What is Deductive reasoning

Deductive reasoning also deductive logic or logical deduction or, informally, "top-down" logic is the process of reasoning from one or more statements (premises) to reach a logically certain conclusion

Deductive reasoning" refers to the process of concluding that something must be true because it is a special case of a general principle that is known to be true. For example, if you know the general principle that the sum

conclude that the sum of the angles in your triangle is 180 degrees.
Deductive reasoning is logically valid and it is the fundamental method in which mathematical facts are shown to be true.

What is Inductive reasoning

It is the process of reasoning that a general principle is true because the special cases you've seen are true. For example, if all the people you've ever met from a particular town have been very strange, you might then say "all the residents of this town are strange". That is inductive reasoning: constructing a general principle from special cases. It goes in the opposite direction from deductive reasoning.
Please dont confuse inductive reasoning with "mathematical induction" or and "inductive proof", which is something quite different

The Principle of Mathematical Induction

Suppose there is a given statement P(*n*) involving the natural number *n* such that
(*i*) *The statement is true for n* = 1, i.e., *P*(1) *is true, and*
(*ii*) *If the statement is true for n* = *k* (*where k is some positive integer*), *then*
iii) the statement is also true for n = k
+ 1, i.e., *truth of P*(*k*) *implies the*
truth of P
(*k* + 1).
Here the Step 1 is Basis of Induction
Step 2 is called inductive hypotthesis. Here we are assuming the identity to be true as Step 1 is true(special case)
Step 3 is called inductive step. Here we prove the identity is true for k+1 on the basis of inductive hypothesis
So it is a deductive reasoning technique.

Solved Examples:

Given n belongs to the set of Natural Numbers. Prove that using principle of Mathematical induction

$$0+1+2+\cdots+n=\frac{n(n+1)}{2}$$

Solutions:
Let us assume this statement as P(n) Then P(0), we have LHS side as 0 RHS= 0(0+1)/2=0 So LHS=RHS
Let us assum P(n) is true,then
Now we have to prove for P(n+1)
0+1+2+......(n)+(n+1)=\frac{(n+1)[(n+1)+1]}{2}

Now lets us take LHS
= $\frac{n(n+1)}{2}+(n+1)$ as P(n) is assumed to be true
=$\frac{(n+1)[(n+1)+1]}{2}$

So P(n+1) is true if P(n) is true. Now Since P(0) is true, we can say P(n) is good for all natural numbers

Example

To prove by mathematical induction that

$6^{(n+2)} + 7^{(2n+1)}$

is divisible by 43 for each positive integer n.

Solution

Let P(n) be the statement

$P(n)=6^{(n+2)} + 7^{(2n+1)}$

Basis of Induction

$P(1) =6^{(1+1)} + 7^{(2+1)}$

$= 559 = 43 X 13,$

the formula is true for n = 1.

Inductive Hypothesis

Assume that P(n) is true for n = k, that is

$P(k)=6^{(k+2)} + 7^{(2k+1)}=43x$

for some integer x.

Inductive Step

Now show that the formula is true for n = k + 1.

Observe that

$P(k+1)=6^{(k+1+2)} + 7^{(2k+2+1)}$

$= ^{(k+3)} + 7^{(2k+3)}$

$=6^{(k+2)} + 7^2 7^{(2k+1)}$

$=6X6^{(k+2)} + (43+6)X7^{(2k+1)}$

$=6X6^{(k+2)} + 6X7^{(2k+1)}+43X7^{(2k+1)}$

$= 6 X (P(k)) + 43 X 7^{(2k+1)}$

Since each component of this sum is divisible by 43 so is the entire sum and the formula holds for k + 1

Example

Prove using Mathematical induction for $n\geqslant 1$ **$1+4+7+......(3n-2)=\frac{n(3n-1)}{2}$**

Solution

Let us assume this statement as P(n)

Then P(1)

$1=\frac{1(3-1)}{2}$

Inductive Hypothesis

Assume that P(n) is true for n = k, that is

$1+4+7+......(3k-2)=\frac{k(3k-1)}{2}$

Inductive Step

Now show that the formula is true for n = k + 1.

Observe that

$1+4+7+.....[3(k+1)-2]=\frac{(k+1)[3(k+1)-1])}{2}$

Let us take the LHS

$1+4+7+.....[3(k+1)-2]=1+4+7+......+(3k-2)+(3k+1)$

$=\frac{k(3-1)}{2}+(3k+1)$

$=\frac{3k^{2}+5k+2}{2}$

$=\frac{(k+1)[3(k+1)+1]}{2}$

Chapter 4

PRINCIPLE OF MATHEMATICAL INDUCTION

INTRODUCTION

To prove certain results or statements in Algebra, that are formulated in terms of n, where n is a natural number, we use a specific technique called principle of mathematical induction (P.M.I)

Steps of P.M.I

Step I - Let p(n): result or statement formulated in terms of n (given question)

Step II – Prove that P(1) is true

Step III – Assume that P(k) is true

Step IV – Using step III prove that P(k+1) is true

Step V - Thus P(1) is true and P(k+1) is true whenever P(k) is true.

