Simple Harmonic Motion (SHM)

1. Periodic motion

If a particle moves such that it repeats its path regularly after equal intervals of time , it's motion is said to be periodic.

The interval of time required to complete one cycle of motion is called time period of motion.

If a body in periodic motion moves back and forth over the same path then the motion is said to be viberatory or oscillatory.

Examples of such motion are to and fro motion of pendulum , viberations of a tuning fork , mass attached to a spring and many more.

Every oscillatory motion is periodic but every periodic motion is not oscillatory for example motion of earth around the sun is periodic but not oscillatory.

Simple Harmonic Motion (or SHM) is the simplest form of oscillatory motion.

SHM arises when force on oscillating body is directly proportional to the displacement from it's equilibrium position and at any point of motion , this force is directed towards the equilibrium position.

2. Simple Harmonic Motion (or SHM)

SHM is a particular type of motion very common in nature.

In SHM force acting on the particle is always directed towards a fixed point known as equilibrium position and the magnitude of force is directly proportional to the displacement of particle from the equilibrium position and is given by

$$F = -kx$$

where k is the force constant and negative sign shows that fforce opposes increase in x.

This force os known as restoring force which takes the particle back towards the equilibrium position , and opposes increase in displacement.

S.I. unit of force constant k is N/m and magnitude of k depends on elastic properties of system under consideration.

For understanding the nature of SHM consider a block of mass m whose one end is attached to a spring and another end is held stationary and this block is placed on a smooth horizontal surface shown below in the fig.

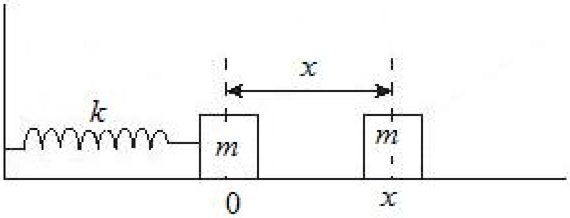


Figure a

Motion of the body can be described with coordinate x taking x=0 i.e. origin as the equilibrium positionwhere the spring is neither stretched or compressed.

We now take the block from it's equilibrium position to a point P by stretching the spring by a distance OP=A and will then release it.

After we release the block at point P, the restoring force acts on the block towards equilibrium position O and the block is then accelerated from point P towards point O as shown below in the fig.

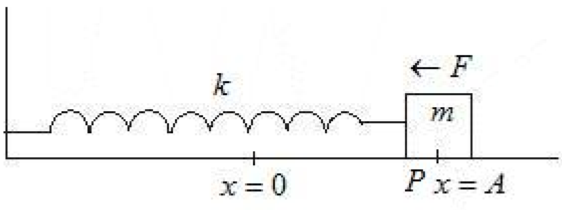


Figure b

Now at equilibrium position this restoring force would become zero but the velocity of block increases as it reaches from point P to O.

When the block reaches point O it's velocity would be maximum and it then starts to move towards left of equilibrium position O.

Now this time while going to the left of equilibrium position spring is compressed and the block moves to the point Q where it's velocity becomes zero.

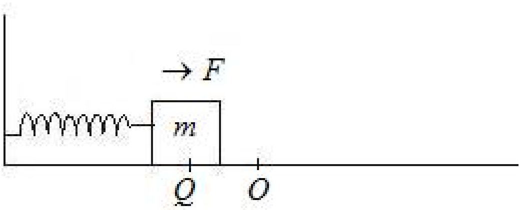


Figure c

The compressed spring now pushes the block towards the right of equilibrium position where it's velocity increases upto point O and decreases to zero when it reaches point P.

This way the block oscillates to and fro on the frictionless surface between points P and Q.

If the distance travelled on both sides of equilibrium position are equal i.e. , OP=OQ then the maximum displacement on either sides of equilibrium are called the Amplitude of oscillations.

Simple Harmonic Motion(SHM) Concept map

Simple Harmonic Motion

Equation of SHM : $d^2x/dt^2 = -kx/m$

If $\phi = \surd(k/m)$ then equation becomes

$d^2x/dt^2 = -\varphi^2 x$

Solution is

$x = A\cos(\omega t + \varphi)$

Velocity of SHM –

$$v = -\omega A\left(1 - \frac{x^2}{A^2}\right)$$

Acceleration -

$a = -\omega^2 A\cos(\omega t + \varphi)$

Total energy in SHM

$$E = \frac{1}{2} m\omega^2 A^2$$

Characteristics of SHM

Amplitude - A is called the amplitude of SHM

Time period - $T = 2\pi\surd(m/k)$

Frequency – $1/T$

Angular frequency – $\omega = 2\pi/T$

Phase - $(\omega t + \varphi)$ is phase of SHM constant φ is known as initial phase

3. Equation of SHM

Consider any particle executing SHM with origin as it's equilibrium position under the influence of restoring force F=

kx , where k is the force constant and x is the displacement of particle from the equilibrium position.

Now since F= -kx is the restoring force and from Newton's law of motion force is give as F=ma , where m is the mass of the particle moving with acceleration a. Thus acceleration of the particle is

$$a=F/m$$

$$=-kx/m$$

but we know that acceleration $a=dv/dt=d^2x/dt^2$

□ $$d^2x/dt^2=-kx/m \qquad (1)$$

This equation 1 is the equation of motion of SHM.

If we choose a constant $\varphi=\sqrt{(k/m)}$ then equation 1 would become

$$d^2x/dt^2=-\varphi^2x \qquad (2)$$

This equation is a differential equation which says that displacement x must be a funcyion of time such that when it's second derivative is calculated the result must be negative constant multiplied by the original function. Sine and cosine functions are the functions satisfying above requirement and are listed as follows

$$x=A \sin\omega t \qquad (3a)$$

$$x=A \cos\omega t \qquad (3b)$$

$$x=A \cos(\omega t+\varphi) \qquad (3c)$$

each one of equation 3a, 3b and 3c can be submitted on the left hand side of equation 2 and can then be solved for varification.

Convinently we choose equation 3c i.e., cosine form for representing displacement of particle at any time t from equilibrium position. Thus,

$$x=A \cos(\omega t+\varphi) \qquad (4)$$

and A , φ and φ are all constants.

Fig below shows the displacement vs. time graph for phase φ=0.

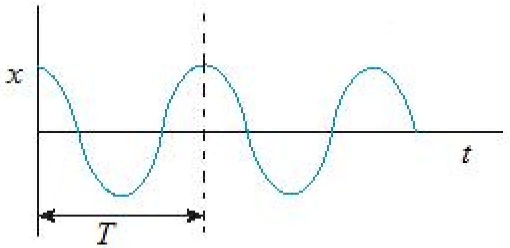


Figure 1

4. Characterstics of SHM

Here in this section we will learn about physical meaning of quantities like A, T, ω and φ.

(a) Amplitude

Quantity A is known as amplitude of motion. it is a positive quantity and it's value depends on how oscillations

were started.
Amplitude is the magnitude of maximum value of displacement on either side from the equilibrium position.
Since maximum and minimum values of any sine and cosine function are +1 and -1 , the maximum and minimum values of x in equation 4 are +A and -A respectively.
Finally A is called the amplitude of SHM.

