Chapter 8

8.1 (d)

8.2 (c)

8.3 (a)

8.4 (c)

8.5 (b)

8.6 (d)

8.7 (d)

8.8 (c)

8.9 (a), (c)

8.10 (a), (c)

8.11 (a), (c), (d)

8.12 (c), (d)

8.13 (c), (d)

8.14 (a), (c), (d)

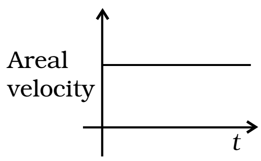
8.15 (a), (c)

8.16 (d)

8.17 Molecules experience the vertically downward force due to gravity just like an apple falling from a tree. Due to thermal motion, which is random, their velocity is not in the vertical direction. The downward force of gravity causes the density of air in the atmosphere close to earth higher than the density as we go up.

8.18 Central force; gravitational force of a point mass, electrostatic force due to a point charge.
Non-central force: spin-dependent nuclear forces, magnetic force between two current carrying loops.

8.19



8.20 It is normal to the plane containing the earth and the sun as areal velocity

$$\frac{\Delta \mathbf{A}}{\Delta t} = \frac{1}{2}\mathbf{r} \times \mathbf{v}\Delta t.$$

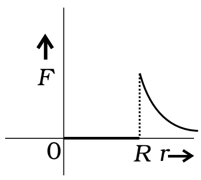
8.21 It remains same as the gravitational force is independent of the medium separating the masses.

8.22 Yes, a body will always have mass but the gravitational force on it can be zero; for example, when it is kept at the centre of the earth.

8.23 No.

8.24 Yes, if the size of the spaceship is large enough for him to detect the variation in g.

8.25

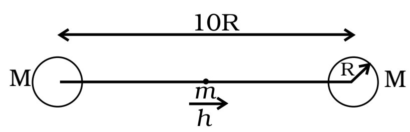


8.26 At perihelion because the earth has to cover greater linear distance to keep the areal velocity constant.

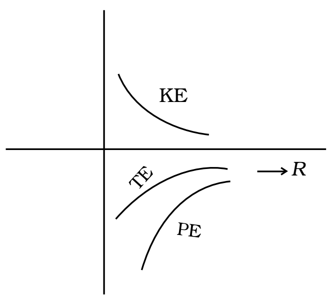
8.27 (a) 90° (b) 0°

8.28 Every day the earth advances in the orbit by approximately 1°. Then, it will have to rotate by 361° (which we define as 1 day) to have sun at zenith point again. Since 361° corresponds to 24 hours; extra 1° corresponds to approximately 4 minute [3 min 59 seconds].

8.29 Consider moving the mass at the middle by a small amount h to the right. Then the forces on it are: $\dfrac{GMm}{(R-h)^2}$ to the right and $\dfrac{GMm}{(R+h)^2}$ to the left. The first is larger than the second. Hence the net force will also be towards the right. Hence the equilibrium is unstable.



8.30



8.31 The trajectory of a particle under gravitational force of the earth will be a conic section (for motion outside the earth) with the centre of the earth as a focus. Only (c) meets this requirement.

8.32 $mgR/2$.

8.33 Only the horizontal component (i.e. along the line joining m and O) will survive. The horizontal component of the force on any point on the ring changes by a factor:

$$\left[\frac{2r}{(4r^2+r^2)^{3/2}}\right] \quad \left[\frac{\mu}{(r^2+r^2)^{3/2}}\right]$$

$$=\frac{4\sqrt{2}}{5\sqrt{5}}.$$

8.34 As r increases:

$U\left(=-\frac{GMm}{r}\right)$ increases.

$v_c\left(=\sqrt{\frac{GM}{r}}\right)$ decreases.

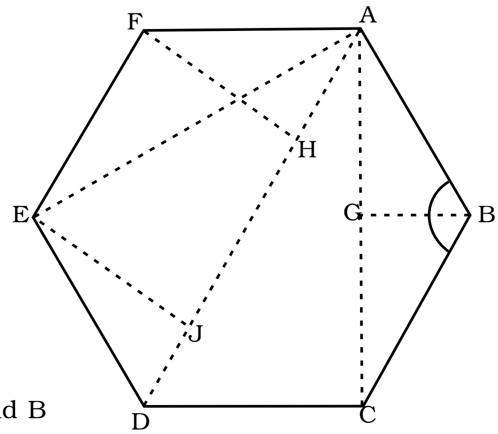
$\omega\left(=\frac{v_c}{r}\times\frac{1}{r^{3/2}}\right)$ decreases.

K decreases because v increases.

E increases because $|U| = 2K$ and $U < O$

l increases because $mvr \propto \sqrt{r}$.

