Introduction to inequalities,symbol and rules of inequality

What is inequalities

In mathematics, an inequality is a relation that holds between two values when they are different

Solving linear inequalities is very similar to solving linear equations, except for one small but important detail: you flip the inequality sign whenever you multiply or divide the inequality by a negative

Symbols used in inequalities

The symbol < means less than. The symbol > means greater than.

The symbol < with a bar underneath means less than or equal to. Usually this is written as $\leq$

The symbol > with a bar underneath means greater than or equal to. Usually this is written as $\geq$

The symbol $\neq$ means the quatities on left and right side are not equal

Examples

1)a < b means a is less then b or b is greater a

2) $a \leq b$ means a is less then or equal to b

3) a > b means a is greater than b

4) $a \geq b$ means a is greater or equal to b

Things are which are safe to do in inequality which does not change in direction

1) addition of same number on both sides

a > b

=>a+c > b +c

2) Substraction of same number on both sides

a > b

=>a-c > b-c

3) Multipication/Division by same positive number on both sides

a > b

if c is positive number then

ac > bc

or

a/c > b/c

Things which changes the direction of the inequality

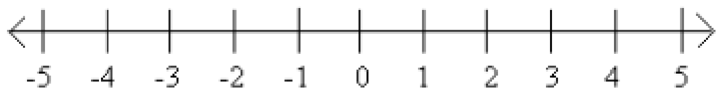
1) swapping the left and right sides

2) Multiplication/Division by negative number on both sides

3) Dont multiple by variable whose values you dont know as you dont know the nature of the variable

Concept Of Number line

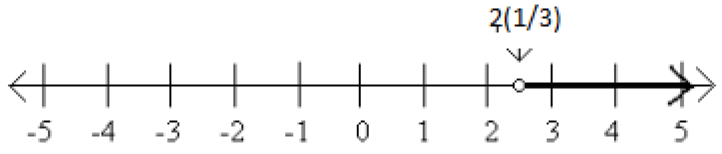
A number line is a horizontal line that has points which correspond to numbers. The points are spaced according to the value of the number they correspond to; in a number line containing only whole numbers or integers, the points are equally spaced.



Number Line

It is very useful in solving problem related to inequalities and also representing it

Suppose x >2(1/ 3), this can represent this on number line like that



Graph of the Inequality x > 2(1/3)

Linear Inequation in One Variable:

A equation of the form

ax+b> 0

or

$ax+b \geq 0$

or

$ax+b< 0$

or

$ax+b \leq 0$

are called the linear equation in One Variable

Example:

$x-2 < 0$

or $3x +10 > 0$

or $10x-17 \geq 0$

Linear Inequation in Two Variable:

A equation of the form

$ax+by> c$

or

$ax+by \geq c$

or

$ax+by< c$

or

$ax+by \leq c$

are called the linear equation in two Variable

Example:

$x-2y < 0$

or $3x +10y > 0$

or $10x-17y \geq 0$

Quadratic Inequation:

We have learned by Quadratic equation is previous chapter

so for a $\neq$ 0

$ax^2+bx+c > 0$

or

$ax^2+bx+c < 0$

or

$ax^2+bx+c \geq 0$

or

$ax^2+bx+c \leq 0$

are called Quadratic Equation in one Variable

Steps to solve the inequalities in one variable.

1) Obtain the linear inequation

2) Pull all the terms having variable on one side and all the constant term on another side of the inequation

3) Simplify the equation in the form given above

$ax > b$

or

$ax \geq b$

or

$ax < b$

or $ax \leq b$

4) Divide the coefficent of the variable on the both side.If the coefficent is positive,direction of the inequality does not changes,but if it is negative, direction of the inequation changed

5) Put the result of this equation on number line and get the solution set in interval form

Example:

$x- 2 > 2x+15$

Solution

$x-2 > 2x+15$

$-17 > x$

Solution set

$(-\infty,-17)$

Some Problems to practice

1. 2x > 9

2) x + 5 > 111

3) 3x < 4,

4) 2(x + 3) < x+ 1

Steps to solve the inequality of the form

(ax+b/cx+d) > k or similar type

1) take k on the LHS

2) Simplify LHS to obtain the inequation in the form

(px+q/ex+f) > 0

Make the coefficent positive if not

3) Find out the end points solving the equatoon px+q=0 and ex+f=0

4) Plot these numbers on the Number line. This divide the number into three segment

5) Start from LHS side of the number and Substitute some value in the equation in all the three segments to find out which segments satisfy the equation

6) Write down the solution set in interval form

Or there is more method to solves these

1) take k on the LHS

2) Simplify LHS to obtain the inequation in the form

(px+q/ex+f) > 0

Make the coefficent positive if not

3) For the equation to satisfy both the numerator and denominator must have the same sign

4) So taking both the part +, find out the variable x interval

5) So taking both the part -, find out the variable x interval
6) Write down the solution set in interval form