(b) Time period

Time interval during which the oscillation repeats itself is known as time period of oscillations and is denoted by T.
Since a particle in SHM repeats it's motion in a regular interval T known as time period of oscillation so displacement x of particle should have same value at time t and t+T. Thus,

$$\cos(\omega t+\varphi)=\cos(\omega(t+T)+\varphi)$$

cosine function $\cos(\omega t+\varphi)$ will repeat it's value if angle $(\omega t+\varphi)$ is increased by 2π or any of it's multiple. As T is the pime period

$$(\omega(t+T)+\varphi)=(\omega t+\varphi)+2\pi$$

or, $$T=2\pi/\text{`} = 2\pi\sqrt{(m/k)} \qquad (5)$$

Equation 5 gives the time period of oscillations.
Now the frequency of SHM is defined as the numberof complete oscillations per unit time i.e., frequency is reciprocal of time period.

$$f=1/T = 1/2\pi(\sqrt{(k/m)}) \qquad (6)$$

Thus, $$\omega=2\text{`}/T = 2\text{`}f \qquad (7)$$

This quantity ω is called the angular frequency of SHM.
S.I. unit of T is s (seconds)
f is Hz (hertz)
ω is rad s^{-1} (radian per second)

(c) Phase

Quantity $(\omega t+\varphi)$ in equation (4) is known as phase of the motion and the constant φ is known as initial phase i.e., phase at time t=0, or phase constant.
Value of phase constant depends on displacement and velocity of particle at time t=0.
The knowledge of phase constant enables us to know how far the particle is from equilibrium at time t=0. For example,
If $\varphi=0$ then from equation 4

$$x=A\cos\omega t$$

that is displacement of oscillating particleis maximum , equal to A at t=0 when the motion was started. Again if $\varphi=\text{`}/2$ then from equation 4

$$x=A\cos(\omega t+\text{`}/2)$$
$$=A\sin\omega t$$

which means that displacement is zero at t=0.
Variation of displacement of particle executing SHM is shown below in the fig.

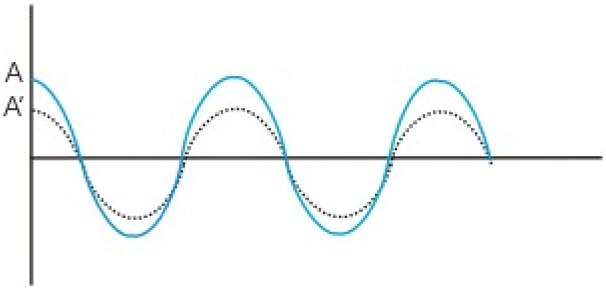


Figure 1a:- Figure shows the displacement Vs time graph for different amplitudes where A>A'

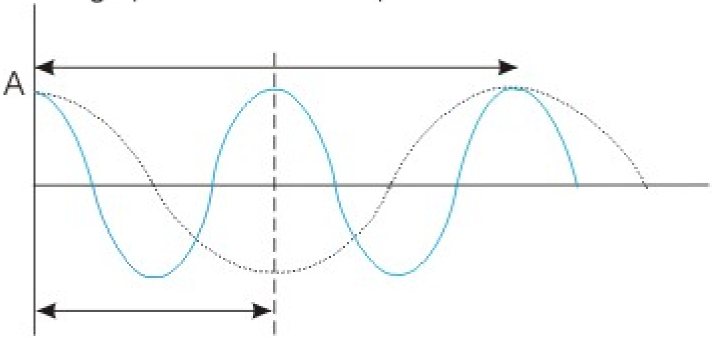


Figure 1b:- Figure shows the graph of SHM with different time periods where T'=T/2

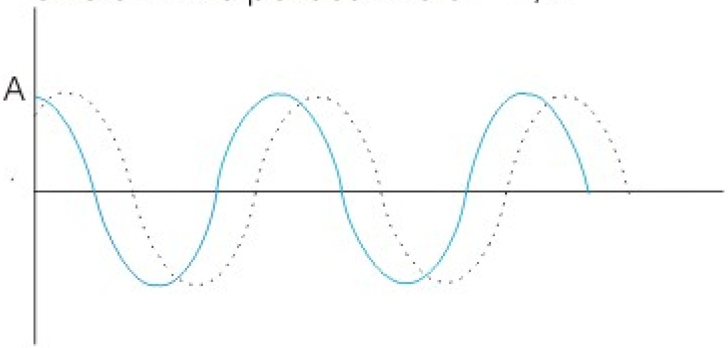


Figure 1c:- Figure shows SHM with different initial phase φ

5.Velocity of SHM

We know that velocity of a particle is given by

$v = dx/dt$

In SHM displacement of particle is given by

$x = A\cos(\omega t+\varphi)$

now differentiating it with respect to t

$v = dx/dt = A\omega(-\sin(\omega t+\varphi))$ (8)

Here in equation 8 quantity $A\omega$ is known as velocity amplitude and velocity of oscillating particle varies between the limits $\pm\omega$.

From trignometry we know that

$\cos^2\theta + \sin^2\theta = 1$

□

$A^2 \sin^2(\omega t+\varphi) = A^2 - A^2\cos^2(\omega t+\varphi)$

Or

$\sin(\omega t+\varphi) = [1 - x^2/A^2]$ (9)

putting this in equation 8 we get,

$$v = -\omega A\left(1-\frac{x^2}{A^2}\right)^{1/2}$$

From this equation 10 we notice that when the displacement is maximum i.e. ±A the velocity v=0, because now the oscillator has to return to change it□s direction.

Figure below shows the variation of velocity with time in SHM with initial phase $\varphi=0$.

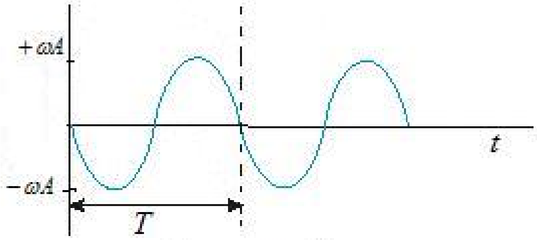


Figure 3

6. Acceleration of SHM

Again we know that acceleration of a particle is given by

$a = dv/dt$

where v is the velocity of particle executing motion.

In SHM velocity of particle is give by,

$v = -\omega\sin(\omega t+\varphi)$

differentiating this we get,

$$a = \frac{d}{dt}(-\omega A\sin(\omega t+\varphi))$$

or,

$a = -\omega^2 A\cos(\omega t+\varphi)$ (11)

Equation 11 gives acceleration of particle executing simple harmonic motion and quantity ω^2 is called acceleration amplitude and the acceleration of oscillating particle varies betwen the limits $\pm\omega^2 A$.

Putting equation 4 in 11 we get

$$a=-\omega^2 x \qquad (12)$$

which shows that acceleration is proportional to the displacement but in opposite direction.

Thus from above equation we can see that when x is maximum (+A or -A), the acceleration is also maximum($-\omega^2 A$ or $+\omega^2 A$)but is directed in direction opposite to that of displacement.

Figure below shows the variation of acceleration of particle in SHM with time having initial phase φ=0.

