Sets

1. Introduction

What is Sets?

Simply put, it's a collection of objects

Examples

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z+ : the set of positive integers

Q+ : the set of positive rational numbers, and

R+ : the set of positive real numbers

Set of even numbers: {..., -4, -2, 0, 2, 4, ...}

Set of odd numbers: {..., -3, -1, 1, 3, ...}

Set of prime numbers: {2, 3, 5, 7, 11, 13, 17, ...}

Positive multiples of 3 that are less than 10: {3, 6, 9}

2. Methods of representing a set

For sets, we simply put each element, separated by a comma, and then put some curly brackets around the whule thing.

Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc

The elements of a set are represented by small letters *a, b, c, x, y, z,* etc

When we say an element a is in a set A, we use the symbul $\in$ to show it.

And if something is not in a set use $\notin$

Example: In a set of even number E, $2 \in E$ but $3 \notin E$

Two Methods are used to represent Sets

(a) Roster forms

In a Roster forms, all the elements in the set is listed.

Example

Set of Vowel ={ a,e,i,o,u}

Some Important points

In roster form, the order in which the elements are listed is immaterial

while writing the set in roster form an element is not generally repeated

(b) Set Builder Form

In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.For example, in the set $\left\{ {2,4,6,8} \right\}$, all the elements possess a common property, namely, each of them is a even number less than 10. Denoting this set by N, we write

$N = \left\{ {x:x{\rm{ is a even number less 10}}} \right\}$

We describe the element of the set by using a symbul x (any other symbul like the letters *y, z,* etc. could be used) which is fullowed by a culon ":" . After the sign of culon, we write the characteristic property possessed by the elements of the set and then enclose the whule description within braces

3. Types of sets

(a) Empty set

A set which does not contain any element is called the *empty set* or the *null set* or the *void set*

It is denoted by $\phi $ or $\left\{ {} \right\}$

It is a set with *no elements*

Examples of empty sets is

$D = \left\{ {x:{x^2} = 9,x{\rm{ is even}}} \right\}$

Here D is the empty set, because the equation ${{x^2} = 9}$ is not satisfied by any even value of x

(b) Finite or infinite set

If M is a set then $n\left(M \right)$ defines the number of distinct elements in the set M.

If $n\left(M \right)$ is zero or finite ,then M is a finite set

If $n\left(M \right)$ is infinite then M is a infinite set

(c) Equal sets

Two sets are said to be equal if they have same members in them.

For A and B to be equal, every member of A should be present in set B and every member of B to be present in set A

It is denoted by equality sign $A = B$

4. Subset and Proper Subset

A set A is said to be a subset of a set B if every element of A is also an element of B .

It is denoted by

$A \subset B$ if whenever $a \in A$, then $a \in B$

If $A \subset B$ and $B \subset A$, then $A = B$.

Every set is subset of itself $A \subset A$

Empty set is subset of every set $\phi \subset A$

If $A \subset B$ and $A \ne B$, then A is proper subset of B. In such a case B is called *superset* of set A

5. Subset of set of the real numbers

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z+ : the set of positive integers

Q+ : the set of positive rational numbers, and

R+ : the set of positive real numbers

$T = \left\{ {x:x \in R{\rm{ and }}x \notin Q} \right\}$, i.e., all real numbers that are not rational

$N \subset Z \subset Q,{\rm{ }}Q \subset R,{\rm{ }}T \subset R,{\rm{ }}N \not\subset T$

6. Interval as subset of R Real Number

$a,b \notin R,b > a$

(a,b)	It is the open interval set between point and b such that All the points between a and b belong to the open interval *(a, b)* but *a, b* themselves do not belong to this interval	$\{ y:a < y < b\}$
$[a,b]$	It is the closed interval set between point and b such that All the points between *a* and *b* belong to the open interval *(a, b)* including *a, b*	$\{ x:a \le x \le b\}$
$[a,b)$	It is the open interval set between point and b such that All the points between *a* and *b* belong to the open interval *(a, b)* including *a,* but not *b*	$\{ x:a \le x < b\}$
(a,b)	It is the open interval set between point and b such that All the points between *a* and *b* belong to the open interval *(a, b)* including *b*, but not *a*	$\{ x:a < x \le b\}$

Power Set

The collection of all subsets of a set X is called the power set of X. It is denoted by $P(X)$. In $P(X)$, every element is a set.

if $X = \left\{ {1,2,3} \right\}$, then

$X\left(A \right) = \left\{ {\phi ,\left\{ 1 \right\},\left\{ 2 \right\},\left\{ 3 \right\},\left\{ {1,2} \right\},\left\{ {1,3} \right\},\left\{ {2,3} \right\},\left\{ {1,2,3} \right\}} \right\}$

