
\begin{aligned}
u & =v_{0} \\
v & =0 \\
s & =s \\
v^{2} & =u^{2}+2 a s \\
0 & =v_{0}{ }^{2}+2 a s \\
2 a s & =-v_{0}{ }^{2}
\end{aligned}
$$

or,
or,
Hence force acting on the body,

$$
\begin{aligned}
F & =m a \\
F_{\text {on body }} & =-\frac{m v_{0}{ }^{2}}{2 s}
\end{aligned}
$$

But from Newton's third law of action and reaction, force applied by body is equal and opposite to the force applied on body

$$
\begin{aligned}
F_{\text {by body }} & =-F_{\text {on body }} \\
& =+\frac{m v_{0}^{2}}{2 s}
\end{aligned}
$$

Therefore work done by body,
or,

$$
\begin{aligned}
& W=\overrightarrow{F \cdot} \rightarrow \\
& W=\frac{m v_{0}^{2}}{2 s} \cdot s \cdot \operatorname{Cos} 0 \\
& W=\frac{1}{2} m v_{0}^{2}
\end{aligned}
$$

or,
Thus K.E. stored in the body is,

$$
\text { K.E. }=\frac{1}{2} m v_{o}^{2}
$$

## KINETIC ENERGY IN TERMS OF MOMENTUM

K.E. of body moving with velocity $v$ is

$$
\text { K.E. }=\frac{1}{2} \mathrm{mv}_{0}^{2}
$$

Multiplying and dividing by m

$$
\begin{aligned}
K & =\frac{1 \mathrm{mv}^{2} \mathrm{xm}}{2 \mathrm{~m}} \\
& =\frac{1 \mathrm{~m}^{2} v^{2}}{2 \mathrm{~m}}
\end{aligned}
$$

But, mv = p (linear momentum)
Therefore,

$$
K=\frac{\mathrm{p}^{2}}{2 \mathrm{~m}}
$$

## POTENTIAL ENERGY

Energy possessed by the body by virtue of its position or state is known as potential energy. Example:- gravitational potential energy, elastic potential energy, electrostatic potential energy etc.

## GRAVITATIONAL POTENTIAL ENERGY

Energy possessed by a body by virtue of its height above surface of earth is known as gravitational potential energy. It is equal to the work done by the body situated at some height in returning back slowly to the surface of earth.

Consider a body of mass m situated at height h above the surface of earth. Force applied by the body in vertically downward direction is

$$
\mathrm{F}=\mathrm{mg}
$$

Displacement of the body in coming back slowly to the surface of earth is

$$
s=h
$$

Hence work done by the body is

|  | $W=F s C o s \theta$ |
| :--- | :--- |
| or, | $W=F s C o s 0$ |
| or, | $W=m g h$ |

This work was stored in the body in the form of gravitational potential energy due to its position. Therefore
G.P.E = mgh

## ELASTIC POTENTIAL ENERGY

Energy possessed by the spring by virtue of compression or expansion against elastic force in the spring is known as elastic potential energy.

## Spring

It is a coiled structure made up of elastic material \& is capable of applying restoring force \& restoring torque when disturbed from its original state. When force ( $F$ ) is applied at one end of the string, parallel to its length, keeping the other end fixed, then the spring expands (or contracts) \& develops a restoring force $\left(F_{R}\right)$ which balances the applied force in equilibrium.

On increasing applied force spring further expands in order to increase restoring force for balancing the applied force. Thus restoring force developed within the spring is directed proportional to the extension produced in the spring.


## $F_{R} \propto x$

or, $\mathbf{F}_{\mathbf{R}}=\mathbf{k X}$ (k is known as spring constant or force constant)

If $x=1, F_{R}=k$
Hence force constant of string may be defined as the restoring force developed within spring when its length is changed by unity.

But in equilibrium, restoring force balances applied force.

$$
F=F_{R}=k x
$$

If $x=1, F=1$
Hence force constant of string may also be defined as the force required to change its length by unity in equilibrium.

## Mathematical Expression for Elastic Potential Energy



Consider a spring of natural length ' $L$ ' \& spring constant ' $k$ ' its length is increased by $x_{0}$. Elastic potential energy of stretched spring will be equal to total work done by the spring in regaining its original length.