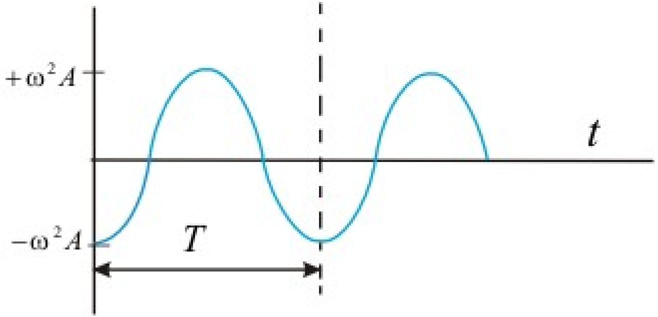


Figure :- 4

7. Total energy in SHM

When a system at rest is displaced from its equilibrium position by doing work on it, it gains potential energy and when it is released, it begins to move with a velovity and acquires kinetic energy.
If m is the mass of system executing SHM then kinetic energy of system at any instant of time is

$$K=(1/2)mv^2 \qquad (13)$$

putting equation 8 in 13 we get,

$$KE=\frac{1}{2}m(-\omega A\sin(\omega t+\phi))^2$$
$$=\frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi) \qquad (14)$$

From equation (14) we see that Kinetic Energy of system varies periodically i.e., it is maximum (= $(1/2)m\omega^2A^2$) at the maximum value of velocity ($\pm\omega A$) and at this time displacement is zero.
When displacement is maximum ($\pm A$), velocity of SHM is zero and hence kinetic energy is also zero and at these extreme points where kinetic energy K=0, all the energy is potential.
At intermediate positions of lying between 0 and $\pm A$, the energy is partly kinetic and partly potential.
To calculate potential energy at instant of time consider that x is the displacement of the system from its equilibrium at any time t.
We know that potential energy of a system is given by the amount of work required to move system from position 0 to x under the action of applied force.
Here force applied on the system must be just enough to oppose the restoring force -kx i.e., it should be equal to kx.
Now work required to give infinitesimal displacement is dx=kx dx.
Thus, total work required to displace the system from 0 to x is

$$=\int_0^x kx\,dx=\frac{1}{2}kx^2$$

thus,

$$PE=\frac{1}{2}kx^2$$
$$=\frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi) \qquad (15)$$

where, from equation 5 $\omega=\sqrt{(k/m)}$ and displacement $x=A\cos(\omega t+\varphi)$.

E=KE+PE

$$=\frac{1}{2}m\omega^2A^2[\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi)]$$

or,

$$E=\frac{1}{2}m\omega^2A^2$$

Thus total energy of the oscillator remains constant as displacement is regained after every half cycle.

If no energy is dissipated then all the potential energy becomes kinetic and vice versa.

Figure below shows the variation of kinetic energy and potential energy of harmonic oscillator with time where phase φ is set to zero for simplicity.

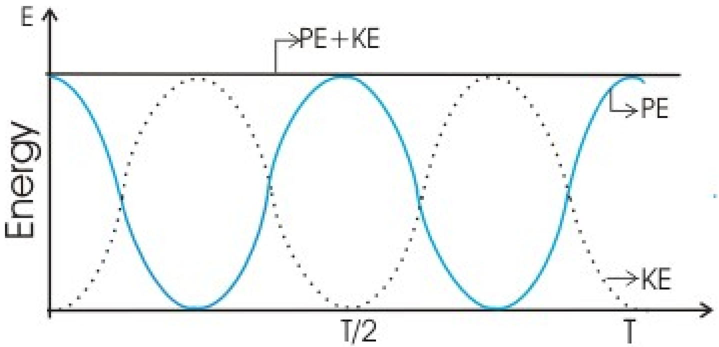


Figure 5:- Energy exchange in SHM

8. Some simple systems executing SHM

(A) Motion of a body suspended from a spring

Figure (6a) below shows a spring of negligible mass, spring constant k and length l suspended from a rigid support.

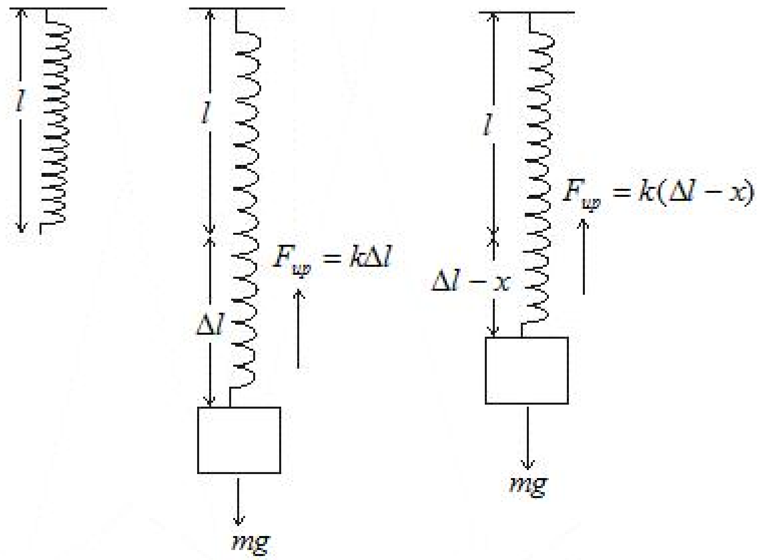


Figure 6(a) Figure 6(b) Figure 6(c)

If the spring is extended by an amount Δl ater attachment of block of mass m then in its equilibrium position upward force equals

$F_{up}=k\Delta l$

also in this equilibrium position

$F_{up}=mg$

or, $k\Delta l=mg$

Again the body is displaced in upwards direction such that it is at a distance x above equilibrium position as shown in figure 6(c).

Now extansion of spring would be (Δl-x), thus upward force now exerted on the body is

$F_{up}=k(\Delta l-x)$

Weight of the body now tends to pull the spring downwards with a force equal to its weight. Thus resultant force on the body is

$F=k(\Delta l-x)-mg$

$=mg-kx-mg$

or,

$F=-kx$ (17)

From equation 17 we see that resultant force on the body is proportional to the displacement of the body from its equilibrium position.

If such a body is set into vertical oscillations it oscillates with an angular frequency

$\omega=\sqrt{(k/m)}$ (18)

(B) Simple pendulum

Simple pendulum consists of a point mass suspended by inextensible weightless string in a uniform gravitational field.

Simple pendulum can be set into oscillatory motion by pulling it to one side of equilibrium position and then releasing it.

In case of simple pendulum path ot the bob is an arc of a circle of radius l, where l is the length of the string.

We know that for SHM F=-kx and here x is the distance measured along the arc as shown in the figure below.

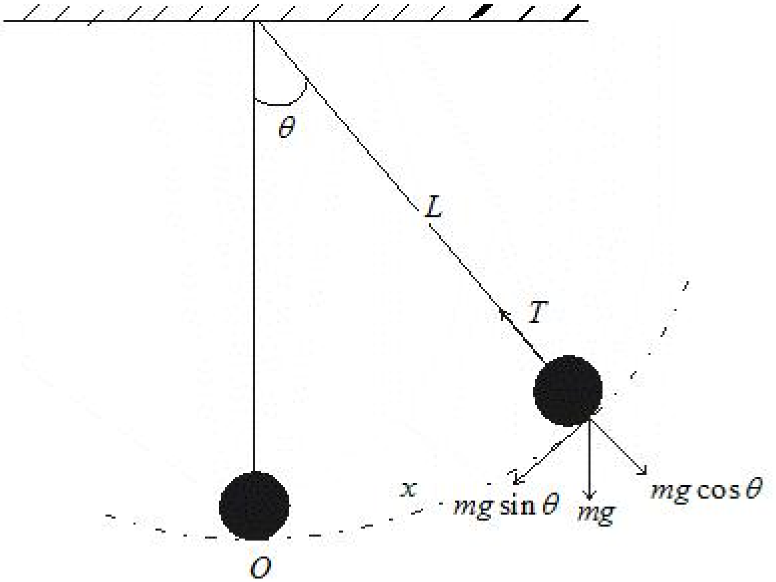


Figure 7 Force on bob of simple pendulum

When bob of the simple pendulum is displaced from its equilibrium position O and is then released it begins to oscillate.

Suppose it is at P at any instant of time during oscillations and θ be the angle subtended by the string with the vertical.

mg is the force acting on the bob at point P in vertically downward direction.

Its component mgcosθ is balanced by the tension in the string and its tangential component mgsinθ directs in the direction opposite to increasing θ .

