Chapter 9

9.1 (b)

9.2 (d)

9.3 (d)

9.4 (c)

9.5 (b)

9.6 (a)

9.7 (c)

9.8 (d)

9.9 (c), (d)

9.10 (a), (d)

9.11 (b), (d)

9.12 (a), (d)

9.13 (a), (d)

9.14 Steel

9.15 No

9.16 Copper

9.17 Infinite

9.18 Infinite

9.19 Let Y be the Young's modulus of the material. Then

$$Y = \frac{f/\pi r^2}{l/L}$$

Let the increase in length of the second wire be l'. Then

$$\frac{\dfrac{2f}{4\pi r^2}}{l'/2L} = Y$$

$$\text{Or, } l' = \frac{1}{Y}\frac{2f}{4\pi r^2}2L = \frac{l}{L}\frac{\pi r^2}{f}\times\frac{2f}{4\pi r^2}2L = l$$

9.20 Because of the increase in temperature the increase in length per unit length of the rod is

$$\frac{\Delta l}{l_0} = \alpha\Delta T = 10^{-5}\times 2\times 10^{-2} = 2\times 10^{-3}$$

Let the compressive tension on the rod be T and the cross sectional area be a, then

$$\frac{T/a}{\Delta l/l_0} = Y$$

$$\therefore T = Y\frac{\Delta l}{l_o}\times a = 2\times 10^{11}\times 2\times 10^{-3}\times 10^{-4}$$

$$= 4\times 10^4\,\text{N}$$

9.21 Let the depth be h, then the pressure is

$$P = \rho gh = 10^3 \times 9.8 \times h$$

$$\text{Now } \left|\frac{P}{\Delta V/V}\right| = B$$

$$\therefore P = B\frac{\Delta V}{V} = 9.8\times 10^8\times 0.1\times 10^{-2}$$

$$\therefore h = \frac{9.8\times 10^8\times 0.1\times 10^{-2}}{9.8\times 10^3} = 10^2\,\text{m}$$

9.22 Let the increase in length be Δl, then

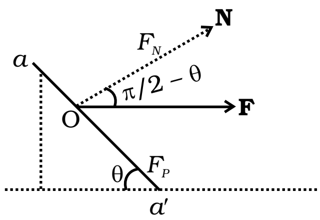
$$\frac{800}{(\pi\times 25\times 10^{-6})/(\Delta l/9.1)} = 2\times 10^{11}$$

$$\therefore \Delta l = \frac{9.1\times 800}{\pi\times 25\times 10^{-6}\times 2\times 10^{11}}\,\text{m}$$

$$\simeq 0.5\times 10^{-3}\,\text{m}$$

9.23 As the ivory ball is more elastic than the wet-clay ball, it will tend to retain its shape instantaneously after the collision. Hence, there will be a large energy and momentum transfer compared to the wet clay ball. Thus, the ivory ball will rise higher after the collision.

9.24 Let the cross sectional area of the bar be A. Consider the equilibrium of the plane aa'. A force **F** must be acting on this plane making an angle $\frac{\pi}{2}-\theta$ with the normal ON. Resolving **F** into components, along the plane and normal to the plane



$$F_P = F\cos\theta$$

$$F_N = F\sin\theta$$

Let the area of the face aa' be A', then

$$\frac{A}{A'} = \sin\theta$$

$$\therefore A' = \frac{A}{\sin\theta}$$

The tensile stress $T = \frac{F\sin\theta}{A'} = \frac{F}{A}\sin^2\theta$ and the shearing stress $Z = \frac{F\cos\theta}{A'} = \frac{F}{A}\cos\theta\,\sin\theta = \frac{F\,\sin 2\theta}{2A}$. Maximum tensile stress is when $\theta = \pi/2$ and maximum shearing stress when $2\theta = \pi/2$ or $\theta = \pi/4$.

9.25 (a) Consider an element dx at a distance x from the load (x = 0). If $T(x)$ and $T(x + dx)$ are tensions on the two cross sections a distance dx apart, then

$$T(x+dx) - T(x) = \mu g dx \text{ (where } \mu \text{ is the mass/length)}$$

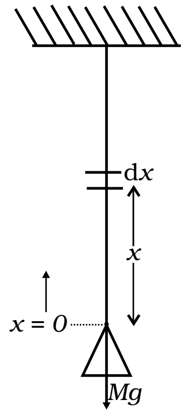
$$\frac{dT}{dx}dx = \mu g dx$$

$$\Rightarrow T(x) = \mu g x + C$$

$$\text{At } x = 0, T(0) = Mg \Rightarrow C = Mg$$

$$\therefore T(x) = \mu g x + Mg$$

Let the length dx at x increase by dr, then



$$\frac{T(x)/A}{dr/dx} = Y$$

or, $\dfrac{dr}{dx} = \dfrac{1}{YA} T(x)$

$$\Rightarrow r = \frac{1}{YA}\int_0^L (\mu g x + Mg)dx$$

$$= \frac{1}{YA}\left[\frac{\mu g x^2}{2} + Mgx\right]_0^L$$

$$= \frac{1}{YA}\left[\frac{mgl}{2} + MgL\right]$$

(m is the mass of the wire)

