Chapter 10

10.1 (c)

10.2 (d)

10.3 (b)

10.4 (a)

10.5 (c)

10.6 (a), (d)

10.7 (c), (d)

10.8 (a), (b)

10.9 (c), (d)

10.10 (b), (c)

10.11 No.

10.12 No.

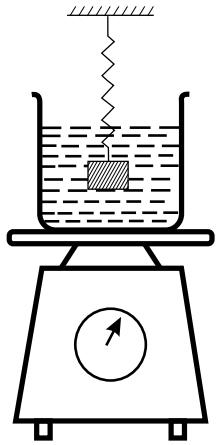
10.13 Let the volume of the iceberg be V. The weight of the iceberg is $\rho_i Vg$. If x is the fraction submerged, then the volume of water displaced is xV. The buoyant force is $\rho_w xVg$ where ρ_w is the density of water.

$$\rho_i Vg = \rho_w xVg$$

$$\therefore x = \frac{\rho_i}{\rho_w} = 0.917$$

10.14 Let x be the compression on the spring. As the block is in equilibrium

$$Mg - (kx + \rho_w Vg) = 0$$



where ρ_w is the density of water and V is the volume of the block. The reading in the pan is the force applied by the water on the pan i.e.,

$m_{vessel} + m_{water} + \rho_w Vg.$

Since the scale has been adjusted to zero without the block, the new reading is $\rho_w Vg$.

10.15 Let the density of water be ρ_w.

Then $\rho a L^3 + \rho L^3 g = \rho_w x L^3 (g + a)$

$$\therefore x = \frac{\rho}{\rho_w}$$

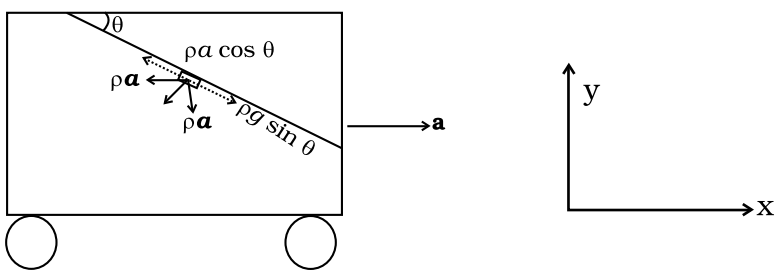
Thus, the fraction of the block submerged is independent of any acceleration, whether gravity or elevator.

10.16 The height to which the sap will rise is

$$h = \frac{2T\cos 0°}{\rho g r} = \frac{2(7.2\times10^{-2})}{10^3\times9.8\times2.5\times10^{-5}} \simeq 0.6\text{m}$$

This is the maximum height to which the sap can rise due to surface tension. Since many trees have heights much more than this, capillary action alone cannot account for the rise of water in all trees.

10.17 If the tanker acclerates in the positive x direction, then the water will bulge at the back of the tanker. The free surface will be such that the tangential force on any fluid parcel is zero.



Consider a parcel at the surface, of unit volume. The forces on the fluid are

$-\rho g\,\hat{\mathbf{y}}$ and $-\rho a\,\hat{\mathbf{x}}$

The component of the weight along the surface is $\rho g \sin\theta$

The component of the acceleration force along the surface is

$\rho a \cos\theta$

$\therefore \rho g \sin\theta = \rho a \cos\theta$

Hence, $\tan\theta = a/g$

10.18 Let v_1 and v_2 be the volume of the droplets and v of the resulting drop.

Then $v = v_1 + v_2$

$$\Rightarrow r^3 = r_1^3 + r_2^3 = (0.001 + 0.008)\,\text{cm}^3 = 0.009\,\text{cm}^3$$

$$\therefore r \simeq 0.21\,\text{cm}$$

$$\therefore \Delta U = 4\pi T\left(r^2 - \left(r_1^2 + r_2^2\right)\right)$$

$$= 4\pi \times 435.5 \times 10^{-3}\ (0.21^2 - 0.05) \times 10^{-4}\,\text{J}$$

$$\simeq -32 \times 10^{-7}\,\text{J}$$

10.19 $R^3 = Nr^3$

$$\Rightarrow r = \frac{R}{N^{1/3}}$$

$$\Delta U = 4\pi T(R^2 - Nr^2)$$

Suppose all this energy is released at the cost of lowering the temperature. If s is the specific heat then the change in temperature would be,

$$\Delta\theta = \frac{\Delta U}{ms} = \frac{4\pi T(R^2 - Nr^2)}{\frac{4}{3}\pi R^3 \rho s}, \text{where } \rho \text{ is the density.}$$

$$\therefore \Delta\theta = \frac{3T}{\rho s}\left(\frac{1}{R} - \frac{r^2}{R^3}N\right)$$

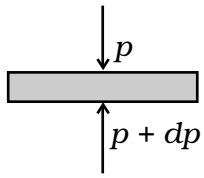
$$= \frac{3T}{\rho s}\left(\frac{1}{R} - \frac{r^2 R^3}{R^3 r^3}\right) = \frac{3T}{\rho s}\left(\frac{1}{R} - \frac{1}{r}\right)$$

10.20 The drop will evaporate if the water pressure is more than the vapour pressure. The membrane pressure (water)

$$p = \frac{2T}{r} = 2.33 \times 10^3\,\text{Pa}$$

$$\therefore r = \frac{2T}{p} = \frac{2(7.28 \times 10^{-2})}{2.33 \times 10^3} = 6.25 \times 10^{-5}\,\text{m}$$