If in the process of regaining its natural length, at any instant extension in the spring was $x$ then force applied by spring is

$$
F=k x
$$

If spring normalizes its length by elementary distance dx opposite to x under this force then work done by spring is

$$
\mathrm{dW}=\mathrm{F} \cdot(-\mathrm{dx}) \cdot \operatorname{Cos} 0
$$

(force applied by spring F and displacement -dx taken opposite to extension x are in same direction)

$$
\mathrm{dW}=-\mathrm{kxdx}
$$

Total work done by the spring in regaining its original length is obtained in integrating dW from $\mathrm{x}_{0}$ to 0

$$
\begin{array}{ll} 
& W=\int_{x_{0}}^{0}-k x d x \\
\text { or, } & W=-k\left[x^{2} / 2\right]_{0}^{x_{0}} \\
\text { or, } & W=-k\left(0^{2} / 2-x_{0}^{2} / 2\right) \\
\text { or, } & W=-k\left(0-x_{0}^{2} / 2\right) \\
\text { or, } & W=\frac{1}{2} k x_{0}^{2}
\end{array}
$$

This work was stored in the body in the form of elastic potential energy.

$$
\text { E.P.E }=\frac{1}{2} k x_{0}^{2}
$$

## WORK ENERGY THEOREM

It states that total work done on the body is equal to the change in kinetic energy.(Provided body is confined to move horizontally and no dissipating forces are operating).


Consider a body of man moving with initial velocity $\mathrm{v}_{1}$. After travelling through displacement $s$ its final velocity becomes $v_{2}$ under the effect of force $F$.

$$
\begin{aligned}
& u=v_{1} \\
& \mathrm{v}=\mathrm{v}_{2} \\
& \mathrm{~s}=\mathrm{s} \\
& \mathrm{v}^{2}=\mathrm{u}^{2}+2 \mathrm{as} \\
& \mathrm{v}_{2}{ }^{2}=\mathrm{v}_{1}{ }^{2}+2 \mathrm{as} \\
& 2 \mathrm{as}=\mathrm{v}_{2}^{2}-\mathrm{v}_{1}^{2} \\
& \mathrm{a}=\underline{v}_{2}^{2}-\mathrm{v}_{1}^{2} \\
& 2 \mathrm{~s}
\end{aligned}
$$

Applying,
or,
or,
Hence external force acting on the body is

$$
\begin{aligned}
& \mathrm{F}=\mathrm{ma} \\
& \mathrm{~F}=\mathrm{m} \underline{\mathrm{v}}_{2}{\frac{2}{2}-\mathrm{v}_{1}{ }^{2}}^{2}
\end{aligned}
$$

Therefore work done on body by external force
or,

$$
\begin{aligned}
& W=\vec{F} \cdot \vec{s} \\
& W=m \underline{v}_{2} \underline{\underline{2}}_{2 s}^{2 s}{ }^{2} \cdot s \cdot \operatorname{Cos} 0
\end{aligned}
$$

or,

$$
\begin{aligned}
& \mathrm{W}=\frac{1}{2} \mathrm{mv}_{2}^{2}-\frac{1}{2} \mathrm{mv}_{1}^{2} \\
& \mathrm{~W}=\mathrm{K}_{2}-\mathrm{K}_{1}
\end{aligned}
$$

or,
or,
$\mathbf{W}=\Delta \mathbf{K}$

## PRINCIPLE OF CONSERVATION OF ENERGY



It states that energy can neither be creased neither be destroyed. It can only be converted from one form to another.
Consider a body of man $m$ situated at height $h$ \& moving with velocity $\mathrm{v}_{\mathrm{o}}$. Its energy will be

$$
\mathrm{E}_{1}=\mathrm{P}_{1}+\mathrm{K}_{1}
$$

or,

$$
E_{1}=m g h+1 / 2 m v_{0}^{2}
$$

If the body falls under gravity through distance $y$, then it acquires velocity $\mathrm{v}_{1}$ and its height becomes ( h -y)
$\mathrm{u}=\mathrm{v}$ 。
$s=y$
$\mathrm{a}=\mathrm{g}$
$\mathrm{v}=\mathrm{v}_{1}$
From

$$
\begin{aligned}
& v^{2}=u^{2}+2 a s \\
& v_{1}{ }^{2}=v_{0}{ }^{2}+2 g y
\end{aligned}
$$

Energy of body in second situation

$$
E_{2}=P_{2}+K_{2}
$$

or,

$$
E_{2}=m g(h-y)+1 / 2 m v^{2}
$$

or,

$$
E_{2}=m g(h-y)+1 / 2 m\left(v_{0}^{2}+2 g y\right)
$$

$$
\mathrm{E}_{2}=\mathrm{mgh}-\mathrm{mgy}+1 / 2 m v_{0}{ }^{2}+\mathrm{mgy}
$$

or,

$$
E_{2}=m g h+1 / 2 m v_{0}^{2}
$$

Now we consider the situation when body reaches ground with velocity $\mathrm{v}_{2}$
$\mathrm{u}=\mathrm{v}$ 。
$\mathrm{s}=\mathrm{h}$
$\mathrm{a}=\mathrm{g}$
$\mathrm{v}=\mathrm{v}_{2}$

From

Energy of body in third situation

|  | $\mathrm{E}_{3}=\mathrm{P}_{3}+\mathrm{K}_{3}$ |
| :--- | :--- |
| or, | $\mathrm{E}_{3}=\mathrm{mg} 0+1 / 2 \mathrm{mv}_{2}{ }^{2}$ |
| or, | $\mathrm{E}_{3}=0+1 / 2 \mathrm{~m}\left(\mathrm{v}_{0}{ }^{2}+2 \mathrm{gh}\right)$ |
| or, | $\mathrm{E}_{3}=1 / 2 \mathrm{mv}_{\mathrm{o}}{ }^{2}+\mathbf{m g h}$ |

From above it must be clear that $\mathbf{E}_{1}=E_{2}=E_{3}$. This proves the law of conservation of energy.

