Chapter 7

7.1 (d)

7.2 (c)

7.3 The initial velocity is $\mathbf{v}_i = v\hat{\mathbf{e}}_y$ and, after reflection from the wall, the final velocity is $\mathbf{v}_f = -v\hat{\mathbf{e}}_y$. The trajectory is described as $\mathbf{r} = y\hat{\mathbf{e}}_y + a\hat{\mathbf{e}}_z$. Hence the change in angular momentum is $\mathbf{r} \times m(\mathbf{v}_f - \mathbf{v}_i) = 2mva\hat{\mathbf{e}}_x$. Hence the answer is (b).

7.4 (d)

7.5 (b)

7.6 (c)

7.7 When b $\rightarrow 0$, the density becomes uniform and hence the centre of mass is at $x = 0.5$. Only option (a) tends to 0.5 as b $\rightarrow$ 0.

7.8 (b) ω

7.9 (a), (c)

7.10 (a), (d)

7.11 All are true.

7.12 (a) False, it is along $\hat{\mathbf{k}}$.

(b) True

(c) True

(d) False, there is no sense in adding torques about 2 different axes.

7.13 (a) False, perpendicular axis theorem is applicable only to a lamina.

(b) True

(c) False, z and z" are not parallel axes.

(d) True.

7.14 When the vertical height of the object is very small as compared to earth's radius, we call the object small, otherwise it is extended.

(a) Building and pond are small objects.

(b) A deep lake and a mountain are examples of extended objects.

7.15 $I = \sum m_i r_i^2$. All the mass in a cylinder lies at distance R from the axis of symmetry but most of the mass of a solid sphere lies at a smaller distance than R.

7.16 Positive slope indicates anticlockwise rotation which is traditionally taken as positive.

7.17 (a) ii, (b) iii, (c) i, (d) iv

7.18 (a) iii, (b) iv (c) ii (d) i.

7.19 No. Given $\sum_i \mathbf{F}_i \neq 0$

The sum of torques about a certain point '0

$\sum_i \mathbf{r}_i \times \mathbf{F}_i = 0$

The sum of torques about any other point O′,

$\sum_i (\mathbf{r}_i - \mathbf{a}) \times \mathbf{F}_i = \sum_i \mathbf{r}_i \times \mathbf{F}_i - \mathbf{a} \times \sum_i F_i$

Here, the second term need not vanish.

7.20 The centripetal acceleration in a wheel arise due to the internal elastic forces which in pairs cancel each other; being part of a symmetrical system.

In a half wheel the distribution of mass about its centre of mass (axis of rotation) is not symmetrical. Therefore, the direction of angular momentum does not coincide with the direction of angular velocity and hence an external torque is required to maintain rotation.

7.21 No. A force can produce torque only along a direction normal to itself as $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{f}$. So, when the door is in the xy-plane, the torque produced by gravity can only be along $\pm z$ direction, never about an axis passing through y direction.

7.22 Let the C.M. be 'b' . Then, $\dfrac{(n-1)mb + ma}{mn} = 0 \Rightarrow b = -\dfrac{1}{n-1}a$

7.23 (a) Surface density $\sigma = \dfrac{2M}{\pi a^2}$

$$\bar{x} = \frac{\int x dm}{\int dm} = \int_{r=0}^{a}\int_{\theta=0}^{\pi} r\cos\theta\, \sigma r dr d\theta \Big/ \int_{r=0}^{a}\int_{\theta=0}^{\pi} \sigma r dr d\theta$$

$$= \frac{\int_0^a r^2 dr \, \sin\theta\big|_0^\pi}{\int_0^a r dr \int_0^\pi d\theta} = 0$$

$$\bar{y} = \frac{\int y dm}{\int dm} = \int_{\theta=0}^{\pi}\int_{r=0}^{a} r\sin\theta\, \sigma r dr d\theta \Big/ \int_{r=0}^{a}\int_{\theta=0}^{\pi} \sigma r dr d\theta$$

$$= \frac{\int_0^a r^2 dr \int_{\theta=0}^{\pi} \sin\theta d\theta}{\int_0^a r dr \int_0^\pi d\theta} = \frac{a^3}{3}\frac{[-\cos\theta]_0^\pi}{(a^2/2)\pi} = \frac{a}{3}\frac{4}{\pi} = \frac{4a}{3\pi}.$$

(b) Same procedure, as in (a) except θ goes from 0 to $\pi/2$ and

$$\sigma = \frac{4M}{\pi a^2}.$$

7.24 (a) Yes, because there is no net external torque on the system. External forces, gravitation and normal reaction, act through the axis of rotation, hence produce no torque.