Thus restoring force is given by

$F=-mg\sin\theta$ (19)

The restoring force is proportional to sinθ not to The restoring force is proportional to sinθ, so equation 19 does not represent SHM.

If the angle θ is small such that sinθ very narly equals θ then above equation 19 becomes

$F=-mg\theta$

since $x=l\theta$ then,

$F=-(mgx)/l$

where x is the displacement OP along the arc. Thus,

$F=-(mg/l)x$ (20)

From above equation 20 we see that restoring forcr is proportional to coordinate for small displacement x , and the constant (mg/l) is the force constant k.

Time period of a simple pendulum for small amplitudes is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m}{\left(mg/l\right)}}$$

or.

$$T = 2\pi\sqrt{\frac{g}{l}} \qquad (21)$$

Corresponding frequency relations are

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \qquad (22)$$

and angular frequency

$\omega=\sqrt{(g/l)}$ (23)

Notice that the period of oscillations is independent of the mass m of the pendulumand for small oscillations pperiod of pendulum for given value of g is entirely determined by its length.

(c) The compound pendulum

Compound pendulum is a rigid body of any shape, capable of oscillating about a horizontal axis passing through it.

Figure below shows vertical section of rigid body capable of oscillating about the point A.

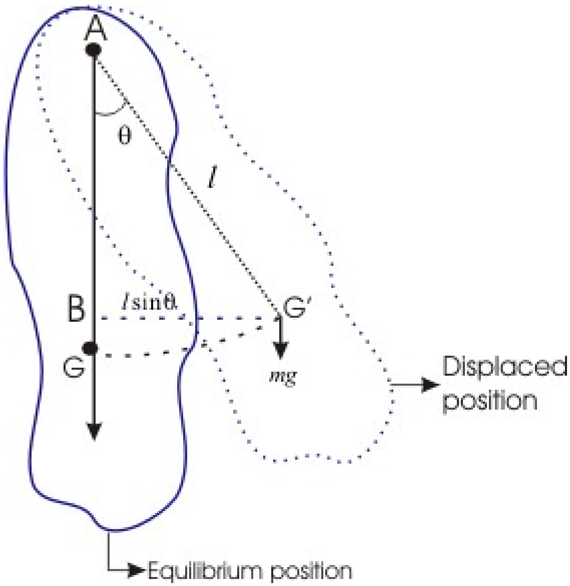


Figure 8

Distance l between point A and the centre of gravity G is called length of the pendulum.

When this compound pendulum is given a small angulr displacement θ and is then released it begins to oscillate about point A.

At angular displacement θ its center of gravity now takes new position at G'.

Weight of the body and its reaction at the support constitute a reactive couple or torque given by

$\tau = -mg\, G'B$

$= -mgl\sin\theta$ (24)

Equation 24 gives restoring couple which tends to bring displaced body to its original position.

If α is the angular acceleration produced in this body by the couple and I is the moment of inertia of body about horizontal axis through A then the couple is

$I\alpha = -mgl\sin\theta$

if θ is very small then we can replace sinθ□θ, so that

$\alpha = -(mgl/I)\theta$ (25)

From above equation (25) we se that pendulum is executing Simple Harmonic Motion with time period

$$T = 2\pi\sqrt{\frac{I}{mgl}} \qquad (26)$$

9. Damped Oscillations

Fractional force, acting on a body opposite to the direction of its motion , is called damping force.

Damping force reduces the velocity and the Kinetic Energy of the moving body.

Damping or dissipative forces generally arises due to the viscosity or friction in the medium and are non conservative in nature.

When velocities of body are not high, damping force is found to be proportional to velocity v of the particle i.e.,

$F_d=-\gamma v$ (27)

where, γ is the damping constant.

If we take damping into consideration for an oscillator then oscillator experiences

(i) Restoring Force :- $F=-kx$

(ii) Damping Force :- $F_d=-\gamma v$

where, x is th edisplacement of oscillating system and v is the velocity of this dispalcement.

Thus equation of motion of damped harmonic oscillator is

$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=0$$

$$\frac{d^2x}{dt^2}+2r\frac{dx}{dt}+\omega^2x=0 \qquad (28)$$

where, $r=(\gamma/2m)$ and $\omega^2=k/m$

Solution of above equation is of the form

$x=Ae^{-rt}\cos(\omega't+\varphi)$ (29)

where,

$\omega'=\sqrt{(\omega^2-r^2)}$ (30)

is the angular frequency of the damped oscillator.

In equation 29 , x is a function of time but it is not a periodic function and because of the damping factor e^{-rt} this function decreases continously with time.

10. Driven or Forced Harmonic oscillator

If an extra periodic force is applied on a damped harmonic oscillator, then the oscillating system is called driven or forced harmonic oscillator, and its oscillations are called forced oscillations.

Such external periodic force can be represented by

$F(t)=F_0\cos\omega_f t$ (31)

where, F_0 is the amplitude of the periodic force and ω_f is th e frequency of external force causing oscillations.

Differential equation of motion under forced oscillations is

$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F_0\cos\omega_f t \qquad (32)$$

In this case particle will neither oscillate with its free undamped frequency nor with damped angular frequency rather it would be forced to oscillate with angular frequency ω_f of applied force.

When damped oscillator is is set in forced motion, the initial motion is combination of damped oscillation and forced oscillations .

After certain amount of time the amplitude of damped oscillations die out or becomec so small that they can be ignoredand only forced oscillationd remainand the motion is thus said to reached steady state.

Solution of equation 32 is

$$x=A\cos(\omega_f t+\varphi) \qquad (33)$$

where A is the amplituse of oscollation of forced oscillator and φ is the initial phase.

In case of forced oscillations both amplitude A and initial phase φ are fixed quantities depending on frequency ω_f of applied force.

Calculations show sthat amplitude

$$A=\frac{F_0}{\{m^2(\omega^2-\omega_f^2)+\omega_f^2\gamma\}^{1/2}} \qquad (34)$$

and initial phase

$$\tan\varphi=-v_0/(\omega_f x_0)$$

where, x_0 is displacement of particle at time t=0, the moment driven force is applied and v_0 is the velocity of the particle at time t=0.

When ω_f is very close to ω, then $m(\omega^2-(\omega_f)^2$ would be much less than $\omega_f\gamma$, for any reasonable value of γ, then equation 34 becomes

$$A=F_0/\gamma\omega_f \qquad (35)$$

Thus the maximum possible amplitude for a given driven frequency is governed by the driving frequency and the damping ,and is never infinity.

This phenomenon of increase in amplitude when the driving force is close to natural frequency of oscillator is called RESONANCE.

Thus resonance occurs when frequency of applied force becomes equal to natural frequency of the oscillator without damping.

Oscillations and Waves

- **Periodic Motion**: A motion which repeats itself over and over again after a regular interval of time.
- **Oscillatory Motion**: A motion in which a body moves back and forth repeatedly about a fixed point.
- **Periodic function**: A function that repeats its value at regular intervals of its argument is called periodic function. The following sine and cosine functions are periodic with period T.

  $f(t) = \sin\frac{2\pi t}{T}$ and $g(t) = \cos\frac{2\pi t}{T}$

  These are called Harmonic Functions.

  Note :- All Harmonic functions are periodic but all periodic functions are not harmonic.