## CONSERVATIVE FORCE

Forces are said to be conservative in nature if work done against the forces gets conversed in the body in form of potential energy. Example:gravitational forces, elastic forces \& all the central forces.

## PROPERTIES OF CONSERVATIVE FORCES

1. Work done against these forces is conserved \& gets stored in the body in the form of P.E.
2. Work done against these forces is never dissipated by being converted into nonusable forms of energy like heat, light, sound etc.
3. Work done against conservative forces is a state function \& not path function i.e. Work done against it, depends only upon initial \& final states of body \& is independent of the path through which process has been carried out.
4. Work done against conservative forces is zero in a complete cycle.

## TO PROVE WORK DONE AGAINST CONSERVATIVE FORCES IS A STATE FUNCTION

Consider a body of man $m$ which is required to be lifted up to height $h$. This can be done in 2 ways.
(i) By directly lifting the body against gravity
(ii) By pushing the body up a smooth inclined plane.

Min force required to lift the body of mass $m$ vertically is

$$
\mathrm{F}=\mathrm{mg}
$$

And displacement of body in lifting is

$$
s=h
$$

Hence work done in lifting is

$$
\begin{aligned}
& \mathrm{W}_{1}=\mathrm{FsCos} 0^{\circ}{ }_{\text {(since force and displacement are in same direction) }} \mathrm{mg} \\
& \mathbf{W}_{1}=\mathbf{m g h}
\end{aligned}
$$

Now we consider the same body lifted through height $h$ by pushing it up a smooth inclined plane


Min force required to push the body is

$$
F=m g \operatorname{Sin} \theta
$$

And displacement of body in lifting is

$$
s=\frac{h}{\underline{\sin \theta}}
$$

Hence work done in pushing is

$$
\begin{aligned}
& \mathrm{W}_{2}=\mathrm{FsCos} 0 \\
& \mathrm{~W}_{2}=\mathrm{mg} \operatorname{Sin} \theta \cdot \frac{\mathrm{~h}}{\operatorname{Sin} \theta} \cdot 1 \\
& \mathbf{W}_{2}=\mathbf{m g h}
\end{aligned}
$$

or,

From above $\mathrm{W}_{1}=\mathrm{W}_{2}$ we can say that in both the cases work done in lifting the body through height ' h ' is same.

## To Prove That Work Done Against Conservative Forces Is Zero In A Complete Cycle



Consider a body of man m which is lifted slowly through height h \& then allowed to come back to the ground slowly through height $h$.

For work done is slowly lifting the body up, Minimum force required in vertically upward direction is

$$
F=m g
$$

Vertical up displacement of the body is

$$
s=h
$$

Hence work done is

|  | $\mathrm{W}=\mathrm{Fs} \operatorname{Cos} \theta$ |
| :---: | :---: |
| or, | $\mathrm{W}_{1}=\mathrm{FsCos} 0$ (since force and displacement are in same direction) |
| or, | $\mathbf{W}_{\mathbf{I}}=\mathbf{m g h} \mathbf{( s i n c e ~ f o r c e ~ a n d ~ d i s p l a c e m e n t ~ a r e ~ i n ~ s a m e ~ d i r e c t i o n ) ~}^{\text {a }}$ |
|  |  |
| Minimum force required in vertically upward direction is |  |
|  |  |
| Vertical down displacement of the body is |  |
|  | $s=h$ |
| Hence work done is |  |
| or, | $\mathrm{W}_{2}=\mathrm{FsCos} 180$ (since force and displacement are in opposite direction) |
| or, $\quad \mathrm{W}_{2}=-\mathrm{mgh}$ |  |
| Hence total work done against conservative forces in a complete cycle is |  |
| or, | $\mathrm{W}=\mathrm{W}_{1}+\mathrm{W}_{2}$ |
|  | $\mathrm{W}=(\mathrm{mgh})+(-\mathrm{mgh})$ |
| or, | w $=0$ |

## NON-CONSERVATIVE FORCES

Non conservative forces are the forces, work done against which does not get conserved in the body in the form of potential energy.

## PROPERTIES OF NON-CONSERVATIVE FORCES

1. Work done against these forces does not get conserved in the body in the form of P.E.
2. Work done against these forces is always dissipated by being converted into non usable forms of energy like heat, light, sound etc.
3. Work done against non-conservative force is a path function and not a state function.
4. Work done against non-conservative force in a complete cycle is not zero.

