Chapter 14

14.1 (b)

14.2 (b)

14.3 (d)

14.4 (c)

14.5 (c)

14.6 (d)

14.7 (b)

14.8 (a)

14.9 (c)

14.10 (a)

14.11 (b)

14.12 (a), (c)

14.13 (a), (c)

14.14 (d), (b)

14.15 (a), (b), (d)

14.16 (a), (b), (c)

14.17 (a), (b) (d)

14.18 (a), (c), (d)

14.19 (i) (A),(C),(E),(G) (ii) (B), (D), (F), (H)

14.20 $2kx$ towards left.

14.21 (a) Acceleration is directly proportional to displacement.

(b) Acceleration is directed opposite to displacement.

14.22 When the bob of the pendulum is displaced from the mean position so that $\sin\theta$

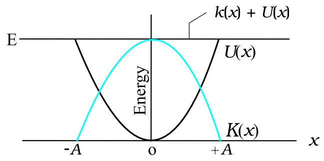
$\cong \theta$

14.23 $+\omega$

14.24 Four

14.25 -ve

14.27



(b)

14.28 $l_m = \frac{1}{6} l_E = \frac{1}{6}\,\text{m}$

14.29 If mass m moves down by h, then the spring extends by $2h$ (because each side expands by h). The tension along the string and spring is the same.

In equilibrium

$$mg = 2\,(k.\ 2h)$$

where k is the spring constant.

On pulling the mass down by x,

$$F = \cancel{mg} - 2k\left(\cancel{2h} + 2x\right)$$

$$= -\,4kx$$

So. $T = 2\pi\sqrt{\dfrac{m}{4k}}$

14.30 $y = \sqrt{2}\sin\left(\omega t - \pi/4\right); T = 2\pi/\omega$

14.31 $\dfrac{A}{\sqrt{2}}$

14.32 $U = U_o\left(1 - \cos\alpha x\right)$

$$F = \frac{-dU}{dx} = \frac{-d}{dx}\left(U_o - U_o\cos\alpha x\right)$$

$$= -U_o\alpha\sin\alpha x$$

$$\simeq -U_o\alpha\alpha x \quad (\text{for small}\ \alpha x, \sin\alpha x \sim \alpha x)$$

$$= -U_o\alpha^2 x$$

We know that $F = -kx$

So, $k = U_o\alpha^2$

$$T = 2\pi\sqrt{\frac{m}{U_o\alpha^2}}$$

14.33 $x = 5 \sin 5t$.

14.34 $\theta_1 = \theta_o \sin(\omega t + \delta_1)$

$\theta_2 = \theta_o \sin(\omega t + \delta_2)$

For the first, $\theta = 2°, \therefore \sin(\omega t + \delta_1) = 1$

For the 2nd, $\theta = -1°, \therefore \sin(\omega t + \delta_2) = -1/2$

$\therefore \omega t + \delta_1 = 90°, \ \omega t + \delta_2 = -30°$

$\therefore \delta_1 - \delta_2 = 120°$

14.35 (a) Yes.

(b) Maximum weight = $Mg + MA\omega^2$

$$= 50 \times 9.8 + 50 \times \frac{5}{100} \times (2\pi \times 2)^2$$

= 490 + 400 = 890N.

Minimum weight = $Mg - MA\omega^2$

$$= 50 \times 9.8 - 50 \times \frac{5}{100} \times (2\pi \times 2)^2$$

= 490–400

= 90 N.

Maximum weight is at the topmost position,

Minimum weight is at the lowermost position.

14.36 (a) 2cm (b) 2.8 s^{-1}

14.37 Let the log be pressed and let the vertical displacement at the equilibrium position be x_o.

At equilibrium

mg = Buoyant force

$= Ax_o\rho g$

When it is displaced by a further displacement x, the buoyant force is $A(x_o + x)\rho g$.

Net restoning force

= Buoyant force – weight

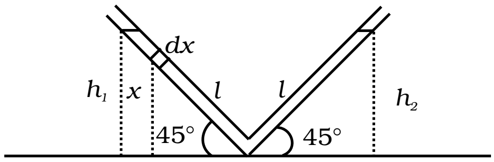
$= A(x_o + x)\rho g - mg$

$= (A\rho g)x$. i.e. proportional to x.

$$\therefore T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

14.38 Consider the liquid in the length dx. It's mass is $A\rho dx$ at a height x.

PE = $A\rho dx\, gx$



The PE of the left column

$$= \int_o^{h_1} A\rho gxdx$$

$$= A\rho g \left.\frac{x^2}{2}\right|_o^{h_1} = A\rho g \frac{h_1^{\;2}}{2} = \frac{A\rho g l^2 \sin^2 45°}{2}$$

Similarly, P.E. of the right column $= A\rho g \dfrac{h_2^{\;2}}{2} = \dfrac{A\rho g l^2 \sin^2 45°}{2}$

$h_1 = h_2 = l \sin 45°$ where l is the length of the liquid in one arm of the tube.