One of the simplest periodic functions is given by

$f(t) = A\cos\omega t \quad [\omega = 2\pi/T]$

If the argument of this function *ωt* is increased by an integral multiple of 2π radians, the value of the function remains the same. The function f(t) is then periodic and its period, T is given by

$T = \frac{2\pi}{\omega}$

Thus the function f(t) is periodic with period T

$f(t) = f(t + T)$

Linear combination of sine and cosine functions

$f(t) = A\sin\omega t + B\cos\omega t$

A periodic function with same period T is given as

$A = D\cos\phi$ and $B = D\sin\phi$

$\therefore f(t) = D \sin(\omega t + \phi)$

$\therefore D = \sqrt{A^2 + B^2}$ and $\phi = \tan^{-1}\frac{x}{a}$

- **Simple Harmonic Motion (SHM):** A particle is said to execute SHM if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from mean position and is always directed towards mean position.
  Restoring Force ∝ Displacement

$$F \propto x$$

$$F = -kx$$

  Where 'k' is force constant.
- **Amplitude**: Maximum displacement of oscillating particle from its mean position.
  $x_{Max} = \pm A$
- **Time Period:** Time taken to complete one oscillation.
- **Frequency:** $= \frac{1}{T}$. Unit of frequency is Hertz (Hz).

$$1 \text{ Hz} = 1\ s^{-1}$$

- **Angular Frequency:** $\omega = \frac{2\pi}{T} = 2\pi v$

$$\text{S.I unit } \omega = \text{rad } s^{-1}$$

- **Phase:**
  1. The Phase of Vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant.
     It is denoted by ø.
  2. **Initial phase or epoch**: The phase of particle corresponding to time t = 0.
     It is denoted by ø.
- **Displacement in SHM :**

$$x = A\,cos\,(\omega t + \phi_0)$$

Where, x = Displacement,

A = Amplitude

ωt = Angular Frequency

ϕ_0 = Initial Phase.

Case 1: When Particle is at mean position x = 0

$$v = -\omega\sqrt{A^2 - 0^2} = -\omega A$$

$$v_{max} = \omega A = \frac{2\pi}{T} A$$

Case 2: When Particle is at extreme position x = $\pm A$

$$v = -\omega\sqrt{A^2 - A^2} = 0$$

<u>Acceleration</u>

Case 3: When particle is at mean position x = 0,

acceleration = $-\omega^2(0)$ = 0.

Case 4: When particle is at extreme position then

$x = A$ acceleration = $-\omega^2 A$

- **Formula Used** :

1. $x = A\cos(\omega t + \phi_0)$

2. $v = \frac{dx}{dt} = -\omega\sqrt{A^2 - x^2}$, $v_{max} = \omega A$.

3. $a = \frac{dv}{dt} = \omega^2 A\cos(\omega t + \phi_0)$

   $$= -\omega^2 x$$

   $a_{max} = \omega^2 A$

4. *Restoring force* $F = -kx = -m\omega^2 x$

   Where k = force constant & $\omega^2 = \frac{k}{m}$

5. *Angular freq.* $\omega = 2\pi\nu = 2\pi/T$

6. *Time Period* $T = 2\pi\sqrt{\frac{\textbf{Displacement}}{\textbf{Acceleration}}} = 2\pi\sqrt{\frac{x}{a}}$

7. *Time Period* $T = 2\pi\sqrt{\frac{\textbf{Inertia Factor}}{\textbf{Spring Factor}}} = 2\pi\sqrt{\frac{m}{k}}$

8. P.E at displacement 'y' from mean position

   $E_P = \frac{1}{2} ky^2 = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 A^2 \sin^2\omega t$

9. K.E. at displacement 'y' from the mean position

$E_K = \frac{1}{2}k(A^2 - y^2) = \frac{1}{2}m\omega^2(A^2 - y^2)$

$= \frac{1}{2}m\omega^2A^2\cos^2\omega t$

10. Total Energy at any point

$E_T = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2 = 2\pi^2mA^2v^2$

11. Spring Factor $K = F/y$

12. Period Of oscillation of a mass 'm' suspended from a massless spring of force constant '*k*'

$$T = 2\pi\sqrt{\frac{m}{k}}$$

For two springs of spring factors k_1 and k_2 connected in parallel effective spring factor

$$k = k_1 + k_2 \qquad \therefore T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

13. For two springs connected in series, effective spring factor 'k' is given as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad Or \quad k = \frac{k_1k_2}{k_1+k_2}$$

$$T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

Note:- When length of a spring is made 'n' times its spring factor becomes $\frac{1}{n}$ times and hence time period increases $\sqrt{n}$ times.

14. When spring is cut into 'n' equal pieces, spring factor of each part becomes '*nk*'.

$$T = 2\pi\sqrt{\frac{m}{nk}}$$

15. Oscillation of simple pendulum

$$T = 2\pi\sqrt{l/g}$$

$$\nu = \frac{1}{2\pi}\sqrt{g/l}$$

16. For a liquid of density ρ contained in a U-tube up to height 'h'

$$T = 2\pi\sqrt{h/g}$$

17. For a body dropped in a tunnel along the diameter of earth

$T = 2\pi\sqrt{R/g}$, where **R** = Radius of earth

18. **Resonance**: If the frequency of driving force is equal to the natural frequency of the oscillator itself, the amplitude of oscillation is very large then such oscillations are called resonant oscillations and phenomenon is called resonance.

Waves

Angular wave number: It is phase change per unit distance.

i.e. $k = \frac{2\pi}{\lambda}$, S.I unit of k is radian per meter.

Relation between velocity, frequency and wavelength is given as :- $V = \upsilon\lambda$

Velocity of Transverse wave:-

(i)In solid molecules having modulus of rigidity 'η' and density 'ρ' is

$$V = \sqrt{\frac{\eta}{\rho}}$$

(ii)In string for mass per unit length 'm' and tension 'T' is $V = \sqrt{\frac{T}{m}}$

Velocity of longitudinal wave:-

(i) in solid $V = \sqrt{\frac{Y}{\rho}}$, Y= young's modulus

(ii) in liquid $V = \sqrt{\frac{K}{\rho}}$, K= bulk modulus

(iii) in gases $V = \sqrt{\frac{K}{\rho}}$, K= bulk modulus

According to Newton's formula: When sound travels in gas then changes take place in the medium are isothermal in nature. $V = \sqrt{\frac{P}{\rho}}$

According to Laplace: When sound travels in gas then changes take place in the medium are adiabatic in nature.

$V = \sqrt{\frac{P\gamma}{\rho}}$ 'Where $\gamma = \frac{Cp}{Cv}$

Factors effecting velocity of sound :-

(i) Pressure – No effect

(ii) Density – $V \alpha \frac{1}{\sqrt{\rho}}$ or $\frac{V1}{V2} = \sqrt{\frac{\rho 1}{\rho 2}}$

(iii) Temp- $V \alpha \sqrt{T}$ or $\frac{V1}{V2} = \sqrt{\frac{T1}{T2}}$

Effect of humidity :– sound travels faster in moist air

(iv) Effect of wind –velocity of sound increasing along the direction of wind.