## PROVE THAT WORK DONE AGAINST NON-CONSERVATIVE FORCES IS A PATH FUNCTION

Consider a body of mass ( m ) which is required to be lifted to height ' h ' by pushing it up the rough incline of inclination.


Minimum force required to slide the body up the rough inclined plane having coefficient of kinetic friction $\mu$ with the body is

$$
F=m g \operatorname{Sin} \theta+f_{k}
$$

or,

$$
\begin{aligned}
& F=m g \operatorname{Sin} \theta+\mu N \\
& F=m g \operatorname{Sin} \theta+\mu m g \operatorname{Cos} \theta \\
& \text { incline in moving throunh }
\end{aligned}
$$

or,

$$
\text { incline in moving through height } h \text { is }
$$

$$
s=\frac{h}{\operatorname{Sin} \theta}
$$

Hence work done in moving the body up the incline is

$$
\mathrm{W}=\mathrm{F} . \mathrm{s} . \operatorname{Cos} 0_{\text {(since force and displacement are in opposite direction) }}
$$

or,
or,

$$
W=(m g \operatorname{Sin} \theta+\mu m g \operatorname{Cos} \theta) \cdot \frac{h}{\operatorname{Sin} \theta} \cdot 1
$$

$$
W=m g h+\frac{\mu m g h}{\operatorname{Tan} \theta}
$$

Similarly if we change the angle of inclination from $\theta$ to $\theta_{1}$, then work done will be

$$
W_{1}=m g h+\frac{\mu m g h}{\operatorname{Tan} \theta_{1}}
$$

This clearly shows that work done in both the cases is different \& hence work done against non-conservative force in a path function and not a state function i.e. it not only depends upon initial \& final states of body but also depends upon the path through which process has been carried out.

## To Prove That Work Done Against Non-conservative Forces In A Complete Cycle Is Not Zero

Consider a body displaced slowly on a rough horizontal plane through displacement $s$ from $A$ to $B$.


Minimum force required to move the body is

$$
\mathrm{F}=\mathrm{f}_{\mathrm{k}}=\mu \mathrm{N}=\mu \mathrm{mg}
$$

Work done by the body in displacement s is $\mathrm{W}=\mathrm{F} . \mathrm{s} . \operatorname{Cos} 0_{\text {(since force and displacement are in same direction) }}$
or,
$\mathrm{W}=\boldsymbol{\mu} \mathrm{mgs}$
Now if the same body is returned back from B to A


Minimum force required to move the body is $\mathrm{F}=\mathrm{f}_{\mathrm{k}}=\mu \mathrm{N}=\mu \mathrm{mg}$
Work done by the body in displacement s is

$$
\mathrm{W}=\mathrm{F} . \mathrm{s} . \operatorname{Cos} 0 \text { (since force and displacement are in same direction) }
$$

or,

$$
\mathrm{W}=\mu \mathrm{mgs}
$$

Hence total work done in the complete process

$$
\mathrm{W}=\mathrm{W}_{1}+\mathrm{W}_{2}=2 \mu \mathrm{mgs}
$$

Note - When body is returned from B to A friction reverse its direction.

## POWER

Rate of doing work by a body with respect to time is known as power.

## Average Power

It is defined as the ratio of total work done by the body to total time taken.

$$
P_{\text {avg }}=\frac{\text { Total work done }}{\text { Total time taken }}=\frac{\Delta \mathbf{W}}{\Delta \mathbf{t}}
$$

## Instantaneous Power

Power developed within the body at any particular instant of time is known as instantaneous power.

Or
Average power evaluated for very short duration of time is known as instantaneous power.

$$
P_{\text {inst }}=\operatorname{Lim}_{\Delta t \rightarrow 0} P_{\text {avg }}
$$

or,

$$
P_{\text {inst }}=\operatorname{Lim}_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}
$$

$$
\begin{array}{ll} 
& P_{\text {inst }}=\frac{d W}{d t} \\
\text { or, } & P_{\text {inst }}=\frac{\mathrm{dP} \cdot \overrightarrow{\mathrm{~s}}}{\mathrm{dt}} \\
\text { or, } & P_{\text {inst }}=\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{ds}} \\
\text { or, } & P_{\text {inst }}=\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathbf{v}}
\end{array}
$$

## EFFICIENCY

It is defined as the ratio of power output to power input.
Or
It is defined as the ratio of energy output to energy input.
Or
I It is defined as the ratio of work output to work input.

$$
\eta=\frac{P_{\text {output }}}{P_{\text {Input }}}=\frac{E_{\text {output }}}{E_{\text {Input }}}=\frac{W_{\text {Ooutput }}}{W_{\text {Input }}}
$$

## PERCENTAGE EFFICIENCY

Percentage Efficiency = Efficiency x 100

$$
\text { Percentage Efficiency }=\eta=\frac{P_{\text {Ooutput }}}{P_{\text {Input }}}=\frac{E_{\text {Ooutput }}}{E_{\text {Input }}}=\frac{\mathbf{W}_{\text {Output }}}{W_{\text {Input }}} \times 100
$$

## COLLISION

Collision between the two bodies is defined as mutual interaction of the bodies for a short interval of time due to which the energy and the momentum of the interacting bodies change.

