Gravitation

(1) Introduction :

In our dairy life we have noticed things falling freely downwords towards earth when thrown upwards or dispped from some hight.

Fact that all bodies irrespective of their masses are accelerated towards the earth mith a constant aceeleation was first recognized by galive (1564-1642)

The motion of celesh'al bodies such as moon. earth plametes etc. and attrachieve of moon towards earth and arth towards sun is an interasting subject of study since long time.

Now the question's what is the force that produces such acceleration is which earth attract all bodies towards the centre and what is the law governing this force.

Is this law is same for both earthly and weshal bodies.

Answer to this question was given by Newton as he declared that "laws of nature are same for earthly and weshal Bodies".

The force between any object falling freely towards earth and that between earth and moon are gowerned by the same laws.

Johnaase kepler (1571-1631) Studied the planetary motion in detail and formulated his three laws of planetary motion, which were available Universal law of grawitation.

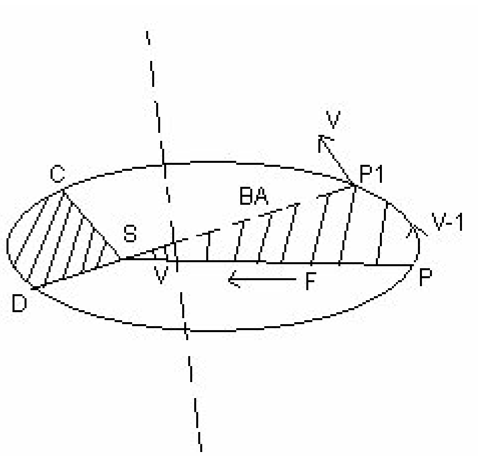
(2) Kepler's Law :-

Kepler's law of planetary motion are :-

(i)Law of orbits :-

Each planet revolues around the sun in an elliptical orbit with sun at one of the foci of the ellipse asd shown in fig (a) below.

Fig (a) An ellipse traced by planet sevolary round the sun.



AO = a - Sewi major axis

BO = b - Sewi minor axis

P - hearest point between planet and sun k/as perihetion

A - farthest point between planet and sun apheiton.

(ii)Law of areas :-

The line joining planet and the sun sweeps equal area in equal intervals of time" [fig b]

This law follows from the observation that when planet is nearer to the sun its velocity increases and It appears to be slower when it is farther from the sun.

(iii)Law of periods :-

The squre of time period of any planet about the sun is propotional to the cube of the semi-major axis."

If T is the time period of semi major axis a of elliptical orbit then.

T^2 x a^3 (1)

If T_1 and T_2 are time periods of any two planets and a_1 and a_2 being their semi major axis resp. then

$T_1{}^2$ x $a_1{}^3 = a_1{}^3$

$T_2{}^2$ x $a_2{}^3$ $a_2{}^3$ (2)

This question (2) can be used to find the time period of a planet, when the time period of the other planet and the semi-major axis of orbits of two planets.

(3) Universal law of gravition :-

Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and invessly proportional to the square of distance between them.

Mathematically Newton's gravitation law is if F is the force acting between two bodies of masses M_1 and M_2 and the distance between them is R then majnitade of force is given as

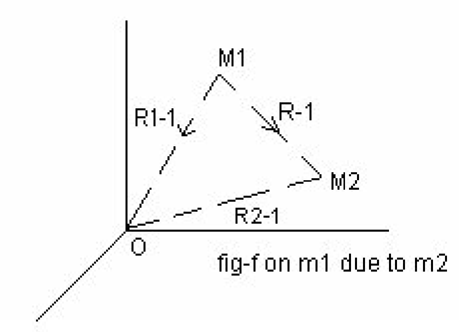


fig-f on m1 due to m2

$$F=\frac{Gm_{1}m_{2}}{r^{2}}$$

In vector notation

$$F=\frac{Gm_{1}m_{2}(-\hat{\boldsymbol{\mathbf{r}}})}{r^{2}}$$

$$F=-\frac{Gm_{1}m_{2}\hat{\boldsymbol{\mathbf{r}}}}{r^{2}}$$ where G- universal gravitational constant

$$\hat{\boldsymbol{\mathbf{r}}}$$ - unit vector from m_1 to m_2 and $$\hat{\boldsymbol{\mathbf{r}}} = \hat{\boldsymbol{\mathbf{r_{1}}}}^ - \hat{\boldsymbol{\mathbf{r_{2}}}} $$

Gravitational force constant is

SI - $G= 6.67 \times 10^{-11}$ nm^2 kg^{-2}

CGS - $G= 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}

Deminsional formula of C1 is $[m^{-1}L^3T^{-2}]$

(4) Acceleration due to gravity of earth :-

Earth attracts every object lying an its surface towards its centre with a force known as gravitational towards its centre with a force known as gravitational pull or gravity.