Wave equation:– if wave is travelling along + x-axis

(i) Y=A sin $(\omega t - kx)$, Where, $k = \frac{2\pi}{\lambda}$

(ii) Y=A sin $2\pi (\frac{t}{T} - \frac{x}{\lambda})$

(iii) Y=A sin $\frac{2\pi}{T}$ (vt-x)

If wave is travelling along –ve x- axis

(iv) Y=A sin $(\omega t + kx)$,Where , $k = \frac{2\pi}{\lambda}$

(v) Y=A sin $2\pi (\frac{t}{T} + \frac{x}{\lambda})$

(vi) Y=A sin $\frac{2\pi}{T}$ (vt+x)

Phase and phase difference

Phase is the argument of the sine or cosine function representing the wave.

$$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Relation between phase difference ($\Delta\phi$) and time interval (Δt) is $\boldsymbol{\Delta\phi = -\frac{2\pi}{T}\Delta t}$

Relation between phase difference (Δp) and path difference (Δx) is $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$

Equation of stationary wave:-

(1) Y_1 =a sin $2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (incident wave)

Y_1 = $\pm$ a sin $2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$ (reflected wave)

Stationary wave formed

Y = Y_1+Y_2 = $\pm$ 2a cos $\frac{2\pi x}{\lambda}$ sin $\frac{2\pi t}{T}$

(2) For (+ve) sign antinodes are at x= 0, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ……

And nodes at x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

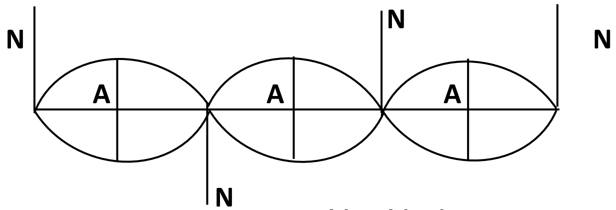
(3) For (-ve) sign antinodes are at x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.....

Nodes at x= 0, $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ……

(4)Distance between two successive nodes or antinodes are $\frac{\lambda}{2}$ and that between nodes and nearest antinodes is $\frac{\lambda}{4}$.

(5) Nodes-point of zero displacement-

Antinodes- point of maximum displacement-



N = Nodes

A = Antinodes

Mode of vibration of strings:-

a) $\upsilon = \frac{p}{2L}\sqrt{\frac{T}{m}}$ Where ,T= Tension

M= mass per unit length

υ = frequency, V=velocity of second, p = 1, 2, 3, …

b) When stretched string vibrates in P loops $\upsilon_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\upsilon$

c) For string of diameter D and density ρ

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

d) Law of length $\nu \propto \frac{1}{L}, \nu L = constant$

ORGANPIPES

1. In an organ pipe closed at one end only odd harmonics are present

$\upsilon_1 = \frac{V}{4L}$ (fundamental)

$\upsilon_2 = 3\upsilon$ (third harmonic or first overtone)

$\upsilon_3 = 5\upsilon$

$\upsilon_n = (2n-1)\ \upsilon$

2. In an open organ pipe at both ends both odd and even harmonics are present.

$\upsilon'_1 = \frac{V}{2L} = \upsilon'$ (first harmonic)

$\upsilon'_2 = 2\upsilon'$ (second harmonic or first overtone)

$\upsilon'3 = 3\upsilon'$

$\upsilon' n = (2n-1)\, \upsilon'$

3. Resonance tube: If L_1 and L_2 are the first and second resonance length with a tuning fork of frequency 'υ'then the speed of sound. v = $4v(L_1 + 0.3D)$

Where ,D=internal diameter of resonance tube

$$v = 2v(L_2 - L_2)$$

End correction=0.3D= $\frac{L2 - L1}{2}$

Beats formation

1. Beat frequency = No. of beats per second = Difference in frequency of two sources.

$$b = \nu_1 - \nu_2$$

2. $\nu_2 = \nu_1 \pm b$
3. If the prong of tuning fork is filed, its frequency increases. If the prong of a tuning fork is loaded with a little way, its frequency decreases. These facts can be used to decide about + or – sign in the above equation.

Doppler effect in sound

1. If V, Vo, Vs, and Vm are the velocity of sound , observes, source and medium respectively, then the apparent frequency

$$\upsilon 1 = \frac{V + Vm - Vo}{V + Vm - Vs} \times \upsilon$$

2. If the medium is at rest (v_m= o),then

$$\upsilon' = \frac{V - Vo}{V - Vs} \times \upsilon$$

3. All the velocity are taken positive with source to observer (S $\rightarrow$ O) direction and negative in the opposite (O $\rightarrow$ S) direction

(Questions)

(1 marks questions)

1. Which of the following relationships between the acceleration 'a' and the displacement 'x' of a particle involve simple harmonic motion?

(a) a=0.7x (b) $a=-200x^2$ (c) a = -10x (d) $a=100x^3$

Ans: - (c) reprent SHM.

2. Can a motion be periodic and not oscillatory?

Ans: - Yes, for example, uniform circular motion is periodic but not oscillatory.

3. Can a motion be periodic and not simple harmonic? If your answer is yes, give an example and if not, explain why?

Ans:- Yes, when a ball is doped from a height on a perfectly elastic surface ,the motion is oscillatory but not simple harmonic as restoring force F=mg=constant and not $F \alpha -x$, which is an essential condition for S.H.M.

4. A girl is swinging in the sitting position. How will the period of the swing change if she stands up?

Ans:-The girl and the swing together constitute a pendulum of time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

As the girl stands up her centre of gravity is raised. The distance between the point of suspension and the centre of gravity decreases i.e. length 'l' decreases .Hence the time period 'T' decreases.

5. The maximum velocity of a particle, executing S.H.M with amplitude of 7mm is 4.4 m/s. What is the period of oscillation?

Ans: - $V_{max} = \omega A = \frac{2\pi}{T} A$, $T = \frac{2\pi A}{Vmax} = \frac{2\times 22\times .007}{7\times 4.4} = 0.01s$

6. Why the longitudinal wave are also called pressure waves?

Ans: - Longitudinal wave travel in a medium as series of alternate compressions and rare fractions i.e. they travel as variations in pressure and hence are called pressure waves.

7. How does the frequency of a tuning fork change, when the temperature is increased?

Ans: -As the temperature is increased, the length of the prong of a tuning fork increased .This increased the wavelength of a stationary waves set up in the tuning fork. As frequency,

$\upsilon = \frac{1}{\lambda}$, So the frequency of tuning fork decreases.

8. An organ pipe emits a fundamental node of a frequency 128Hz. On blowing into it more strongly it produces the first overtone of the frequency 384Hz. What is the type of pipe –Closed or Open?

Ans: - The organ pipe must be closed organ pipe, because the frequency the first overtone is three times the fundamental frequency.

9. All harmonic are overtones but all overtones are not harmonic. How?

Ans: -The overtones with frequencies which are integral multiple of the fundamental frequency are called harmonics. Hence all harmonic are overtones. But overtones which are non-integrals multiples of the fundamental frequency are not harmonics.

10. What is the factor on which pitch of a sound depends?

Ans: - The pitch of a sound depends on its frequency.

(2 Marks questions)

1. At what points is the energy entirely kinetic and potential in S.H.M? What is the total distance travelled by a body executing S.H.M in a time equal to its time period, if its amplitude is A?

Ans. The energy is entirely kinetic at mean position i.e. at y=0. The energy is entirely potential at extreme positions i.e.

$$y = \pm A$$

Total distance travelled in time period T = 2A + 2A = 4A.

2. A simple pendulum consisting of an inextensible length 'l' and mass 'm' is oscillating in a stationary lift. The lift then accelerates upwards with a constant acceleration of 4.5 m/s^2. Write expression for the time period of simple pendulum in two cases. Does the time period increase, decrease or remain the same, when lift is accelerated upwards?

Ans. When the lift is stationary, T = $2\pi\sqrt{\frac{l}{g}}$

When the lift accelerates upwards with an acceleration of 4.5 m/s^2

T ´= $2\pi\sqrt{\frac{l}{g+45}}$

Therefore, the time period decreases when the lift accelerates upwards.

3. Does the function y = sin^{2}ωt represent a periodic or a S.H.M? What is period of motion?

Ans. Displacement y = sin^{2}ωt

Velocity v = $\frac{dy}{dt}$ = 2sin ωt × cosωt × ω

v = ω sin 2ωt

Acceleration a = $\frac{dv}{dt}$ = ω×cos 2ωt × 2ω

a = 2 ω^{2}cos 2ωt.