## Types of Collision

There are basically three types of collisions-
i) Elastic Collision - That is the collision between perfectly elastic bodies. In this type of collision, since only conservative forces are operating between the interacting bodies, both kinetic energy and momentum of the system remains constant.
ii) Inelastic Collision - That is the collision between perfectly inelastic or plastic bodies. After collision bodies stick together and move with some common velocity. In this type of collision only momentum is conserved. Kinetic energy is not conserved due to the presence of non-conservative forces between the interacting bodies.
iii) Partially Elastic or Partially Inelastic Collision - That is the collision between the partially elastic bodies. In this type of collision bodies do separate from each other after collision but due to the involvement of non-conservative inelastic forces kinetic energy of the system is not conserved and only momentum is conserved.

## Collision In One Dimension - Analytical Treatment






Consider two bodies of masses $m_{1}$ and $m_{2}$ with their center of masses moving along the same straight line in same direction with initial velocities $u_{1}$ and $u_{2}$ with $m_{1}$ after $m_{2}$. Condition necessary for the collision is $u_{1}>u_{2}$ due to which bodies start approaching towards each other with the velocity of approach $u_{1}-u_{2}$.
Collision starts as soon as the bodies come in contact. Due to its greater velocity and inertia $m_{1}$ continues to push $m_{2}$ in the forward direction whereas $m_{2}$ due to its small velocity and inertia pushes $m_{1}$ in the backward direction. Due to this pushing force involved between the two colliding bodies they get deformed at the point of contact and a part of their kinetic energy gets consumed in the deformation of the bodies. Also $\mathrm{m}_{1}$ being pushed opposite to the direction of the motion goes on decreasing its velocity and $m_{2}$ being pushed in the direction of motion continues increasing its velocity. This process continues until both the bodies acquire the same common velocity v . Up to this stage there is maximum deformation in the bodies maximum part of their kinetic energy gets consumed in their deformation.

## Elastic collision



In case of elastic collision bodies are perfectly elastic. Hence after their maximum deformation they have tendency to regain their original shapes, due to which they start pushing each other. Since $m_{2}$ is being pushed in the direction of motion its velocity goes on increasing and $m_{1}$ being pushed opposite to the direction of motion its velocity goes on decreasing. Thus condition necessary for separation i.e. $\mathbf{v}_{\mathbf{2}}>\mathbf{v}_{\mathbf{1}}$ is attained and the bodies get separated with velocity of separation $\mathbf{v}_{\mathbf{2}}-\mathbf{v}_{\mathbf{1}}$.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is again returned back to the system when the bodies regain their original shapes. Hence in such collision energy conservation can also be applied along with the momentum conservation.
Applying energy conservation

$$
\begin{align*}
E_{i} & =E_{f} \\
\frac{1}{2} m_{1} u_{1}^{2}+\frac{1}{2} m_{2} u_{2}^{2} & =\frac{1}{2} m_{1} v_{1}^{2}+\frac{1}{2} m_{2} v_{2}^{2} \\
m_{1}\left(u_{1}^{2}-v_{1}^{2}\right) & =m_{2}\left(v_{2}^{2}-u_{2}^{2}\right)  \tag{i}\\
m_{1}\left(u_{1}-v_{1}\right)\left(u_{1}+v_{1}\right) & =m_{2}\left(v_{2}-u_{2}\right)\left(v_{2}+u_{2}\right)
\end{align*}
$$

Applying momentum conservation

$$
\begin{align*}
p_{i} & =p_{\mathrm{f}} \\
m_{1} u_{1}+m_{2} u_{2} & =m_{1} v_{1}+m_{2} v_{2} \\
m_{1}\left(u_{1}-v_{1}\right) & =m_{2}\left(v_{2}-u_{2}\right) . \tag{ii}
\end{align*}
$$

Dividing equation (i) by (ii)

$$
u_{1}+v_{1}=v_{2}+u_{2}
$$

or, $\quad \mathbf{v}_{\mathbf{2}}-\mathbf{v}_{\mathbf{1}}=\mathbf{u}_{\mathbf{1}}-\mathbf{u}_{\mathbf{2}}$
$\begin{aligned} \text { or, } & \text { Velocity of separation } \\ \text { or, } & v_{2}=v_{1}+u_{1}-u_{2}\end{aligned}$
Putting this in equation (i)

$$
v_{1}=\frac{\left(m_{1}-m_{2}\right) u_{1}}{\left(m_{1}+m_{2}\right)}+\frac{2 m_{2} u_{2}}{\left(m_{1}+m_{2}\right)}
$$

