Chapter 12

12.1 (c) adiabatic

A is isobaric process, D is isochoric. Of B and C, B has the smaller slope (magnitude) hence is isothermal. Remaing process is adiabatic.

12.2 (a)

12.3 (c)

12.4 (b)

12.5 (a)

12.6 (b)

12.7

(a), (b) and (d).

12.8 (a), (d)

12.9 (b), (c)

12.10 (a), (c)

12.11 (a), (c)

12.12 If the system does work against the surroundings so that it compensates for the heat supplied, the temperature can remain constant.

12.13 $U_p - U_Q$ = W.D. in path 1 on the system + 1000 J

= W.D. in path 2 on the system + Q

$$Q = (-100 + 1000)\text{J} = 900\,\text{J}$$

12.14 Here heat removed is less than the heat supplied and hence the room, including the refrigerator (which is not insulated from the room) becomes hotter.

12.15 Yes. When the gas undergoes adiabatic compression, its temperature increases.

$dQ = dU + dW$

As $dQ = 0$ (adiabatic process)

so $dU = -dW$

In compression, work is done on the system So, dW = -ve

$\Rightarrow dU = +$ ve

So internal energy of the gas increases, i.e. its temperature increases.

12.16 During driving, temperature of the gas increases while its volume remains constant.

So according to Charle's law, at constant V, $P \alpha T$.

Therefore, pressure of gas increases.

12.17 $\dfrac{Q}{Q_1} = \dfrac{T_2}{T_1} = \dfrac{3}{5}$, $Q_1 - Q_2 = 10^3\,\text{J}$

$$Q_1\left(1 - \frac{3}{5}\right) = 10^3\,\text{J} \Rightarrow Q_1 = \frac{5}{2} \times 10^2\,\text{J} = 2500\text{ J},\ Q_2 = 1500\text{J}$$

12.18 $5 \times 7000 \times 10^3 \times 4.2\text{J} = 60 \times 15 \times 10 \times \text{N}$

$$N = \frac{21 \times 7 \times 10^6}{900} = \frac{147}{9} \times 10^3 = 16.3 \times 10^3 \text{ times.}$$

12.19 $P(V+\Delta v)^{\gamma} = (P+\Delta p)V^{\gamma}$

$$P\left[1+\gamma\frac{\Delta v}{V}\right] = P\left(1+\frac{\Delta p}{P}\right)$$

$$\gamma\frac{\Delta v}{V} = \frac{\Delta p}{P};\ \frac{dv}{dp} = \frac{V}{\gamma p}$$

$$\text{W.D.} = \int_{P_1}^{P_2} p\,dv = \int_{P_1}^{P_2} p\frac{V}{\gamma p}dp = \frac{(P_2-P_1)}{\gamma}V$$

12.20 $\eta = 1-\frac{270}{300} = \frac{1}{10}$

Efficiency of refrigerator $= 0.5\eta = \frac{1}{20}$

If Q is the heat/s transferred at higher temperture then $\frac{W}{Q} = \frac{1}{20}$

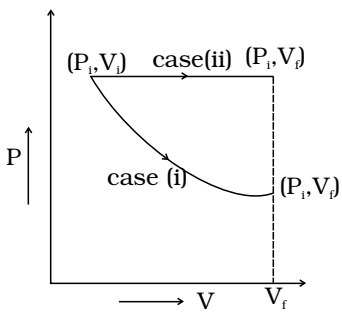
or Q = 20W = 20kJ,

and heat removed from lower temperture = 19 kJ.

12.21 $\frac{Q_2}{W} = 5$, $Q_2 = 5\text{W}, Q_1 = 6\text{W}$

$\frac{T_2}{T_1} = \frac{5}{6} = \frac{T}{300}$, $T_2 = 250\text{K} = -23^{\circ}\text{C}$

12.22 The *P-V* digram for each case is shown in the figure. In case (i) $P_i\ V_i = P_f\ V_f$; therfore process is isothermal. Work done = area under the PV curve so work done is more when the gas expands at constant pressure.