Whenever force acts on any body it produces acceleration and in case of gravitation this acceleration produced under effect of gravity is known as accelesation due to gravity (g)

Value of accelesation due to gravity is independent of mass of the body and its value near surface of earth is 9.8 ms^{-2}

Expression for acceleration due to gravity

Consider mass of earth to be as M_E and its redius be R_E Suppose a body of mass M (much smaller then that fo earth) is kept at the earth surface. Force eseerted by earth on the body of mass m is

$$F=\frac{-GMM_{E}}{R_{E}^{2}}$$

The force for the body due to earth produces acceleration due to gravity

(g) in the motion of the body. From Newton's Second law of motion

f=mg

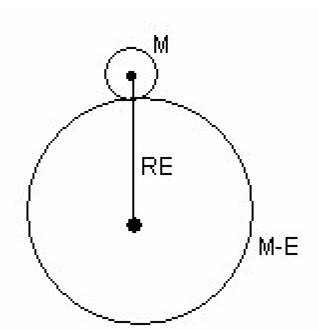
from (4) and (s)

$$g=\frac{-GM_{E}}{R_{E}^{2}}$$

which is acceleration due to gravity at earth's surface.

(5) Acceleration due to gravity below and above the earth surface :-

(i) Above earth's surface



An object of mass m is placed at hight h above the earth's active in this object is

$$F=\frac{-GMM_{E}}{(R_{E}+h)^{2}}$$

From this it can be concluded that value of g decreases as distance above surface of earth increases now,

$$g=\frac{-GM_{E}}{R_{E}(1+\frac{h}{R_{E}})^{2}}$$

$$g=\frac{g_{0}}{(1+\frac{h}{R_{E}})^{2}}$$

$$g_{0}=\frac{-GM_{E}}{R_{E}^{2}}$$

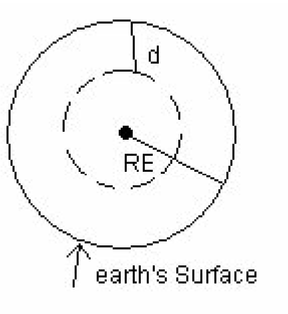
eqn (7) tells us that for small hight h above surface of earth. value of g decreases by factor $(1-2h/R_E)$

- for h<<R

$g=g_o(1+h/R_E)^{-2}$

$g=g_o(1-2h/R_E)$ Expanding by Binomial theriom

(ii) Below the earth's surface



If one goes inside the earth surface the value of g again decreases

P = density of material of earth them

$m=(4/3)(R_E)^3P$

From this acceleration due to gravity at earth□s Surface is

$$g_{0}=\frac{G(4/3)R_{E}^{3}P}{R_{E}^{2}}$$

$g=(4/3)GR_EP$ ---------(8)

g-acceleration due to gravity at depth D below earth's surface

-Body at depth d will experience force only due to portion of reduce (R_E-d) of earth's

-outer spherical shell of thickness d will not exeperience any force

-M is mas of the portion of earth with radius (R_E-d) then

$$g=\frac{-GM}{r^{2}}$$

$M=(4/3)(R_E-d)^3$ P

$$g=\frac{G(4/3)(R_{E}-d)^{3}P}{(R_{E}-d)^{2}}$$

$g= (4/3)G (R_E-d)P$ (9)

Dividing epn (9) by (8)

$g/g_o= (1-d/R_E)$

or $g=go (1-d/R_E)$

nbsp; (10)

from epn (10) is cleaer that acceeleration due to gravity also decreases with depth.

GRAVITATION

CONCEPTS

- **Kepler's law of planetry motion**

(a) Kepler's first law (law of orbit): Every planet revolves around the sun in an elliptical orbit with the sun is situated at one focus of the ellipse.

(b) Kepler's second law (law of area): The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time , i.e., the areal velocity of the planet around the sun is constant.

(c) Kepler's third law (law of period): The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semimajor axis of the elliptical orbit of the planet around the sun.

- Gravitation is the name given to the force of attraction acting between any two bodies of the universe.