Also, note that $n\left[{X\left(A \right)} \right] = 8 = {2^3}$

In general, if X is a set with $n\left(X \right) = m$, then it can be shown that

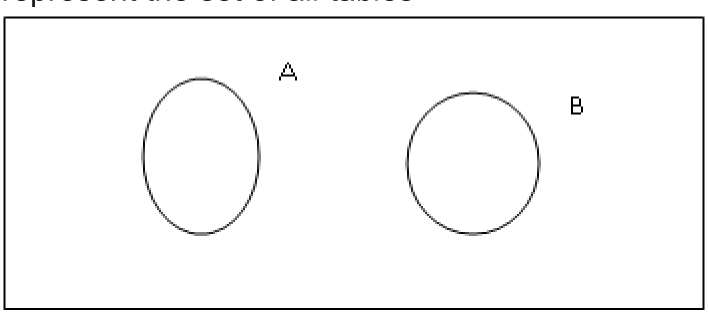
$n\left[{P\left(A \right)} \right] = {2^m}$

Universal Set

A Universal is the set of all elements under consideration, denoted by capital U.

Venn diagram

Venn diagrams were introduced in 1880 by John Venn (1834–1923). These diagrams consist of rectangles and closed curves usually circles. The universal set is represented usually by a rectangle and its subsets by circles. Venn diagrams normally comprise overlapping circles. The interior of the circle symbolically represents the elements of the set, while the exterior represents elements that are not members of the set. For instance, in a two-set Venn diagram, one circle may represent the group of all wooden objects, while another circle may represent the set of all tables

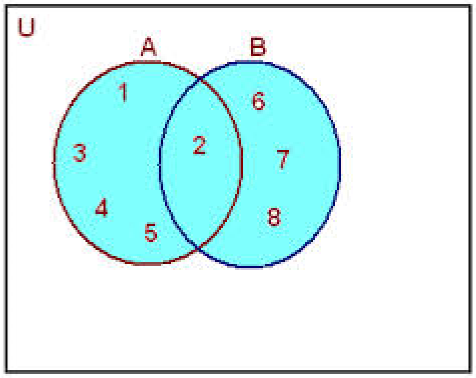


Union of Sets

The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.

$A \cup B = \left\{ {x:x \in A{\rm{ or }}x \in B} \right\}$

Venn Digram



A Union B

Some Properties of the Operation of Union

Commutative law : $X \cup Y = Y \cup X$

Associative law : $\left({X \cup Y} \right) \cup Z = X \cup \left({Y \cup Z} \right)$

Law of identity element, ϕ is the identity of $\cup$: $X \cup \phi = X$

Idempotent law : $X \cup X = X$

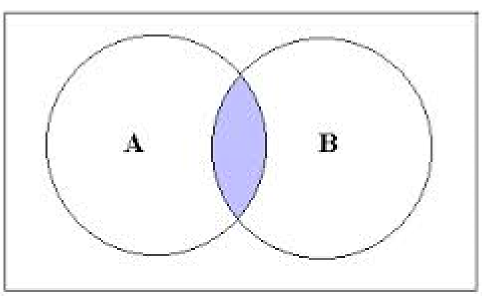
Law of U : $U \cup X = U$

Intersection of Sets

The Intersection of two sets A and B is the set C which consists of all those elements which are present in both A and B . In symbols, we write.

$A \cap B = \left\{ {x:x \in A{\rm{ and }}x \in B} \right\}$

Venn Digram



Some Properties of Operation of Intersection

Commutative law : $X \cap Y = Y \cap X$

Associative law : $\left({X \cap Y} \right) \cap Z = X \cap \left({Y \cap Z} \right)$

Law of $\cap$ and U : $\phi \cap X = \phi$, $U \cap X = X$

Idempotent law : $X \cap X = X$

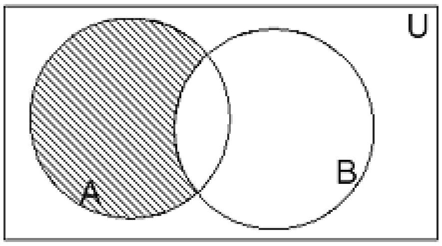
Distributive law : $X \cap \left({Y \cup Z} \right) = \left({X \cap Y} \right) \cup \left({X \cap Z} \right)$

Difference of set

in A but not in B . In symbols, we write,

$A - B = \left\{ {x:x \in A{\rm{ and }}x \notin B} \right\}$

Venn Digram



Some Properties of Operation of Difference

$A - B \ne B - A$

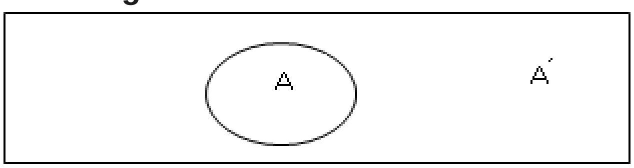
The sets $\left({A - B} \right)$, $\left({A \cap B} \right)$ and $\left({B - A} \right)$ are mutually disjoint sets.