As the acceleration is not proportional to displacement y, the given function does not represent SHM. It represents a periodic motion of angular frequency 2ω.

∴ Time Period T = $\frac{2\pi}{Angular\ freq.}$ = $\frac{2\pi}{2\omega}$ = $\frac{\pi}{\omega}$

4. All trigonometric functions are periodic, but only sine or cosine functions are used to define SHM. Why?

Ans. All trigonometric functions are periodic. The sine and cosine functions can take value between -1 to +1 only. So they can be used to represent a bounded motion like SHM. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both negative and positive). So these functions cannot be used to represent bounded motion like SHM.

5. A simple Harmonic Motion is represented by $\frac{d^2x}{dt^2}$+αx = 0. What is its time period?

Ans. $\frac{d^2x}{dt^2} = -\alpha x$ Or a = – αx

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{x}{\alpha x}} = \frac{2\pi}{\sqrt{\alpha}}$$

$$\mathbf{T = \frac{2\pi}{\sqrt{\alpha}}}$$

6. The Length of a simple pendulum executing SHM is increased by 2.1%. What is the percentage increase in the time period of the pendulum of increased length?

Ans. Time Period, $T = 2\pi\sqrt{\frac{l}{g}}$ i.e. $T \propto \sqrt{l}$.

The percentage increase in time period is given by

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100 \text{ (for small variation)}$$

$= \frac{1}{2} \times 2.1\%$

$= 1.05\%$

7. A simple Harmonic motion has an amplitude A and time period T. What is the time taken to travel from x = A to x = A/2.

Ans. Displacement from mean position = $A - {}^{A}/_{2} = {}^{A}/_{2}$.

When the motion starts from the positive extreme position, y = A cosωt.

$$\therefore \frac{A}{2} = A\cos\frac{2\pi}{T}t.$$

$\cos\frac{2\pi}{T}t = ½ = \cos\frac{\pi}{3}$

or $\frac{2\pi}{T}t = \frac{\pi}{3}$

$\therefore t = {}^{T}/_{6}$

8. An open organ pipe produces a note of frequency 5/2 Hz at 15^{0}C, calculate the length of pipe. Velocity of sound at 0^{0}C is 335 m/s.

Ans. Velocity of sound at 15^{0}C

$V=V_0+0.61xt =335+0.61x15 =344.15$ m/s. (Thermal coefficient of velocity of sound wave is .61/^{0}C)

Fundamental frequency of an organ pipe

$$\upsilon = \frac{V}{4L} \quad , \quad \therefore L = \frac{V}{4\upsilon} = \frac{344.15}{4\times512} = 0.336m$$

9. An incident wave is represented by Y(x, t)=20sin(2x-4t).Write the expression for reflected wave

(i) From a rigid boundary

(ii) From an open boundary.

Ans.(i) The wave reflected from a rigid boundary is

$$Y (x, t) = -20\sin (2x+4t)$$

(i)The wave reflected from an open boundary is

$$Y (x, t) = 20\sin (2x+4t)$$

Explain why

(i) in a sound wave a displacement node is a pressure antinode and vice- versa

(ii) The shape of pulse gets- distorted during propagation in a dispersive medium.

Ans. (i) At a displacement node the variations of pressure is maximum. Hence displacement node is the a pressure antinode and vice-versa.

(ii)When a pulse passes through a dispersive medium the wavelength of wave changes.

So, the shape of pulse changes i.e. it gets distorted.

(3 Marks Questions)

1. The speed of longitudinal wave `V` in a given medium of density ρ is given by the formula, use this formula to explain why the speed of sound in air.

   (a) is independent at pressure

   (b) increases with temperature and

   (c) increases with humidity

2. Write any three characteristics of stationary waves.

Ans. (i) in stationary waves, the disturbance does not advance forward. The conditions of crest and trough merely appear and disappear in fixed position to be followed by opposite condition after every half time period. (ii) The distance between two successive nodes or antinodes is equal to half the wavelength. (iii) The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

3. Show that the speed of sound in air increased by .61m/s for every 1^0 C rise of temperature.

Ans. $V \alpha \sqrt{T}$

$$\frac{Vt}{Vo} = \sqrt{\frac{t+273}{0+273}}$$

$V_t = V_0 \left(1+\frac{t}{273}\right)^{1/2} = V_0 \left(1+\frac{1}{2}.\frac{t}{273}\right)$

$V_t = V_0 + \frac{V\circ \times t}{546}$

At, 0^0C speed of sound in air is 332 m/s.

$\therefore V_t - V_0 = \frac{332\times t}{546}$

When t= 1^0C , $V_t - V_0$ = 0.61m/s.

4. Find the ratio of velocity of sound in hydrogen gas $\Upsilon = \frac{7}{5}$ to that in helium gas $\Upsilon = \frac{5}{3}$ at the same temperature. Given that molecular weight of hydrogen and helium are 2 and 4 respectively.

Ans. $V = \sqrt{\frac{\gamma RT}{M}}$

At constant temperature,

$$\frac{V_H}{V_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}}\frac{M_H}{M_{He}}} = \sqrt{\frac{715}{5/3}}.\frac{4}{2} = 1.68.$$

5. The equation of a plane progressive wave is, $y = 10Sin2\pi(t - 0.005x)$ where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

Ans. Given, y = $10Sin2\pi(t - 0.005x)$............. (1)

Standard equation for harmonic wave is, $y = A\ Sin2\pi(\frac{t}{T} - \frac{x}{\lambda})$.............. (2)

Comparing eqn (1) & (2), $A = 10,\ \frac{1}{t} = 1,\ \frac{1}{\lambda} = 0.005$

(i) Amplitude A = 10cm

(ii) Frequency $\nu = \frac{1}{T} = 1Hz$

(iii) Wavelength $\lambda = \frac{1}{0.005} = 200cm$

(iv) Velocity v = $\nu\,\lambda$ = 1 x 200 = 200cm/s

6. Write displacement equation respecting the following condition obtained in SHM.

Amplitude = 0.01m

Frequency = 600Hz

Initial phase = $\frac{\pi}{6}$

Ans. Y = A Sin (2π$\nu t + \phi_o$)

= 0.01 Sin $(1200\pi t + \frac{\pi}{6})$

7. The amplitude of oscillations of two similar pendulums similar in all respect are 2cm & 5cm respectively. Find the ratio of their energies of oscillations.

Ans. $\frac{E_1}{E_2} = (\frac{A_1}{A_2})^2 = \left(\frac{2}{3}\right)^2 = 4:25$

8. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion?

Ans. A periodic motion repeats after a definite time interval T. So,

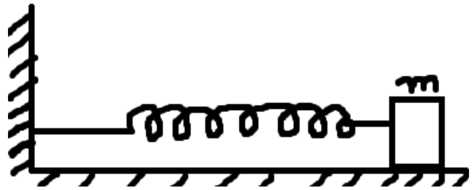
$$y(t) = y(t+T) = y(t+2T)\ etc.$$

9. A spring of force constant 1200N/m is mounted horizontal table. A mass of 3Kg is attached to the free end of the spring, pulled sideways to a distance of 2.0cm and released.

(i) What is the frequency of oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass?