- Newton's law of gravitation: It states that gravitational force of attraction acting between two point mass bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, i.e., $F=Gm_1m_2/r^2$, where G is the universal gravitational constant.

- Gravitational constant (G): It is equal to the force of attraction acting between two bodies each of unit mass, whose centres are placed unit distance apart. Value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula $G =[M^{-1}L^3T^{-2}]$. In SI unit, the value of $G =6.67X10^{-11}Nm^2kg^{-2}$.

- Gravity: It is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth. Gravity is the measure of weight of the body. The weight of a body of mass m=mass X acceleration due to gravity=mg. The unit of weight of a body will be the same as those of force.

- **Acceleration due to gravity (g):** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone. It is vector quantity. The value of g changes with height, depth, rotation of earth the value of g is zero at the centre of the earth. The value of g on the surface of earth is 9.8 ms^{-2}. The acceleration due to gravity (g) is related with gravitational constant (G) by the relaion, $g=GM/R^2$ where M and R are the mass and radius of the earth.

- **Variation of acceleration due to gravity**:

(a) Effect of altitude, $g'=Gr^2/(R+h)^2$ and $g'=g(1-2h/R)$

The first is valid when h is comparable with R and the second relation is valid when $h<<R$.

The value of g decreases with increase in h.

(b) Effect of depth $g'=g(1-d/R)$

The acceleration due to gravity decreases with increase in depth d and becomes zero at the center of earth.

(c) Effect of rotation of earth: $g'=g-R\,\omega^2 cos^2\lambda$

The acceleration due to gravity on equator decreases on account of rotation of earth and increase with the increase in latitude of a place.

- Gravitational field: It is the space around a material body in which its gravitational pull can be experienced by other bodies. The strength of gravitational field at a point is the measure of gravitational intensity at that point. The intensity of gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field. The intensity of gravitational field at a point distance r from the center of the body of mass M is given by

$E=GM/r^2$=acceleration due to gravity.

- **Gravitational potential:** The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Gravitational potential at a point, V=work done(W)/test mass(m_0)= -GM/r. $V=\frac{W}{m_0}= -\frac{GM}{r}$

Gravitational intensity (I) is related to gravitational potential (V) at a point by the relation, E= -dV/dr

- **Gravitational potential energy of a body,** at a point in the gravitational field of another body is defined as the amount of work done in bringing the given body from infinity to that point without acceleration.

  Gravitational potential energy U=gravitational potential X mass of body $=-\frac{GM}{r}$ X m.

- **Inertial mass of a body** is defined as the force required to produce unit acceleration in the body.

  Gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity.

  Inertial mass of a body is identical to the gravitational mass of that body. The main difference is that the gravitational mass of a body is affected by the presence of other bodies near it. Whereas the inertial mass of a body remains unaffected by the presence of other bodies near it.

- **Satellite:** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

(a) Orbital speed of satellite is the speed required to put the satellite into given orbit around earth.

- Time period of satellite(T): It is the time taken by satellite to complete one revolution around the earth. $T=\frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$

- Height of satellite above the earth surface:

$$h=\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3}-R$$

- Total energy of satellite, E=P.E +K.E=$\frac{-GMm}{2(R+h)}$

Blinding energy of satellite = -E = GM m/(R+h)