Compliment of set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of UU which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U. Thus,

$A' = \left\{ {x:x \in U{\rm{ and }}x \notin A} \right\}$,obviously $A' = U - A$

Venn Digram



Some Properties of compliment_of_sets

Complement laws:

$A \cup A' = U$

$A \cap A' = \phi$

De Morgan’s law:

$\left({A \cup B} \right)' = A' \cap B'$

$\left({A \cap B} \right)' = A' \cap B'$

Law of double complementation: $\left({A'} \right)' = A$

Laws of empty set and universal set : $\phi ' = U$ and $U' = \phi$

Cardinality of the set

The cardinality of the set defines the number of element in the Set

If A is the set, Cardinality of the set is defined as $n(A)$

For $A = \{ 1,2,3\}$ then $n(A) = 3$

Set Relations

Joined Set	Disjoined Set
Set having common elements	Set having no common elements

Important Operation on Cardinality

If $n\left({X \cap Y} \right) \ne 0$

$n\left({X \cup Y} \right) = n\left(X \right) + n\left(Y \right) - n\left({X \cap Y} \right)$

If $n\left({X \cap Y} \right) = 0$

$n\left({X \cup Y} \right) = n\left(X \right) + n\left(Y \right)$

Chapter 1
Sets

Concept: Representation of a set
Concepts:- Different types of sets – Subsets- Power sets – Universal set – Operations on sets – Compliment of a set – Practical Problems.

Text book questions

Ex: 1	Questions 3, 4,5
Ex: 2	Questions 1, 2
Ex: 3	Questions 4, 5,6*,7*
Ex: 4	Questions 4, 6, 9
Ex: 5	Questions 4*, 5*
Misc.Ex:	Questions 8, 9, 11, 15**, 16**
Example	Question: 34**

Note: * *Important*

** *Very Important*

Extra/HOT questions

1. Write the following sets in set builder form
   I) {1/4, 2/5, 3/6, 4/7, 5/8}
   II) { ...., -5, 0, 5, 10, ...}
   III) {-4, 4}
2. Let A, B and C are three sets then prove the following:
   i) $A - (A \cap B) = A - B$
   ii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
   iii) $A - (B \cap C) = (A - B) \cup (A - C)$
   iv) $A \cap (B - C) = (A \cap B) - (A \cap C)$
3. Draw Venn diagrams for the following sets:
   i) $(A - B)' \cap A$
   ii) $(A \cap B \cap C)'$
   iii) $(A \cap B)'\ if\ A \subset B$
   iv) $(A - B) \cap (A \cup B)$
   v) $(A \cap B)'\ if\ A\ and\ B\ are\ disjoint\ sets$

vi) $(A \cup B \cup C)'$

vii) $(A - B) \cup (A \cap B)$

4. In a survey of 100 students, the number of students studying the various languages were found to be English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, Number of no language 24. Find

   i) How many students were studying Hindi?

   ii) How many students were studying English and Hindi

   [Ans:18,3]

5. In a survey of 25 students it was found that 15 had taken Maths, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Maths and chemistry, 9 had taken Maths and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken:

   i) Only Chemistry

   ii) Only Maths

   iii) Only Physics

   iv) Physics and Chemistry but not Maths

   v) Maths and Physics but not Chemistry

   vi) Only one of the subject

   vii) At least one of the subjects

   viii) None of the subjects [Ans:5, 4, 2, 1, 6, 11, 23, 2]

6. Of the members of three athletic team in a certain school, 21 are in the Basketball Team, 26 in the Hockey team and 29 in the Football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the three. How many members are there in all?

   [Ans:43]

7. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A & B, 10 read magazine B&C and 3 read all the three. Find:

   i) How many read none of the magazines?

   ii) How many read magazine C only?

   iii) How many read magazine A only?

   iv) How many read magazine B & C but not A ?

   [Ans:18,32,13,0]

8. Let A and B be two finites sets such that n(A) = m and n(B) = n. If the ratio of number of elements of power sets of A and B is 64 and

n(A) + n(B) = 32. Find the value of m and n.

[Ans:19, 23]

9. In a survey of 400 students of a school, 100 were listed as smokers and 150 as chewers of Gum, 75 were listed as both smokers and gum chewers. Find out how many students are neither smokers nor gum chewers. [Ans:225]
10. In a university out of 100 teachers, 15 like reading newspapers only, 12 like learning computers only and 8 like watching movies only on TV in the spare time. 40 like reading news papers and watching movies, 20 like learning computer and watching movies, 10 like reading news paper and learning computer, 65 like watching movies. Draw a Venn diagram and show the various portions and hence evaluate the numbers of teachers who:
    i) Like reading newspapers
    ii) Like learning computers
    iii) Did not like to do any of the things mentioned above. [62, 39, 1]