Similarly we can prove

$$
v_{2}=\frac{\left(m_{2}-m_{1}\right) u_{2}}{\left(m_{1}+m_{2}\right)}+\frac{2 m_{1} u_{1}}{\left(m_{1}+m_{2}\right)}
$$

Case 1- If the bodies are of same mass,

$$
\begin{aligned}
\mathrm{m}_{1}=\mathrm{m}_{2} & =\mathrm{m} \\
\mathbf{v}_{\mathbf{1}} & =\mathbf{u}_{\mathbf{2}} \\
\mathbf{v}_{\mathbf{2}} & =\mathbf{u}_{\mathbf{1}}
\end{aligned}
$$

Hence in perfectly elastic collision between two bodies of same mass, the velocities interchange.ie. If a moving body elastically collides with a similar body at rest. Then the moving body comes at rest and the body at rest starts moving with the velocity of the moving body.

Case 2- If a huge body elastically collides with the small body, $\mathrm{m}_{1} \gg \mathrm{~m}_{2}$
$m_{2}$ will be neglected in comparison to $m_{1}$

$$
\begin{aligned}
& v_{1}=\frac{\left(m_{1}-0\right) u_{1}}{\left(m_{1}+0\right)}+\frac{2.0 \cdot u_{2}}{\left(m_{1}+0\right)} \\
& \mathbf{v}_{\mathbf{1}}=u_{1}
\end{aligned}
$$

and

$$
\begin{aligned}
& \mathrm{v}_{2}=\frac{\left(0-m_{1}\right) \mathrm{u}_{2}+\frac{2 \mathrm{~m}_{1} \mathrm{u}_{1}}{\left(\mathrm{~m}_{1}+0\right)}}{\left(\mathrm{m}_{1}+0\right)} \\
& \mathbf{v}_{2}=-\mathrm{u}_{2}+2 \mathrm{u}_{1} \\
& \mathbf{v}_{2}=2 \mathrm{u}_{1}
\end{aligned}
$$

If, $u_{2}=0$
Hence if a huge body elastically collides with a small body then there is almost no change in the velocity of the huge body but if the small body is initially at rest it gets thrown away with twice the velocity of the huge moving body.eg. collision of truck with a drum.

Case 3- If a small body elastically collides with a huge body, $m_{2} \gg m_{1}$
$m_{1}$ will be neglected in comparison to $m_{2}$

$$
\mathrm{v}_{1}=\frac{\left(0-\mathrm{m}_{2}\right) u_{1}}{\left(0+\mathrm{m}_{2}\right)}+\frac{2 \mathrm{~m}_{2} \mathrm{u}_{2}}{\left(0+\mathrm{m}_{2}\right)}
$$

or,
If

$$
\begin{aligned}
& \mathrm{v}_{\mathbf{1}}=-\mathrm{u}_{1}+2 \mathrm{u}_{2} \\
& \mathrm{u}_{2}=0 \\
& \mathrm{v}_{\mathbf{1}}=-\mathrm{u}_{1}
\end{aligned}
$$

and

$$
\begin{aligned}
& \mathrm{v}_{2}=\frac{\left(m_{2}-0\right) \mathrm{u}_{2}}{\left(0+\mathrm{m}_{2}\right)}+\frac{2 \cdot 0 \cdot \mathrm{u}_{1}}{\left(0+\mathrm{m}_{2}\right)} \\
& \mathbf{v}_{\mathbf{2}}=\mathrm{u}_{\mathbf{2}}
\end{aligned}
$$

Hence if a small body elastically collides with a huge body at rest then there is almost no change in the velocity of the huge body but if the huge body is initially at rest small body rebounds back with the same speed.eg. collision of a ball with a wall.

## Inelastic collision

In case of inelastic collision bodies are perfectly inelastic. Hence after their maximum deformation they have no tendency to regain their original shapes, due to which they continue moving with the same common velocity.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is permanently consumed in the deformation of the bodies against non-conservative inelastic forces. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.
Applying momentum conservation

$$
\begin{aligned}
& p_{i}=p_{f} \\
\text { or, } \quad & m_{1} u_{1}+m_{2} u_{2}
\end{aligned}=m_{1} v+m_{2} v .
$$

## Partially Elastic or Partially Inelastic Collision

In this case bodies are partially elastic. Hence after their maximum deformation they have tendency to regain their original shapes but not as much as perfectly elastic bodies. Hence they do separate but their velocity of separation is not as much as in the case of perfectly elastic bodies i.e. their velocity of separation is less than the velocity of approach.