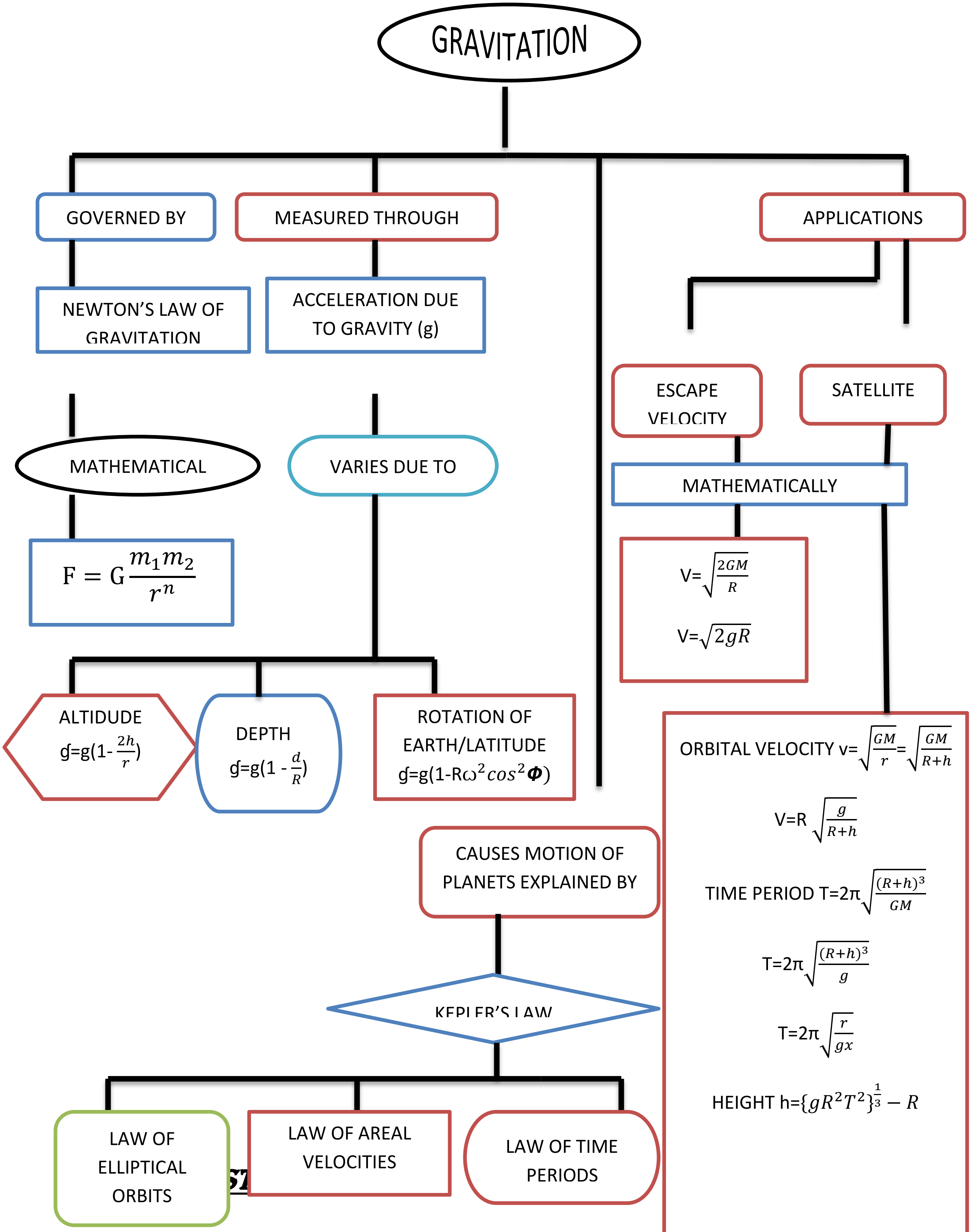
- Geostationary satellite: A satellite which revolves around the earth with the same angular speed in the same direction as is done by the earth around its axis is called geostationary or geosynchronous satellite. The height of geostationary satellite is = 36000 km and its orbital velocity = 3.1 km s^{-1}.

- Polar satellite: It is that satellite which revolves in polar orbit around earth ,i.e. , polar satellite passes through geographical north and south poles of earth once per orbit.

- Escape speed: The escape speed on earth is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth(or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. Escape velocity v_e is given by, $v_e = \sqrt{\frac{2GM}{R}}=\sqrt{2gR}$. For earth, the value of escape speed is 11.2kms^{-1}.

- For a point close to the earth's surface , the escape speed and orbital speed are related as $v_e = \sqrt{2}\, v_o$

- Weightlessness: It is a situation in which the effective weight of the body becomes zero.



GRAVITATION
GOVERNED BY
MEASURED THROUGH
APPLICATIONS
NEWTON'S LAW OF GRAVITATION
ACCELERATION DUE TO GRAVITY (g)
ESCAPE VELOCITY
SATELLITE
MATHEMATICAL
VARIES DUE TO
MATHEMATICALLY
$F = G\frac{m_1 m_2}{r^n}$
V=$\sqrt{\frac{2GM}{R}}$
V=$\sqrt{2gR}$
ALTIDUDE
ɡ=g(1- $\frac{2h}{r}$)
DEPTH
ɡ=g(1 - $\frac{d}{R}$)
ROTATION OF EARTH/LATITUDE
ɡ=g(1-Rω$^2 cos^2 \boldsymbol{\phi}$)
ORBITAL VELOCITY v=$\sqrt{\frac{GM}{r}}$=$\sqrt{\frac{GM}{R+h}}$
V=R $\sqrt{\frac{g}{R+h}}$
TIME PERIOD T=2π$\sqrt{\frac{(R+h)^3}{GM}}$
T=2π$\sqrt{\frac{(R+h)^3}{g}}$
T=2π$\sqrt{\frac{r}{gx}}$
HEIGHT h={$gR^2T^2\}^{\frac{1}{3}} - R$
CAUSES MOTION OF PLANETS EXPLAINED BY
KEPLER'S LAW
LAW OF ELLIPTICAL ORBITS
LAW OF AREAL VELOCITIES
LAW OF TIME PERIODS

Q1.When a stone of mass m is falling on the earth of mass M; find the acceleration of earth if any?