(b) By angular momentum conservation

$$I\omega = I_1\omega_1 + I_2\omega_2$$

$$\therefore \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(c) $K_f = \frac{1}{2}(I_1 + I_2)\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)^2} = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$

$K_i = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$

$\Delta K = K_f - K_i = -\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$

(d) The loss in kinetic energy is due to the work against the friction between the two discs.

7.25 (a) Zero (b) Decreases (c) Increases (d) Friction (e) $v_{cm} = R\omega$.

(f) Acceleration produced in centre of mass due to friction:

$$a_{cm} = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g.$$

Angular acceleration produced by the torque due to friction,

$$\alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{I}$$

$$\therefore v_{cm} = u_{cm} + a_{cm}t \Rightarrow v_{cm} = \mu_k gt$$

and $\omega = \omega_o + \alpha t \Rightarrow \omega = \omega_o - \frac{\mu_k mgR}{I}t$

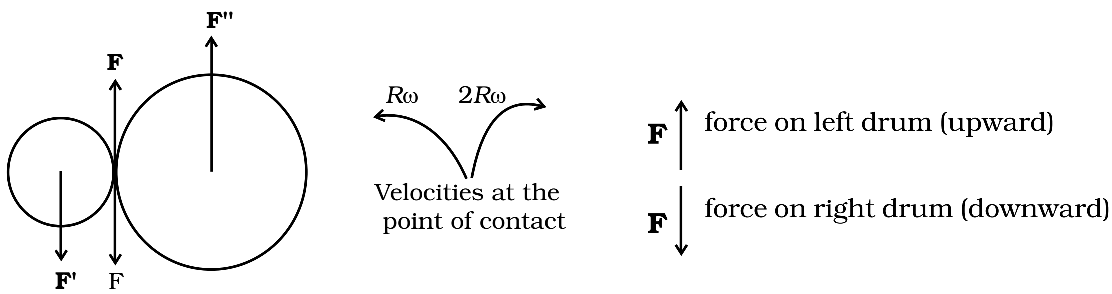
For rolling without slipping,

$$\frac{v_{cm}}{R} = \omega_o - \frac{\mu_K mgR}{I}t$$

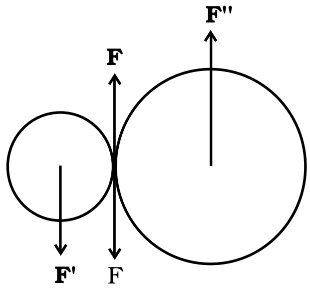
$$\frac{\mu_K gt}{R} = \omega_o - \frac{\mu_K mgR}{I}t$$

$$t = \frac{R\omega_o}{\mu_k g\left(1 + \frac{mR^2}{I}\right)}$$

7.26 (a)



(b) $F' = F = F''$ where F and F'' and external forces through support.



$F_{net} = 0$

External torque = $F \times 3R$, anticlockwise.

(c) Let ω_1 and ω_2 be final angular velocities (anticlockwise and clockwise respectively)

Finally there will be no friction.

Hence, $R\,\omega_1 = 2\,R\,\omega_2 \Rightarrow \dfrac{\omega_1}{\omega_2} = 2$

7.27 (i) Area of square = area of rectangle $\Rightarrow c^2 = ab$

$$\frac{I_{xR}}{I_{xS}} \times \frac{I_{yR}}{I_{yS}} = \frac{b^2}{c^2} \times \frac{a^2}{c^2} = \left(\frac{ab}{c^2}\right)^2 = 1$$

(i) and (ii) $\dfrac{I_{yR}}{I_{yS}} > \dfrac{I_{xR}}{I_{xS}} \Rightarrow \dfrac{I_{yR}}{I_{yS}} > 1$

and $\dfrac{I_{xR}}{I_{xS}} < 1$.

(iii) $I_{zr} - I_{ZS} \propto (a^2 + b^2 - 2c^2)$

$$= a^2 + b^2 - 2ab > 0$$

$\therefore (I_{zR} - I_{zS}) > 0$

$\therefore \dfrac{I_{zR}}{I_{zS}} > 1.$

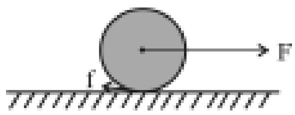
7.28 Let the accelaration of the centre of mass of disc be 'a', then

$Ma = F\text{-}f$ (1)

The angular accelaration of the disc is $\alpha = a/R$. (if there is no sliding).

Then

$\left(\dfrac{1}{2}MR^2\right)\alpha = Rf$ (2)



$\Rightarrow Ma = 2f$

Thus, $f = F/3$. Since there is no sliding,

$\Rightarrow f \le \mu mg$

$\Rightarrow F \le 3\mu Mg.$