In such collision the part of kinetic energy of the bodies which has been consumed in the deformation of the bodies is only slightly returned back to the system. Hence in such collision energy conservation can-not be applied and only momentum conservation is applied.

$$
\left(v_{2}-v_{1}\right)<\left(u_{1}-u_{2}\right)
$$

## Collision In Two Dimension - Oblique Collision



Before Collision


Collision Starts


After Collision

When the centers of mass of two bodies are not along the same straight line, the collision is said to be oblique. In such condition after collision bodies are deflected at some angle with the initial direction. In this type of collision momentum conservation is applied separately along x-axis and y-axis. If the collision is perfectly elastic energy conservation is also applied.

Let initial velocities of the masses $m_{1}$ and $m_{2}$ be $u_{1}$ and $u_{2}$ respectively along x-axis. After collision they are deflected at angles $\theta$ and $\varnothing$ respectively from $x$-axis, on its either side of the $x$ axis.

Applying momentum conservation along $x$-axis

$$
p_{f}=p_{i}
$$

$$
m_{1} v_{1} \operatorname{Cos} \theta+m_{2} v_{2} \operatorname{Cos} \varnothing=m_{1} u_{1}+m_{2} u_{2}
$$

Applying momentum conservation along y-axis

$$
\begin{aligned}
p_{\mathrm{f}} & =p_{i} \\
m_{1} v_{1} \operatorname{Sin} \theta-m_{2} \mathbf{v}_{2} \operatorname{Sin} \varnothing & =m_{1} 0+m_{2} 0 \\
m_{1} v_{1} \operatorname{Sin} \theta-m_{2} v_{2} \operatorname{Sin} \varnothing & =0 \\
\mathbf{m}_{\mathbf{1}} \mathbf{v}_{\mathbf{1}} \operatorname{Sin} \theta & =\mathbf{m}_{\mathbf{2}} \mathbf{v}_{\mathbf{2}} \operatorname{Sin} \varnothing
\end{aligned}
$$

$$
\text { or, } \quad m_{1} v_{1} \operatorname{Sin} \theta-m_{2} v_{2} \operatorname{Sin} \varnothing=0
$$

or,
In case of elastic collision applying energy conservation can also be applied

$$
\mathrm{K}_{\mathrm{f}}=\mathrm{K}_{\mathrm{i}}
$$

$$
\frac{1}{2} m_{1} u_{1}^{2}+\frac{1}{2} m_{2} u_{2}^{2}=\frac{1}{2} m_{1} v_{1}^{2}+\frac{1}{2} m_{2} v_{2}^{2}
$$

## Coefficient Of Restitution

It is defined as the ratio of velocity of separation to the
velocity of approach.

$$
e=\frac{\text { Velocity of separation }}{\text { Velocity of approach }}
$$

$$
e=\frac{\left(v_{2}-v_{1}\right)}{\left(u_{1}-u_{2}\right)}
$$

Case-1 For perfectly elastic collision, velocity of separation is equal to velocity of approach, therefore

$$
e=1
$$

Case-2 For perfectly inelastic collision, velocity of separation is zero, therefore e=0
Case-3 For partially elastic or partially inelastic collision, velocity of separation is less than velocity of approach, therefore

```
e<1
```


## MEMORY MAP



## Very Short Answer Type 1 Mark Questions

1. Define the conservative and non-conservative forces? Give example of each?
2. A light body and a heavy body have same linear momentum. Which one has greater K.E?
(Ans: Lighter body has more K.E.)
3.If the momentum of the body is doubled by what percentage does its K.E changes?
(300\%)
3. A truck and a car are moving with the same K.E on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance?
4. What happens to the P.E of a bubble when it rises up in water?
5. Define spring constant of a spring?
6. What happens when a sphere collides head on elastically with a sphere of same mass initially at rest?
7. Derive an expression for K.E of a body of mass moving with a velocity v by calculus method.
8. After bullet is fired, gun recoils. Compare the K.E. of bullet and the gun.
(K.E. of bullet > K.E. of gun)
9. In which type of collision there is maximum loss of energy?