Ans. Force exerted by falling stone on earth, F=mg

Acceleration of earth=$\frac{F}{M}$=$\frac{mg}{M}$

Q2.Why G is called a universal constant?

Ans. It is so because the value of G is same for all the pairs of the bodies (big or small) situated anywhere in the universe.

Q3.According to Kepler's second law the radius vector to a planet from the sun sweeps out equal area in equal interval of time. The law is a consequence of which conservation law.

Ans. Law of Conservation of angular momentum.

Q4.What are the factors which determine ; Why some bodies in solar system have atmosphere and others don't have?

Ans. The ability of a body (planet) to hold the atmosphere depends on acceleration due to gravity.

Q5.What is the maximum value of gravitational potential energy and where?

Ans. The value of gravitational potential energy is negative and it increases as we move away from the earth and becomes maximum (zero) at infinity.

Q6.The gravitational potential energy of a body at a distance r from the center of earth is U. What is the weight of the body at that point?

Ans. U=$\frac{GMm}{r}$=$(\frac{GM}{r^2})$ r m=g r m= (mg) r

Q7.A satellite revolving around earth loses height. How will its time period be changed?

Ans. Time period of satellite is given by; T=2$\pi\sqrt{\frac{(R+h)^3}{GM}}$. Therefore ,T will decrease, when h decreases.

Q8.Should the speed of two artificial satellites of the earth having different masses but the same orbital radius, be the same?

Ans.Yes it is so because the orbital speed of a satellite is independent of the mass of a satellite. Therefore the speeds of the artificial satellite will be of different masses but of the same orbital radius will be the same.

Q9.Can a pendulum vibrate in an artificial satellite?

Ans. No, this is because inside the satellite, there is no gravity ,i.e., g=0.

As t = 2π$\sqrt{l/g}$, hence, for g=0 , t = ∞ . Thus, the pendulum will not vibrate.

Q10.Why do different planets have different escape speed?

Ans. As, escape speed =$\sqrt{2GM/R}$, therefore its value are different for different planets which are of different masses and different sizes.

2 MARKS QUESTIONS

Q1.Show that weight of all body is zero at Centre of earth?

Ans. The value of acceleration due to gravity at a depth d below the surface of earth of radius R is given by g̍=g(1-d/R).At the center of earth, (dept)d=R; so, g̍ =0.The weight of a body of mass m at the centre of earth =mg'=m x 0=0.

Q2.If a person goes to a height equal to radius of the earth from its surface. What would be his weight relative to that on the earth.

Ans. At the surface of the earth, weight W=mg=GM m/R^2.

At height h =R , weight W'=mg'=$\frac{GM\ m}{(R+h)^2}$=$\frac{GM\ m}{(R+R)^2}$ $\frac{W'}{W}$=$\frac{R^2}{(2R)^2}$ =$\frac{1}{4}$ W'=$\frac{W}{4}$

It means the weight would reduce to one-fourth of the weight on the surface of earth.

Q3.What will be the effect on the time period of a simple pendulum on taking to a mountain?

Ans. The time period of a pendulum, T=2π$\sqrt{l/g}$, i.e., T=∝1/$\sqrt{g}$.As the value of g is less at mountain than at plane, hence time period of simple pendulum will be more at mountain than at plane though the change will be very small.

Q4.A satellite is revolving around the earth, close to the surface of earth with a kinetic energy E. How much kinetic energy should be given to it so that it escapes from the surface of earth?

Ans. Let $v_{0,}v_e$ be the orbital and escape speeds of the satellite, then v_e=$\sqrt{2v_{0,}}$.

Energy in the given orbit,$E_1 = \frac{1}{2}mv_0^2 = E$

Energy for the escape speed,$E_2 = \frac{1}{2}m\, v_e^2 = \frac{1}{2}m(\sqrt{2}v_0^2) = 2E$

Energy required to be supplied =$E_2 - E_1 = E$.