Ans. Here k = 1200N/m, m = 3Kg, A = 2cm = 2×0.01m

(i) $\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2} \times \frac{1}{3.14}\sqrt{\frac{1200}{3}} = 3.2s^{-1}$

(ii) $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1200}{3}} = 20s^{-1}$

Maximum acceleration = $\omega^2 A = (20)^2 \times 2 \times 10^{-2} = 8m/s^2$

(iii) Maximum speed = $\omega A = 20 \times 2 \times 10^{-2} = 0.40m/s$

10. Which of the following function of time represent, (a) simple harmonic (b) periodic but not SHM and (c) non periodic ?

(i) Sinωt- Cosωt (ii) $Sin^3\omega t$ (iii) $3Cos(\frac{\pi}{2} - 2\omega t)(iv)\exp(-\omega^2 t^2)$

Ans. (i) $x(t) = Sin\omega t - Cos\omega t = \sqrt{2Sin(\omega t - \frac{\pi}{2})}$, so the function is in SHM.

(ii) $x(t) = Sin^3\omega t = \frac{1}{4}(3Sin\omega t - Sin3\omega t)$, represent two separate SHM motion but their combination does not represent SHM.

(iii) $x(t) = 3Cos\left(\frac{\pi}{4} - 2\omega t\right) = 3Cos(2\omega t - \frac{\pi}{4})$, represent SHM.

(iv) $\exp(-\omega^2 t^2) = non\ periodic.$

(5 Marks Questions)

1. (a) A light wave is reflected from a mirror. The incident & reflected wave superimpose to form stationary waves. But no nodes & antinodes are seen, why?

(b) A standing wave is represented by y=2ASinKxCoswt.If one of the component wave is $y_1 = ASin(\omega t - Kx)$, what is the equation of the second component wave?

Ans. (a) As is known, the distance between two successive nodes or two successive antinodes is $\frac{\lambda}{2}$. The wavelength of visible light is of the order of $10^{-7}m$. As such as a small distance cannot be detected by the eye or by a ordinary optical instrument. Therefore, nodes and antinodes are not seen.

(b) As, $2SinACosB = Sin(A+B) + Sin(A-B)$

$$y = 2ASinKxCos\omega t$$

$$= ASin(Kx+\omega t) + ASin(Kx-\omega t)$$

According to superposition principle,

$$y = y_1 + y_2$$

$$and\ y_1 = ASin(\omega t - Kx) = -ASin(Kx - \omega t)$$

$$y_2 = y - y_1 = 2ASinKxCos\omega t + ASin(Kx-\omega t)$$
$$= ASin(Kx+\omega t) + 2ASin(Kx-\omega t)$$
$$= ASin(Kx+\omega t) - 2ASin(\omega t - Kx)$$

2. Discuss Newton's formula for velocity of sound in air. What correction was made to it by Laplace and why?

Ans. According to Newton the change in pressure & volume in air is an isothermal process. Therefore he calculated, $v = \sqrt{\frac{p}{\rho}}$ on substituting the require value he found, the velocity of sound was not in close agreement with the observation value. Then Laplace pointed out the error in Newton's formula. According to Laplace the change in pressure and volume is an adiabatic process. So he calculated the value of sound as, $v = \sqrt{\frac{\Upsilon r}{\rho}}$ on putting require value he found velocity of sound as 332m/s very closed to observed theory.

3. (a) What are beats? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing wave.

(b) Draw fundamental nodes of vibration of stationary wave in (i) closed pipe, (ii) in an open pipe.

4. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4.

5. Explain Doppler's effect of sound. Derive an expression for the apparent frequency where the source and observer are moving in the same direction with velocity Vs and Vo respectively, with source following the observer.

[Ans. = $\nu^{|} = \frac{v-v_o}{v-v_s} * \nu$]

6. For a travelling harmonic wave, $y = 2Cos(10t - 0.008x + 0.35)$ where x & y are in cm and t in second. What is the phase difference between oscillatory motions at two points separated by a distance of (i) 4cm (ii) 0.5m (iii)$\frac{\lambda}{2}$ $(iv)\frac{3\lambda}{4}$?

Ans.$y = 2Cos(10t - 0.008x + 0.35) \dots \dots \dots .(i)$

We know, $y = ACos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right) \dots \dots \dots .(ii)$

From (i) & (ii), $\frac{2\pi}{\lambda} = 0.008, \lambda = \frac{2\pi}{0{,}008} cm = \frac{2\pi}{0.80} m.$

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda} * path\ difference = \frac{2\pi}{\lambda} * \Delta x.$

(i) When $\Delta x = 4cm\ ,\quad \Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 4 = 3.2rad\ .$

(ii) When$\Delta x = 0.5m,\ \Delta\phi = \frac{2\pi}{2\pi} * 0.80 * 0.5 = 0.40rad.$

(iii) When $\Delta x = \frac{\lambda}{2},\ \Delta\phi = \frac{2\pi}{\lambda} * \frac{\lambda}{2} = \pi rad.$

(iv) When$\Delta x = \frac{3\lambda}{4},\ \Delta\phi = \frac{2\pi}{\lambda} * \frac{3\lambda}{4} = \frac{3\pi}{2} rad.$

7. **(i)** A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

(ii) A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly exited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open? (Speed of sound = 340 m/s).

Ans. (i) For the fundamental mode,

$\lambda = 2L = 2 \times 100 = 200$ cm = 2m.

Frequency $\nu = 2.53$ kHz = 2530 Hz

Speed of sound, $v = \nu\lambda = 2530 \times 2 = 5060$ m/s

=5.06 km/s

(ii) Length of pipe L= 20 cm = 0.2 m

Speed of sound v= 340 m/s

Fundamental frequency of closed organ pipe

$\nu = v/4L = \frac{340}{4\times0.2} = 425$ Hz sw can be excited

Fundamental frequency of open organ pipe

$\nu' = \frac{v}{2L} = \frac{340}{2\times0.2} = 850$ Hz

Hence source of frequency 430 Hz will not be in resonance with open organ pipe.

8. A train stands at a platform blowing a whistle of frequency 400 Hz in still air.

(i) What is the frequency of the whistle heard by a man running

(a)Towards the engine 10 m/s.

(b) Away from the engine at 10 m/s?

(ii) What is the speed of sound in each case?

(iii) What is the wavelength of sound received by the running man in each case?

Take speed of sound in still air = 340 m/s.

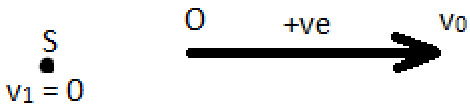
Ans.(i) (a) When the man runs towards the engine

$V_0 = -10$ m/s, $v_1 = 0$

$$v' = \frac{v - v_0}{v - v_s} \times v = \frac{340+10}{340-0} \times 400 = \frac{350}{340} \times 400 = 411.8$$

(b) When the man runs away from the engine

$V_0 = +\,10$ m/s, $\quad v_s = 0$



$$v'' = \frac{v - v_0}{v - v_s} \times v = \frac{340-10}{340-0} \times 400 = \frac{330}{340} \times 400 = 388.2 \text{ Hz}$$

(ii) (a) When the man runs towards the engine , relative velocity of sound

$v' = v + v_0 = 340 + 10 = 350$ m/s

(b) When the man runs away from the engine, relative velocity of sound

$v' = v – v_0 = 340 – 10 = 330$ m/s .

(iii) The wavelength of sound is not affected by the motion of the listener. Its value is

$$\lambda = \frac{v}{\nu} = 340/400 = 0.85\text{m}$$

9. What is a spring factor? Derive the expression for resultant spring constant when two springs having constants k_1 and $k_2$are connected in (i) parallel and (ii) in series.

10. Show that for a particle in linear S.H.M., the average kinetic energy over a period of oscillation is equal to the average potential energy over the same period. At what distance from the mean position is the kinetic energy in simple harmonic oscillator equal potential energy?