Hence by P.M.I, P(n) is true for all natural numbers n

Type I

Eg: Ex 4.1

1) Prove that

$1+3+3^2+\dots\dots\dots+3^{n-1} = \frac{3^n-1}{2}$

Solution:-

Step I : Let P(n): $1+3+3^2+\dots\dots\dots+3^{n-1} = \frac{3^n-1}{2}$

Step II: P(1):

LHS = 1

RHS = $= \frac{3-1}{2} = \frac{2}{2} = 1$

LHS=RHS

Therefore p(1) is true.

Step III: Assume that P(k) is true

i.e $1+3+3^2+\dots\dots\dots+3^{k-1} = \frac{3^k-1}{2}$ ______(1)

Step IV: we have to prove that P(k+1) is true.

ie to prove that $1+3+3^2+\ldots\ldots\ldots+3^{k-1}+3 \;=\; \frac{3^{k+1}-1}{2}$

Proof

LHS $= (1+3+3^2+\ldots\ldots\ldots+3^{k-1}) +3$

$=\frac{3^k-1}{2} + 3^k$ from eq(1)

$= \frac{3^k-1 + 2.3^k}{2}$

$= \frac{3.3^k - 1}{2} \quad = \frac{3^{k+1}-1}{2} \quad$ = RHS

Therefore P(k+1) is true

Step V: Thus P(1) is true and P(k+1) is true whenever P(k) is true. Hence by P.M.I, P(n) is true for all natural number n.

Text book

Ex 4.1

Q. 1,2, 3**(HOT), 4, 5*,6*,7,8,9,10*,11**,12,13**,14**,15,16**,17**, eg 1, eg 3

Type 2

Divisible / Multiple Questions like Q. 20**,21,22**,23 of Ex 4.1
eg 4, eg 6**(HOT)

Q 22. Prove that $3^{2n+2}-8n-9$ is divisible by 8 for all natural number n.

Solution

Step I: Let p(n): $3^{2n+2}-8n-9$ is divisible by 8

Step II: P(1): $3^4 - 8 -9 =81-17 = 64$ which is divisible by 8

Therefore p(1) is true

Step III: Assume that p(k) is true

i.e $3^{2k+2} -8k-9 = 8m;$ m is a natural number.

i.e $3^{2k}.9 = 8m+8k+9$

ie $3^{2k} = \frac{8m +8k+9}{9}$ ______(1)

Step IV: To prove that p(k+1) is true.

ie to prove that 3^{2k+4} -8(k+1) –9 is divisible by 8.

Proof: 3^{2k+4}-8k-17 = $3^{2k}.3^{4}$-8k-17 =$(\frac{8m+8k+9}{9})$ x 3^4 – 8k-17(from eqn (1))

= (8m+8k+9)9-8k-17 = 72m+72k+81-8k-17 = 72m-64k+64 = 8[9m-8k+8] is divisible by 8.

Step V: Thus P(1) is true and P(k+1) is true whenever P(k) is true. hence by P.M.I, P(n) is true for all natural numbers n.

Type III: Problems based on Inequations

Ex 4.1 Q. 18,14, eg 7

(Q 18) Prove that 1+2+3+……+n< $\frac{(2n+1)^2}{8}$

Step I : Let P(n): 1+2+3+……+n< $\frac{(2n+1)^2}{8}$

Step II: P(1): 1< $\frac{9}{8}$ which is true, therefore p(1) is true.

Step III: Assume that P(k) is true.

ie 1+2+3+……+k< $\frac{(2k+1)^2}{8}$ ________(1)

Step IV: We have to prove that P(k+1) is true. ie to

prove that 1+2+3+…..+k+(k+1)< $\frac{(2k+3)^2}{8}$

Proof: Adding (k+1) on both sides of inequation (1)

1+2+3+…..+k+(k+1) < $\frac{(2k+1)^2}{8} + (k+1)$

$= \frac{(4k^2+4k+1)+8k+8}{8}$

$= \frac{4k^2+12k+9}{8}$

$= \frac{(2k+3)^2}{8}$

Therefore $1+2+3+\ldots..+k+(k+1) < \frac{(2k+3)^2}{8}$

P(k+1) is true.

Step V: Thus P(1) is true and P(k+1) is true whenever P(k) is true. Hence by P.M.I, P(n) is true for all natural number n.

HOT/EXTRA QUESTIONS

Prove by mathematical induction that for all natural numbers n.

1) $a^{2n-1} - 1$ is divisible by a-1 (type II)
2) $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ is an integer(HOT)
3) $\sin x + \sin 3x + \ldots\ldots + \sin (2n-1)x = \frac{\sin^2 nx}{\sin x}$ (HOT Type 1)
4) $3^{2n-1}+3^n+4$ is divisible by 2 (type II)
5) Let P(n): n^2+n-19 is prime, state whether P(4) is true or false
6) $2^{2n+3} \leq (n+3)!$ (type III)
7) What is the minimum value of natural number n for which $2^n < n!$ holds true?
8) $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 (type II)

Answers

5) false
7) 4