Q5.A tennis ball and a cricket ball are to be projected out of gravitational field of the earth. Do we need different velocities to achieve so?

Ans. We require the same velocity for the two balls, while projecting them out of the gravitational field. It is so because, the value of escape velocity does not depend upon the mass of the body to be projected [i.e. , v_e=$\sqrt{2gR}$].

Q6.Suppose the gravitational force varies inversely as the nth power of the distance. Show that the time period of a planet in circular orbit of radius R around the sun will be proportional to $R^{(n+1)/2}$.

Ans. $\frac{GM\,m}{R^n} = mR(\frac{2\pi}{T})^2$

$$T^2 = \frac{R \times 4\pi^2 \times R^n}{GM} = \frac{4\pi^2 R^{(n+1)}}{GM}$$

$$T = \frac{2\pi}{\sqrt{GM}} . R^{(n+1)/2}$$

$$T \propto R^{(n+1)/2}$$

Q7.Draw graphs showing the variation of acceleration due to gravity with (a)height above the earth's surface, (b)depth below the Earth's surface.

Ans.(a)The variation of g with height h is related by relation g∝1/r^2 where r=R+h. Thus, the variation of g and r is a parabolic curve.

(b)The variation of g with depth is released by equation g'=g(1-d/R) i.e. g'∝ $(R-d)$ .Thus, the variation of g and d is a straight line.

Q8.Why does moon have no atmosphere?

Ans. Moon has no atmosphere because the value of acceleration due to gravity 'g' on surface of moon is small. Therefore, the value of escape speed on the surface of moon is small. The molecules of atmospheric gases on the surface of the moon have thermal speeds greater than the escape speed. That is why all the molecules of gases have escaped and there is no atmosphere on moon.

Q9.A rocket is fired with a speed v=2$\sqrt{gR}\, near$ the earth's surface and directed upwards. Find its speed in interstellar space.

Ans. Let v be the speed of rocket instellar space.

Using law of conservation of energy, we have$\frac{1}{2}m(2\sqrt{gR})^2$=$\frac{1}{2}mv_e^2 + \frac{1}{2}mv^2$

$$=\frac{1}{2}m(\sqrt{2gR})^2 + \frac{1}{2}mv^2$$

$$v^2 = 4gR - 2gR$$

$$v = \sqrt{2gR}$$

3 marks questions

Q1.Explain how knowledge of g helps us to find (i) mass of earth and (ii)mean density of earth?

Q2. Obtain the expression for orbital velocity, time period, and altitude of a satellite.

Q3. What do you understand by 'Escape velocity'? Derive an expression for it in terms of parameters of given planet.

Q4. What do you understand by gravitational field, Intensity of gravitational field . Prove that gravitational intensity at a point is equal to the acceleration due to gravity at that point.

Q5.A mass M is broken into two parts of masses $m_1 and\, m_2$. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum.

Ans. Let $m_1 = m,$ then $m_2 = M - m$. Gravitational force of attraction between them when placed distance r apart will be $= \frac{Gm(M-m)}{r^2}$.

Differentiating it w.r.t. m, we get

$$\frac{dF}{dm} = \frac{G}{r^2}\left[m\frac{d}{dm}(M-m) + (M-m)\frac{dm}{dm}\right] = \frac{G}{r^2}[m(-1) + M - m] = \frac{G}{r^2}(M-2m)$$

If F is maximum, then $\frac{dF}{dm} = 0$;

Then $\frac{G}{r^2}(M-2m) = 0$ or M=2m or m=$\frac{M}{2}$

Q6.Two particles of equal mass move in a circle of radius r under the action of their mutual gravitational attraction. Find the speed of each particle if its mass is m.

Ans. The two particles will move on a circular path if they always remain dramatically opposite so that the gravitation force on one particle due to other is directed along the radius. Taking into consideration the circulation of one particle we have

$$\frac{mv^2}{r} = \frac{Gmm}{(2r)^2} \text{ or } v = \sqrt{\frac{Gm}{4r}}$$

Q7.The magnitude of gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are $I_1 and\ I_2$ respectively. Find the ratio of (I_1/I_2) if $r_1 > R\ and\ r_2 < R$.

Ans. When $r_1 > R,$ the point lies outside the sphere . Then sphere can be considered to be a point mass body whose whole mass can be supposed to be concentrated at its Centre. Then gravitational intensity at a point distance $r_1$from the Centre of the sphere will be,$I_1 = GM/r_1^2$

When $r_2 < R$, the point P lies inside the sphere. The unit mass body placed at P, will experience gravitational pull due to sphere of radius r_2, whose mass is M'=$\frac{M\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi R^3} = \frac{Mr_2^3}{R^3}$.