10.21 (a) Consider a horizontal parcel of air with cross section A and height dh. Let the pressure on the top surface and bottom surface be p and $p+dp$. If the parcel is in equilibrium, then the net upward force must be balanced by the weight.



i.e. $(p+dp)A - pA = -PgAdh$

$\Rightarrow dp = -\rho g dh.$

(b) Let the density of air on the earth's surface be ρ_o, then

$$\frac{p}{p_o} = \frac{\rho}{\rho_o}$$

$$\Rightarrow \rho = \frac{\rho_o}{p_o} p$$

$$\therefore dp = -\frac{\rho_o g}{p_o} p dh$$

$$\Rightarrow \frac{dp}{p} = -\frac{\rho_o g}{p_o} dh$$

$$\Rightarrow \int_{p_o}^{p} \frac{dp}{p} = -\frac{\rho_o g}{p_o} \int_{o}^{h} dh$$

$$\Rightarrow \ln \frac{p}{p_o} = -\frac{\rho_o g}{p_o} h$$

$$\Rightarrow p = p_o \exp\left(-\frac{\rho_o g}{p_o} h\right)$$

(c) $$\ln \frac{1}{10} = -\frac{\rho_o g}{p_o} h_o$$

$$\therefore h_o = -\frac{p_o}{\rho_o g} \ln \frac{1}{10}$$

$$= \frac{p_o}{\rho_o g} \times 2.303$$

$$= \frac{1.013 \times 10^5}{1.29 \times 9.8} \times 2.303 = 0.16 \times 10^5 \text{m} = 16 \times 10^3 \text{m}$$

(d) The assumption $p \propto \rho$ is valid only for the isothermal case which is only valid for small distances.

10.22 (a) 1 kg of water requires L_v k cal

$\therefore M_A$ kg of water requires $M_A L_v$ k cal

Since there are N_A molecules in M_A kg of water the energy required for 1 molecule to evaporate is

$$u = \frac{M_A L_v}{N_A} \text{J}$$

$$= \frac{18 \times \overset{90}{\cancel{540}} \times 4.2 \times 10^3}{\cancel{6} \times 10^{26}} \text{J}$$

$$= 90 \times 18 \times 4.2 \times 10^{-23} \text{J}$$

$$\simeq 6.8 \times 10^{-20} \text{J}$$

(b) Consider the water molecules to be points at a distance d from each other.

N_A molecules occupy $\frac{M_A}{\rho_w} l$

Thus, the volume around one molecule is $\frac{M_A}{N_A \rho_w} l$

The volume around one molecule is $d^3 = (M_A/N_A \rho_w)$

$$\therefore d = \left(\frac{M_A}{N_A \rho_w}\right)^{1/3} = \left(\frac{18}{6 \times 10^{26} \times 10^3}\right)^{1/3}$$

$$= (30 \times 10^{-30})^{1/3} \text{ m} \simeq 3.1 \times 10^{-10} \text{m}$$

(c) 1 kg of vapour occupies 1601×10^{-3} m^3.

$\therefore$ 18 kg of vapour occupies $18 \times 1601 \times 10^{-3}$ m^3

$\Rightarrow 6 \times 10^{26}$ molecules occupies $18 \times 1601 \times 10^{-3}$ m^3

$\therefore$ 1 molecule occupies $\frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}}$ m^3

If d' is the inter molecular distance, then

$d'^3 = (3 \times 1601 \times 10^{-29})$m^3

$\therefore d' = (30 \times 1601)^{1/3} \times 10^{-10}$ m

$= 36.3 \times 10^{-10}$ m

(d) $F(d'-d) = u \Rightarrow F = \dfrac{u}{d'-d} = \dfrac{6.8\times10^{-20}}{(36.3-3.1)\times10^{-10}} = 0.2048\times10^{-10}\,\text{N}$

(e) $F/d = \dfrac{0.2048\times10^{-10}}{3.1\times10^{-10}} = 0.066\,\text{N m}^{-1} = 6.6\times10^{-2}\,\text{N m}^{-1}$

10.23 Let the pressure inside the balloon be P_i and the outside pressure be P_o

$$P_i - P_o = \frac{2\gamma}{r}$$

Considering the air to be an ideal gas

$P_i V = n_i R T_i$ where V is the volume of the air inside the balloon, n_i is the number of moles inside and T_i is the temperature inside, and $P_o V = n_o R T_o$ where V is the volume of the air displaced and n_o is the number of moles displaced and T_o is the temperature outside.

$n_i = \dfrac{P_i V}{R T_i} = \dfrac{M_i}{M_A}$ where M_i is the mass of air inside and M_A is the molar mass of air and $n_o = \dfrac{P_o V}{R T_o} = \dfrac{M_o}{M_A}$ where M_o is the mass of air outside that has been displaced. If W is the load it can raise, then

$W + M_i g = M_o g$

$\Rightarrow W = M_o g - M_i g$

Air is 21% O_2 and 79% N_2

$\therefore$ Molar mass of air M_A = 0.21×32 + 0.79×28 = 28.84 g.

$$\Rightarrow W = \frac{M_A V}{R}\left(\frac{P_o}{T_o} - \frac{P_i}{T_i}\right) g$$

$$= \frac{0.02884\times\frac{4}{3}\pi\times8^3\times9.8}{8.314}\left(\frac{1.013\times10^5}{293} - \frac{1.013\times10^5}{333} - \frac{2\times5}{8\times313}\right)\text{N}$$

$$\simeq \frac{0.02884\times\frac{4}{3}\pi\times8^3}{8.314}\times1.013\times10^5\left(\frac{1}{293} - \frac{1}{333}\right)\times9.8\text{N}$$

= 3044.2 N.