$$A = \pi \times (10^{-3})^2 \text{m}^2, \; Y = 200 \times 10^9 \text{Nm}^{-2}$$

$$m = \pi \times (10^{-3})^2 \times 10 \times 7860\text{kg}$$

$$\therefore r = \frac{1}{2 \times 10^{11} \times \pi \times 10^{-6}}\left[\frac{\pi \times 786 \times 10^{-7} \times 10 \times 10}{2} + 25 \times 10 \times 10\right]$$

$$= [196.5 \times 10^{-6} + 3.98 \times 10^{-3}] \;\; \sim 4 \times 10^{-3}\text{m}$$

(b) The maximum tension would be at x = L.

$T = \mu gL + Mg = (m+M)g$

The yield force

$$= 250 \times 10^6 \times \pi \times (10^{-3})^2 = 250 \times \pi \, N$$

At yield

$$(m + M)g = 250 \times \pi$$

$$m = \pi \times (10^{-3})^2 \times 10 \times 7860 \;\; << M \;\; \therefore Mg \sim 250 \times \pi$$

Hence, $M = \dfrac{250 \times \pi}{10} = 25 \times \pi \sim 75\text{kg}.$

9.26 Consider an element at r of width dr. Let $T(r)$ and $T(r+dr)$ be the tensions at the two edges.

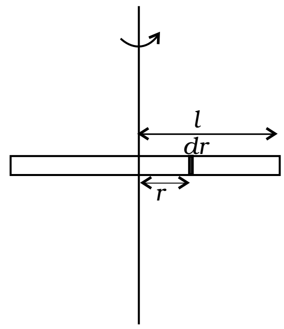
$-T(r+dr) + T(r) = \mu\omega^2 r dr$ where μ is the mass/length

$$-\frac{dT}{dr}dr = \mu\omega^2 r dr$$

At $r = l \quad T = 0$

$$\Rightarrow C = \frac{\mu\omega^2 l^2}{2}$$

$$\therefore T(r) = \frac{\mu\omega^2}{2}(l^2 - r^2)$$



Let the increase in length of the element dr be d (δ)

$$Y = \frac{(\mu\omega^2/2)(l^2 - r^2)/A}{\frac{d(\delta)}{dr}}$$

$$\therefore \frac{d(\delta)}{dr} = \frac{1}{YA}\frac{\mu\omega^2}{2}(l^2 - r^2)$$

$$\therefore d(\delta) = \frac{1}{YA}\frac{\mu\omega^2}{2}(l^2 - r^2)dr$$

$$\therefore \delta = \frac{1}{YA}\frac{\mu\omega^2}{2}\int_0^l (l^2 - r^2)dr$$

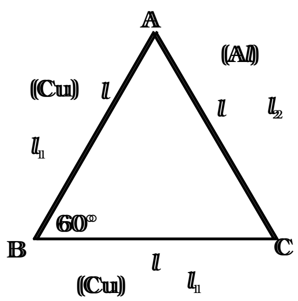
$$= \frac{1}{YA}\frac{\mu\omega^2}{2}\left[l^3 - \frac{l^3}{3}\right] = \frac{1}{3YA}\mu\omega^2 l^3 = \frac{1}{3YA}\mu\omega^2 l^2$$

The total change in length is $2\delta = \frac{2}{3YA}\mu\omega^2 l^2$

9.27 Let l_1 = AB, l_2 = AC, l_3 = BC

$$\cos\theta = \frac{l_3^2 + l_1^2 - l_2^2}{2l_3 l_1}$$

Or, $2l_3 l_1 \cos\theta = l_3^2 + l_1^2 - l_2^2$



Differenciating

$$2(l_3 dl_1 + l_1 dl_3)\cos\theta - 2l_1 l_3 \sin\theta d\theta$$

$$= 2l_3 dl_3 + 2l_1 dl_3 + 2l_1\alpha_1 - 2l_2\alpha_2$$

Now,
$$dl_1 = l_1\alpha_1\Delta t$$
$$dl_2 = l_2\alpha_1\Delta t$$
$$dl_3 = l_3\alpha_2\Delta t$$

and $l_1 = l_2 = l_3 = l$

$$(l^2\alpha_1\Delta t + l^2\alpha_1\Delta t)\cos\theta + l^2\sin\theta d\theta = l^2\alpha_1\Delta t + l^2\alpha_1\Delta t - l^2\alpha_2\Delta t$$