Therefore, the gravitational intensity at P will be ,

$$I_2 = \frac{GMr_2^3}{R^3} \cdot \frac{1}{r_2^2} = \frac{GMr_2}{R^3}$$

$$\frac{I_1}{I_2} = \frac{GM}{r_1^2} \cdot \frac{R^3}{GMr_2} = \frac{R^3}{r_1^2 r_2}$$

Q8.Two bodies of masses $m_1 and\ m_2$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Find their relative velocity of approach at a separation distance r between them.

Ans. Let v_r be the relative velocity of approach of two bodies at a distance r apart. The reduced mass of the system of two particles is , $\mu = \frac{m_1 m_2}{m_1+m_2}$.

According to law of conservation of mechanical energy.

Decrease in potential energy = increase in K.E.

$$0-\left(-\frac{Gm_1m_2}{r}\right)=\frac{1}{2}\mu v_r^2 \text{ or } \frac{Gm_1m_2}{r}=\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_r^2 \text{ or } v_r=\sqrt{\frac{2G(m_1+m_2)}{r}}$$

Q9.Since the moon is gravitationally attracted to the earth, why does it not simply crash on earth?

Ans. The moon is orbiting around the earth in a certain orbit with a certain period . The centripetal force required for the orbital motion is provided to the gravitational pull of earth. The moon can crash into the earth if its tangential velocity is reduced to zero. AS moon has tangential velocity while orbiting around earth, it simply falls around the earth rather than into it and hence cannot crash into the earth.

Q10.What are the conditions under which a rocket fired from earth, launches an artificial satellite of earth?

Ans. Following are the basic conditions: (i) The rocket must take the satellite to a suitable height above the surface of earth for ease of propulsion.

(ii)From the desired height, the satellite must be projected with a suitable speed, called orbital speed.

(iii)In the orbital path of satellite, the air resistance should be negligible so that its speed does not decrease and it does not burn due to the heat produced.

5 MARKS QUESTIONS

Q1.State Kepler's laws of planetary motion. Prove second Kepler's law using concept of conservation of angular motion.

Q2.State universal law of gravitation. What is the significance of this law. Find the expression for acceleration due to gravity.

Q3.Explain the variation of acceleration due to gravity with (I) altitude (ii) depth

Q4. Define gravitational potential energy. Derive the expression for gravitational potential energy. What is the maximum value of gravitational potential energy?

Q5.What is escape speed? Derive the expressions for it. Calculate escape speed for the Earth.

TYPICAL PROBLEMS

Q1.Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Ans. The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius. Consider the motion of one of the particles. The force on the particle is $F = \frac{Gm^2}{4R^2}$. If the speed is v, its acceleration is v^2/R.

Thus by Newton's Law,

$$\frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$V = \sqrt{\frac{Gm}{4R}}$$

Q2.A particle is fired vertically upward with a speed of 3.8km/s. Find the maximum height attained by the particle. Radius of earth=6400km and g at the surface=9.8m/s. Consider only earth's gravitation.

Ans. At the surface of the earth, the potential energy of the earth-particle system is $\frac{GMm}{R}$ with usual symbol. The kinetic energy is 1/2 mv^2 where $v_0 = 9.8km/s$. At the maximum height the kinetic energy is zero. If the maximum height reached is H, the potential energy of the earth-particle system at this instant is $-\frac{GMm}{R+H}$. Using conservation of energy , $-\frac{GMm}{R} + \frac{1mv^2}{2} = -\frac{GMm}{R+H}$

Writing GM=g$R^2 and$ dividing by m,

$$-gR+\frac{v_0^2}{2}=\frac{-gR^2}{R+H}$$

$$\frac{R^2}{R+H}=R-\frac{v_0^2}{2g}$$

$$R+H=\frac{R^2}{R-\frac{v_0^2}{2g}}$$

Putting the value of R,$v_0 and$ g on right side,

$$R+H=\frac{(6400\ km)^2}{6400-\frac{(9.8km/s)^2}{2\ X\ 9.8s^{-2}}}$$

$$=27300km$$

H = (27300 - 6400)km =20900km

3. Derive an expression for the gravitational field due to a uniform rod of length L and mass M at a point on its perpendicular bisector at a distance d from the center.