12.23 (a) Work done by the gas (Let $PV^{1/2} = A$)

$$\Delta W = \int_{V_1}^{V_2} pdv = A\int_{V_1}^{V_2}\frac{dV}{\sqrt{V}} = A\left[\frac{\sqrt{V}}{1/2}\right]_{V_1}^{V_2} = 2A\left(\sqrt{V_2}-\sqrt{V_1}\right)$$

$$= 2P_1V_1^{1/2}\left[V_2^{1/2}-V_1^{1/2}\right]$$

(b) Since $T = pV/nR = \frac{A}{nR}.\sqrt{V}$

Thus, $\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{2}$

(c) Then, the change in internal energy

$$\Delta U = U_2 - W_1 = \frac{3}{2}R(T_2-T_1) = \frac{3}{2}RT_1(\sqrt{2}-1)$$

$$\Delta W = 2A\sqrt{V_1}\left(\sqrt{2}-1\right) = 2RT_1\left(\sqrt{2}-1\right)$$

$$\Delta Q = (7/2)RT_1\left(\sqrt{2}-1\right)$$

12.24 (a) A to B

(b) C to D

(c) $W_{AB} = \int_A^B p\,dV = 0; W_{CD} = 0$.

Similarly. $W_{BC} = \left[\int_B^C pdV = k\int_B^C \frac{dV}{V^r} = k\frac{V^{-r+1}}{-R+1}\right]_{V_B}^{V_C}$

$$= \frac{1}{1-\gamma}(P_cV_c - P_BV_B)$$

Similarly, $W_{DA} = \frac{1}{1-\gamma}(P_AV_A - P_DV_D)$

Now $P_C = P_B\left(\frac{V_B}{V_C}\right)^{\gamma} = 2^{-\gamma}P_B$

Similarly, $P_D = P_A\,2^{-\gamma}$

Totat work done = $W_{BC} + W_{DA}$

$$= \frac{1}{1-\gamma}\left[P_BV_B\left(2^{-\gamma+1}-1\right) - P_AV_A\left(2^{-\gamma+1}-1\right)\right]$$

$$= \frac{1}{1-\gamma}(2^{1-\gamma}-1)(P_B-P_A)V_A$$

$$= \frac{3}{2}\left(1-\left(\frac{1}{2}\right)^{2/3}\right)(P_B-P_A)V_A$$

(d) Heat supplied during process A, B

$dQ_{AB} = dU_{AB}$

$$Q_{AB} = \frac{3}{2}nR(T_B-T_A) = \frac{3}{2}(P_B-P_A)V_A$$

Efficiency = $\frac{\text{Net Work done}}{\text{Heat Supplied}} = \left[1-\left(\frac{1}{2}\right)^{2/3}\right]$

12.25 $Q_{AB} = U_{AB} = \frac{3}{2}R(T_B-T_A) = \frac{3}{2}V_A(P_B-P_A)$

$$Q_{Bc} = U_{BC} + W_{BC}$$

$$= (3/2)\ P_B(V_C - V_B) + P_B(V_C - V_B)$$

$$= (5/2\ )P_B(V_C - V_A)$$

$Q_{CA} = 0$

$Q_{DA} = (5/2)\ P_A\ (V_A - V_D)$

12.26 Slope of $P = f(V)$, curve at (V_o, P_o)

$$= f'(V_o)$$

Slope of adiabat at (V_o, P_o)

$$= k\ (-\gamma)\ V_o^{-1-\gamma} = -\gamma\ P_o/V_o$$

Now heat absorbed in the process $P = f(V)$

$dQ = dV + dW$

$= nC_v dT + P\,dV$

Since $T = (1/nR)\ PV = (1/nR)\ V f(V)$

$$dT = (1/nR)\ [f(V) + V\ f'(V)]\ dV$$

Thus

$$\left.\frac{dQ}{dV}\right|_{V=V_o} = \frac{CV}{R}\left[f(V_o) + V_o f'(V_o)\right] + f(V_o)$$

$$= \left[\frac{1}{\gamma-1} + 1\right] f(V_o) + \frac{V_o f'(V_o)}{\gamma-1}$$

$$= \frac{\gamma}{\gamma-1} P_o + \frac{V_o}{\gamma-1} f'(V_o)$$

Heat is absorbed when $dQ/dV > 0$ when gas expands, that is when

$\gamma P_o + V_o f'(V_o) > 0$

$f'(V_o) > -\gamma P_o/V_o$

12.27 (a) $P_i = P_a$

(b) $P_f = P_a + \frac{k}{A}(V - V_o) = P_a + k(V - V_o)$

(c) All the supplied heat is converted to mechanical energy. No change in internal energy (Perfect gas)

$$\Delta Q = P_a(V - V_o) + \frac{1}{2}k(V - V_o)^2 + C_V(T - T_o)$$

where $T_o = P_a V_o/R$,

$T = [P_a + (R/A)\text{-}(V\text{-}V_o)]V/R$