Total P.E. $= A\rho g h^2 = A\rho g l^2 \sin^2 45° = \dfrac{A\rho g l^2}{2}$

If the change in liquid level along the tube in left side in y, then length of the liquid in left side is $l–y$ and in the right side is $l + y$.

Total P.E. $= A\rho g(l - y)^2 \sin^2 45° + A\rho g(l + y)^2 \sin^2 45°$

Change in PE = $(\text{PE})_f - (\text{PE})_i$

$$= \frac{A\rho g}{2}\left[(l - y)^2 + (l + y)^2 - l^2\right]$$

$$= \frac{A\rho g}{2}\left[\cancel{l^2} + y^2 - 2\cancel{ly} + l^2 + y^2 + 2\cancel{ly} - \cancel{l^2}\right]$$

$$= A\rho g\left\lfloor y^2 + l^2 \right\rfloor$$

Change in K.E. $= \dfrac{1}{2} A\rho 2l\dot{y}^2$

Change in total energy = 0

$$\Delta(P.E) + \Delta(K.E) = 0$$

$$A\rho g\left[l^2 + y^2\right] + A\rho l\dot{y}^2 = 0$$

Differentiating both sides w.r.t. time,

$$A\rho g\left[0+2y\frac{dy}{dt}\right]+2A\rho l\dot{y}\ddot{y}=0$$

$$2A\rho gy+2A\rho l\ddot{y}=0$$

$$l\ddot{y}+gy=0$$

$$\ddot{y}+\frac{g}{l}y=0$$

$$\omega^2=\frac{g}{l}$$

$$\omega=\sqrt{\frac{g}{l}}$$

$$T=2\pi\sqrt{\frac{l}{g}}$$

14.39 Accelertation due to gravity at $P=\frac{g.x}{R}$, where g is the acceleration at the surface.

Force $=\frac{mgx}{R}=-k.x,\quad k=\frac{mg}{R}$

Motion will be SHM with time period $T=\sqrt{\frac{m}{K}}=2\pi\sqrt{\frac{R}{g}}$

14.40 Assume that t = 0 when $\theta=\theta_o$. Then,

$\theta=\theta_o\cos\omega t$

Given a seconds pendulum $\omega=2\pi$

At time t_1, let $\theta=\theta_o/2$

$\therefore\qquad \cos 2\pi t_1=1/2\quad\Rightarrow t_1=\frac{1}{6}$

$\dot{\theta}=-\theta_0 2\pi\sin\ 2\pi t\qquad\left[\dot{\theta}=\frac{d\theta}{dt}\right]$

At $t_1=1/6$

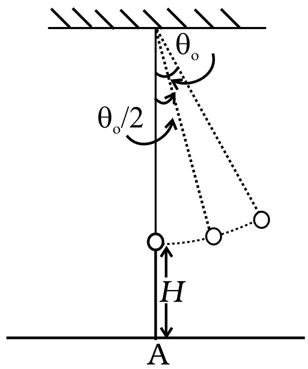
$\dot{\theta}=-\theta_0 2\pi\sin\frac{2\pi}{6}=-\sqrt{3}\pi\theta_0$

Thus the linear velocity is

$u=-\sqrt{3}\pi\theta_0 l$ perpendicular to the string.

The vertical component is

$u_y=-\sqrt{3}\pi\theta_0 l\sin\theta_0$



and the horizontal component is

$u_x = -\sqrt{3}\pi\theta_0 l\cos\theta_0$

At the time it snaps, the vertical height is

$H' = H + l\left(1 - \cos\left(\theta_0 / 2\right)\right)$

Let the time required for fall be t, then

$H' = u_y t + (1/2)\,gt^2$ (notice g is also in the negative direction)

Or, $\frac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 l\sin\theta_0 t - H' = 0$

$$\therefore t = \frac{-\sqrt{3}\pi\theta_0 l\sin\theta_0 \pm \sqrt{3\pi^2\theta_0^2\,e^2\sin^2\theta_0 + 2gH'}}{g}$$

$$\simeq \frac{-\sqrt{3}\pi l\theta_0^2 \pm \sqrt{3\pi^2\theta_0^4\,l^2 + 2gH'}}{g}$$

Neglecting terms of order θ_0^2 and heigher,

$t \simeq \sqrt{\frac{2H'}{g}}$.

Now $H' \simeq H + l(1-1) = H \therefore t \simeq \sqrt{\frac{2H}{g}}$

The distance travelled in the x direction is $u_x t$ to the left of where it snapped.

$X = \sqrt{3}\pi\theta_0 l\cos\theta_0\sqrt{\frac{2H}{g}}$

To order of θ_0,

$X = \sqrt{3}\pi\theta_0 l\sqrt{\frac{2H}{g}} = \sqrt{\frac{6H}{g}}\theta_0 l$.

At the time of snapping, the bob was

$l\sin\theta_0 \simeq l\theta_0$ distance from A.

Thus, the distance from A is

$l\theta_0 - \sqrt{\frac{6H}{g}}l\theta_0 = l\theta_0\,(1 - \sqrt{6H/g})$.