Ans. A small section of rod is considered at 'x' distance mass of the element = (M/L). dx = dm

$$dE_1=\frac{G(dm)X1}{(d^2+x^2)}\quad =2.\frac{G(dm)}{(d^2+x^2)}.\frac{d}{\sqrt{(d^2+x^2}}=\frac{2GMd\ dx}{L(d^2+x^2)(\sqrt{(d^2+x^2)}}$$

Total gravitational field

E=$\int_0^{L/2}\frac{2Gmd\ dx}{L(d^2+x^2)^{3/2}}$

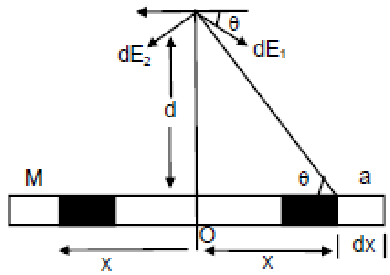
Integrating the above equation it can be found that,

$$E=\frac{2GM}{d\sqrt{L^2+4d^2}}$$

Resultant dE = 2 dE_1 sin θ

$$= 2 \times \frac{G(dm)}{(d^2+x^2)} \times \frac{d}{\sqrt{(d^2+x^2}} = \frac{2 \times GM \times d\ dx}{L(d^2+x^2)(\sqrt{(d^2+x^2)}}$$

Total gravitational field



$$E = \int_0^{L/2} \frac{2Gmd\ dx}{L(d^2+x^2)^{3/2}}$$

Integrating the above equation it can be found that,

$$E = \frac{2GM}{d\sqrt{L^2 + 4d^2}}$$

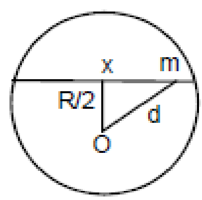
Q4.A tunnel is dug along a diameter of the earth. Find the force on a particle of mass m placed in the tunnel at a distance x from the centre.

Ans. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R^2}{4}\right)} = (\frac{1}{2})\sqrt{4x^2 + R^2}$$

M be the mass of the earth, M' the mass of the sphere of radius d/2.

Then M = (4/3) π$R^3\rho$



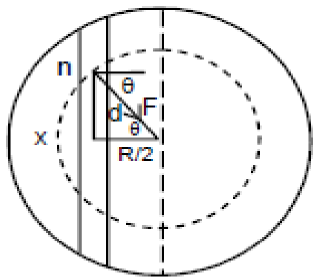
M' = (4/3)π$d^3\tau$

Or $\frac{M\prime}{M} = \frac{d^3}{R^3}$

□ Gravitational force is m,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$

So, Normal force exerted by the wall = F cosθ



$$\frac{GMmd}{R^3} \times \frac{R}{2d} = \frac{GMm}{2R^2}$$

Therefore I think normal force does not depend on x.

Q5. (a) Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation.

(b)If the satellite is directly above the north pole at some instant , find the time it takes to come over equatorial plane. Mass of the earth=$6X10^{24}kg$

Ans.(a) Angular speed f earth & the satellite will be same

$$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$$

Or

$$\frac{1}{24\ X\ 3600} = \frac{1}{2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$$

Or $12|3600 = 3.14\sqrt{\frac{(R+h)^3}{gR^2}}$

Or $$\frac{(R+h)^2}{gR^2} = \frac{(12\ X\ 3600)^2}{(3.14)^2}$$

Or $\frac{(6400+h)^3 X\ 10^9}{9.8X(6400)^2X10^6} = \frac{(12X3600)^2}{(3.14)^2}$

Or $\frac{(6400+h)^3X10^9}{6272X10^9} = 432 \times 10^4$

Or $(6400+h)^3 = 6272\ \times\ 432\ \times\ 10^4$

Or $6400 + h = (6272\ \times\ 432\ \times\ 10^4)^{1/3}$

Or $h = (6272\ \times\ 432\ \times\ 10^4)^{\frac{1}{3}} - 6400$

$= 42300\ m.$

(b)Time taken from north pole to equator = (1/2) t

$$= \left(\frac{1}{2}\right) X6.28 \sqrt{\frac{(43200+6400)^3}{10\ X\ (6400)^2\ X\ 10^6}} = 3.14 \sqrt{\frac{(497)^3 X\ 10^6}{(64)^2\ X\ 10^{11}}}$$

$$= 3.14 \sqrt{\frac{497\ X\ 497\ X\ 497}{64\ X\ 64\ X\ 10^5}} = 6\ hour.$$