8.35 AB = C



$(AC) = 2\ AG = 2.l.\frac{\sqrt{3}}{2} = \sqrt{3}l$

AD = AH + HJ + JD

$$=\frac{l}{2}+l+\frac{l}{2}$$

$$= 2l.$$

$AE = AC = \sqrt{3}l$, AF = l

Force along AD due to m at F and B

$$=Gm^2\left[\frac{1}{l^2}\right]\frac{1}{2}+Gm^2\left[\frac{1}{l^2}\right]\frac{1}{2}=\frac{Gm^2}{l^2}$$

Force along AD due to masses at E and C

$$=Gm^2\frac{1}{3l^2}\cos(30^\circ)+\frac{Gm^2}{3l^2}\cos(30^\circ)$$

$$= \frac{Gm^2}{3l^2}\sqrt{3} = \frac{Gm^2}{\sqrt{3}l^2}.$$

Force due to mass M at D

$$= \frac{Gm^2}{4l^2}.$$

$$\therefore \text{Total Force} = \frac{Gm^2}{l^2}\left[1 + \frac{1}{\sqrt{3}} + \frac{1}{4}\right].$$

8.36 (a) $r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$

$$\therefore h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$$

$$= 4.23\text{x}10^7 - 6.4\text{x}10^6$$

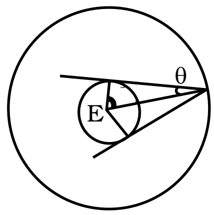
$$= 3.59\times10^7\,\text{m}.$$

(b) $\theta = cos^{-1}\left(\frac{R}{R+h}\right)$

$$= cos^{-1}\left(\frac{1}{1+h/r}\right)$$

$$= cos^{-1}\left(\frac{1}{1+5.61}\right)$$

$$= 81^\circ 18'$$



$$\therefore 2\theta = 162^\circ 36'$$

$$\frac{360^\circ}{2\theta} \approx 2.21;\ \text{Hence minimum number} = 3.$$

8.37 Angular momentem and areal velocity are constant as earth orbits the sun.

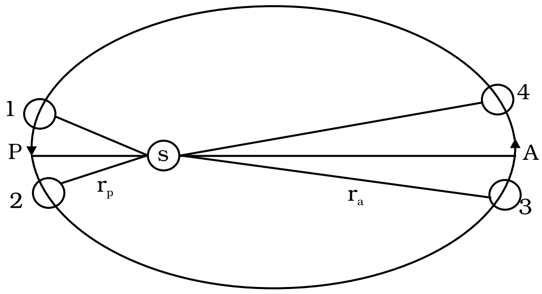
At perigee $r_p^2\omega_p = r_a^2\omega_a$ at apogee.

If 'a' is the semi-major axis of earth's orbit, then $r_p = a(1-e)$ and $r_a = a(1+e)$.

$$\therefore \frac{\omega_p}{\omega_a} = \left(\frac{1+e}{1-e}\right)^2, \quad e = 0.0167$$

$$\therefore \frac{\omega_p}{\omega_a} = 1.0691$$

Let ω be angular speed which is geometric mean of ω_p and ω_a and corresponds to mean solar day,



$$\therefore \left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.0691$$

$$\therefore \frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} = 1.034.$$

If ω corresponds to 1° per day (mean angular speed), then

$\omega_p = 1.034^\circ$ per day and $\omega_a = 0.967^\circ$ per day. Since 361° = 14hrs: mean solar day, we get 361.034° which corresponds to 24 hrs 8.14″ (8.1″ longer) and 360.967° corresponds to 23 hrs 59 min 52″ (7.9″ smaller).

This does not explain the actual variation of the length of the day during the year.

8.38 $r_a = a(1+e) = 6R$

$r_p = a(1-e) = 2R \quad \Rightarrow e = \dfrac{1}{2}$

Conservation of angular momentum:

angular momentum at perigee = angular momentum at apogee

$$\therefore mv_p r_p = mv_a r_a$$

$$\therefore \frac{v_a}{v_p} = \frac{1}{3}.$$

Conservation of Energy:

Energy at perigee = Energy at apogee

$$\frac{1}{2}mv_p^{\,2} - \frac{GMm}{r_p} = \frac{1}{2}mv_a^{\,2} - \frac{GMm}{r_a}$$

$$\therefore v_p^{\,2}\left(1-\frac{1}{9}\right) = -2GM\left[\frac{1}{r_a} - \frac{1}{r_p}\right] = 2GM\left[\frac{1}{r_a} - \frac{1}{r_p}\right]$$

$$v_p = \left[\frac{2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]}{\left[1-\left(v_a/v_p\right)\right]^2}\right]^{1/2} = \left[\frac{\frac{2GM}{R}\left[\frac{1}{2} - \frac{1}{6}\right]}{\left(1-\frac{1}{9}\right)}\right]^{1/2}$$

$$= \left(\frac{2/3}{8/9} \frac{GM}{R} \right)^{1/2} = \sqrt{\frac{3}{4} \frac{GM}{R}} = 6.85 \text{km/s}$$

$v_p = 6.85 \text{km/s}$, $v_a = 2.28 \text{km/s}.$

For $r = 6R,\ v_c = \sqrt{\frac{GM}{6R}} = 3.23 \text{km/s}.$

Hence to transfer to a circular orbit at apogee, we have to boost the velocity by Δ = (3.23 – 2.28) = 0.95 km/s. This can be done by suitably firing rockets from the satellite.