$$\sin\theta\, d\theta = 2\alpha_1\Delta t(1 - \cos\theta) - \alpha_2\Delta t$$

Putting θ = 60^o

$$d\theta\frac{\sqrt{3}}{2} = 2\alpha_1\Delta t \times (1/2) - \alpha_2\Delta t$$

$$= (\alpha_1 - \alpha_2)\Delta t$$

Or, $d\theta = \dfrac{2(\alpha_1 - \alpha_2)\Delta t}{\sqrt{3}}$

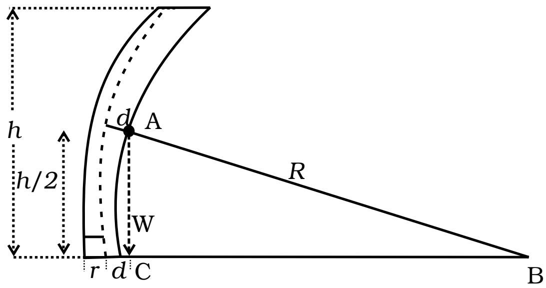
9.28 When the tree is about to buckle

$$Wd = \frac{Y\pi r^4}{4R}$$

If $R \gg h$, then the centre of gravity is at a height $l \simeq \frac{1}{2}h$ from the ground.

From Δ ABC

$$R^2 \simeq (R-d)^2 + \left(\frac{1}{2}h\right)^2$$



If $d \ll R$

$$R^2 \simeq R^2 - 2Rd + \frac{1}{4}h^2$$

$$\therefore d = \frac{h^2}{8R}$$

If w_0 is the weight/volume

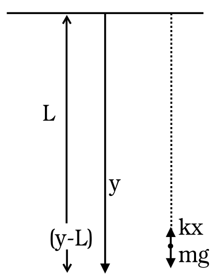
$$\frac{Y\pi r^4}{4R} = w_0(\pi r^2 h)\frac{h^2}{8R}$$

$$\Rightarrow h \simeq \left(\frac{2Y}{w_o}\right)^{1/3} r^{2/3}$$

9.29 (a) Till the stone drops through a length L it will be in free fall. After that the elasticity of the string will force it to a SHM. Let the stone come to rest instantaneously at y.

The loss in P.E. of the stone is the P.E. stored in the stretched string.

$$mgy = \frac{1}{2}k(y-L)^2$$



Or, $mgy = \frac{1}{2}ky^2 - kyL + \frac{1}{2}kL^2$

Or, $\frac{1}{2}ky^2 - (kL+mg)y + \frac{1}{2}kL^2 = 0$

$$y = \frac{(kL+mg) \pm \sqrt{(kL+mg)^2 - k^2L^2}}{k}$$

$$= \frac{(kL + mg) \pm \sqrt{2mgkL + m^2 g^2}}{k}$$

Retain the positive sign.

$$\therefore y = \frac{(kL + mg) + \sqrt{2mgkL + m^2 g^2}}{k}$$

(b) The maximum velocity is attained when the body passes, through the "equilibrium, position" i.e. when the instantaneous acceleration is zero. That is $mg - kx = 0$ where x is the extension from L:

$$\Rightarrow mg = kx$$

Let the velecity be v. Then

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = mg(L + x)$$

$$\frac{1}{2}mv^2 = mg(L + x) - \frac{1}{2}kx^2$$

Now $mg = kx$

$$x = \frac{mg}{k}$$

$$\therefore \frac{1}{2}mv^2 = mg\left(L + \frac{mg}{k}\right) - \frac{1}{2}k\frac{m^2 g^2}{k^2}$$

$$= mgL + \frac{m^2 g^2}{k} - \frac{1}{2}\frac{m^2 g^2}{k}$$

$$\frac{1}{2}mv^2 = mgL + \frac{1}{2}\frac{m^2 g^2}{k}$$

$$\therefore v^2 = 2gL + mg^2 / k$$

$$v = (2gL + mg^2 / k)^{1/2}$$

(c) Consider the particle at an instantaneous position y. Then

$$\frac{md^2 y}{dt^2} = mg - k(y - L)$$

$$\Rightarrow \frac{d^2 y}{dt^2} + \frac{k}{m}(y - L) - g = 0$$

Make a transformation of variables: $z = \frac{k}{m}(y - L) - g$

Then $\frac{d^2z}{dt^2} + \frac{k}{m}z = 0$

$\therefore z = A\cos(\omega t + \phi)$ where $\omega = \sqrt{\frac{k}{m}}$

$\Rightarrow y = \left(L + \frac{m}{k}g\right) + A'\cos(\omega t + \phi)$

Thus the stone performs SHM with angular frequency ω about the point

$y_0 = L + \frac{m}{k}g$