## Very Short Answer Type 2 Marks Questions

1. A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m . If due to air resistance loss of energy is $10 \%$ what is the speed with which the bob arrives the lowest point?
(Ans : 6m/s)
2. Find the work done if a particle moves from position $\overrightarrow{r_{1}}=(4 i+3 j+6 k) m$ to $a$ position $\overrightarrow{r_{2}}=(14 i=13 j=16 k)$ under the effect of force, $\overrightarrow{F=}(4 i+4 j-4 k) N$ ?
(Ans: 40J)
3. 20 J work is required to stretch a spring through 0.1 m . Find the force constant of the spring. If the spring is stretched further through 0.1 m calculate work done?
(Ans : $4000 \mathrm{Nm}-1,60 \mathrm{~J}$ )
4. A pump on the ground floor of a building can pump up water to fill a tank of volume $30 \mathrm{~m}^{3}$ in 15 min . If the tank is 40 m above the ground, how much electric power is consumed by the pump? The efficiency of the pump is $30 \%$.
(Ans : 43.556 kW )
5. Spring of a weighing machine is compressed by 1 cm when a sand bag of mass 0.1 kg is dropped on it from a height 0.25 m . From what height should the sand bag be dropped to cause a compression of 4 cm ?
(Ans: 4m)
6. Show that in an elastic one dimensional collision the velocity of approach before collision is equal to velocity of separation after collision?
7. A spring is stretched by distance $x$ by applying a force $F$. What will be the new force required to stretch the spring by $3 x$ ? Calculate the work done in increasing the extension?
8. Write the characteristics of the force during the elongation of a spring. Derive the relation for the P.E. stored when it is elongated by length. Draw the graphs to show the variation of potential energy and force with elongation?
9. How does a perfectly inelastic collision differ from perfectly elastic collision? Two particles of mass $m_{1}$ and $m_{2}$ having velocities $u_{1}$ and $u_{2}$ respectively make a head on collision. Derive the relation for their final velocities?
10. In lifting a 10 kg weight to a height of $2 \mathrm{~m}, 250$ Joule of energy is spent. Calculate the acceleration with which it was raised? $\left(\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)$
(Ans : $2.5 \mathrm{~m} / \mathrm{s}^{2}$ )

## Short Answer Type 3 Marks Questions

1. An electrical water pump of $80 \%$ efficiency is used to lift water up to a height of 10 m . Find mass of water which it could lift in 1 hrour if the marked power was 500 watt?
2. A cycle is moving up the incline rising 1 in 100 with a const. velocity of $5 \mathrm{~m} / \mathrm{sec}$. Find the instantaneous power developed by the cycle?
3. Find \% change in K.E of body when its momentum is increased by $50 \%$.
4. A light string passing over a light frictionless pulley is holding masses m and 2 m at its either end. Find the velocity attained by the masses after 2 seconds.
5. Derive an expression for the centripetal force experienced by a body performing uniform circular motion.
6 . Find the elevation of the outer tracks with respect to inner. So that the train could safely pass through the turn of radius 1 km with a speed of $36 \mathrm{~km} / \mathrm{hr}$. Separation between the tracks is 1.5 m ?
6. A block of mass $m$ is placed over a smooth wedge of inclination $\theta$. With what horizontal acceleration the wedge should be moved so that the block must remain stationery over it?
7. Involving friction prove that pulling is easier than pushing if both are done at the same angle.
8. In vertical circular motion if velocity at the lowermost point is $\sqrt{ }(6 \mathrm{rg})$ where find the tension in the string where speed is minimum. Given that mass of the block attached to it is m ?
9. A bullet of mass $m$ moving with velocity $u$ penetrates a wooden block of mass $M$ suspended through a string from rigid support and comes to rest inside it. If length of the string is $L$ find the angular deflection of the string.

## Long Answer Type 5 Marks Questions

1. What is conservative force? Show that work done against conservative forces is a state function and not a path function. Also show that work done against it in a complete cycle is zero?
2. A body of man 10 kg moving with the velocity of $10 \mathrm{~m} / \mathrm{s}$ impinges the horizontal spring of spring constant $100 \mathrm{Nm}^{-1}$ fixed at one end. Find the maximum compression of the spring? Which type of mechanical energy conversion has occurred? How does the answer in the first part changes when the body is moving on a rough surface?
3. Two blocks of different masses are attached to the two ends of a light string passing over the frictionless and light pully. Prove that the potential energy of the bodies lost during the motion of the blocks is equal to the gain in their kinetic energies?
4. A locomotive of mass m starts moving so that its velocity v is changing according to the law $\mathrm{v} \sqrt{ }(\mathrm{as})$, where a is constant and s is distance covered. Find the total work done by all the forces acting the locomotive during the first $t$ seconds after the beginning of motion?
5. Derive an expression for the elastic potential energy of the stretched spring of spring constant k . Find the \% change in the elastic potential energy of spring if its length is increased by $10 \%$ ?

## Some Intellectual Stuff

1. A body of mass $m$ is placed on a rough horizontal surface having coefficient of static friction $\mu$ with the body. Find the minimum force that must be applied on the body so that it may start moving? Find the work done by this force in the horizontal displacement s of the body?
2. Two blocks of same mass $m$ are placed on a smooth horizontal surface with a spring of constant $k$ attached between them. If one of the block is imparted a horizontal velocity v by an impulsive force, find the maximum compression of the spring?
3. A block of mass $M$ is supported against a vertical wall by a spring of constant $k$. A bullet of mass m moving with horizontal velocity $\mathrm{v}_{0}$ gets embedded in the block and pushes it against the wall. Find the maximum compression of the spring?
4. Prove that in case of oblique elastic collision of a moving body with a similar body at rest, the two bodies move off perpendicularly after collision?
5. A chain of length $L$ and mass $M$ rests over a sphere of radius $R(L<R)$ with its one end fixed at the top of the sphere. Find the gravitational potential energy of the chain considering the center of the sphere as the zero level of the gravitational potential energy?