Lets take one example to clarify the points
$(x – 3)/(x + 5) > 0$.
Solution
Method A
1) Lets find the end points of the equation
Here it is clearly
$x=3$ and $x=-5$
2) Now plots them on the Number line
3) Now lets start from left part of the most left number
i.e
case 1
$x < -5$,Let takes $x=-6$ $(-6-3)/(-6+5) > 0$
$3 > 0$
So it is good
Case 2
Now take $x=-5$
as $x+5$ becomes zero and we cannot have zero in denominator,it is not the solution
Case 3
Now $x > -5$ and $x < 3$, lets take $x=1$ $(1-3)/(1+5) > 0$
$-1/6 > 0$
Which is not true
Case 4
Now take $x =3$,then
$0> 0$,So this is also not true
case 5
$x> 3$,Lets $x=4$
$(4-3)(4+5) > 0$
$1/9 > 0$
So this is good
So the solution is
$x < -5$ or $x > 3$
or
$(-\infty,-5)\cup (3,\infty)$
Method B
1) the numerator and denominator must have the same sign. Therefore, either
1) $x – 3 > 0$ and $x + 5 > 0$,

or

2) $x - 3 < 0$ and $x + 5 < 0$. Now, 1) implies $x > 3$ and $x > -5$.

Which numbers are these that are both greater than 3 and greater than -5?

Clearly, any number greater than 3 will also be greater than -5. Therefore, 1) has the solution

$x > 3$.

Next, 2) implies

$x < 3$ and $x < -5$.

Which numbers are these that are both less than 3 and less than -5?

Clearly, any number less than -5 will also be less than 3. Therefore, 2) has the solution

$x < -5$.

The solution, therefore, is

$x < -5$ or $x > 3$

Steps to solve Quadratic or polynomial inequalities

$ax^2+bx+c > 0$

or

$ax^2+bx+c < 0$

or

ax^2+bx+c $\geq$ 0

or

ax^2+bx+c $\leq$ 0

1) Obtain the Quadratic equation inequation
2) Pull all the terms having on one side and Simplify the equation in the form given above
3) Simplify the equation in the form given above
4) Find the roots of the quadratic equation using any of the method and write in this form

(x-a)(x-b)

5) Plot these roots on the number line .This divide the number into three segment

5) Start from LHS side of the number and Substitute some value in the equation in all the three segments to find out which segments satisfy the equation

6) Write down the solution set in interval form

Example

1) Simpify the inequality which means factorizing the equation in case of quadratic equalities

example

x2-5x+6 > 0

Which can be simpified as

(x-2)(x-3) > 0

2) Now plot those points on Number line clearly

3) Now start from left of most left point on the Number line and look out the if inequalities looks good or not. Check for greater ,less than and equalities at all the end points

So in above case of

x2-5x+6 > 0

We have two ends points 2 ,3

Case 1

So for x < 2 ,Let take x=1,then (1-2)(1-3) > 0

So This inequalities is good for x < 2

Case 2

Now for x =2,it makes it zero,so not true. Now takes the case of x > 2 but less 3.Lets takes 2.5

(2.5-2)(2.5-3) > 0

-.25 > 0

Which is not true so this solution is not good

Case 3

Now lets take the right most part i.e x > 3

Lets take x=4

(4-2)(4-3) > 0

2> 0

So it is good.

Now the solution can either be represented on number line or we can say like this

$(-\infty,2)\cup (3,\infty)$

Steps to solve the pair of Linear inequation or quadratic equation

1) Solving them is same as steps given above to solve the linear inequation and quadratic equation
2) Solve both the inequation in the pair
3) Look for the intersection of the solution and give the solution in interval set

Some Important points to note

1) We cannot have zero in denominator
2) We should be checking for equalities at all the end points

Absolute value equation:

Absolute value is denoted by |x|. And it is defined as

|x| = x if x $\geq$ 0

=-x if x < 0

So it is always positive.

Examples

|x-4|=2

or x-4=-2 or x-4=2

or x=2 or x =6

Absolute value inequation

|x-2| > 4

|x| < 2

This is a form of Absolute value inequation

Important Formula's

for a and r being positive real number

|x| < a implies that -a< x< a

|x| > a implies that x< -a or x> a

|x| $\geq$ a implies that x $\geq$ a or x $\leq$ a

|x-a| < r implies that a-r < x < a+r

|x-a| > r implies that x < a-r or x > a+r

a< |x| < b implies that x lies in (-b,-a) or (a,b)

a< |x-c| < b implies that x lies in (-b+c,-a+c) or (a+c,b+c)

Examples

1) |x-2| > 4

Solution

we know from the Formula

|x-a| > r implies that x < a-r or x > a+r

So x < -2 or x > 6

2) |x| < 2

Solution:

We know that Formula

|x| < a implies that -a< x< a

So -2 < x < 2

Graphical Solution of Linear inequalities in Two Variable.

A linear equation in two variable is of the form

$ax+by+c=0$

We have already studied in Coordinate geometry that this can be represented by a straight line in x-y plane. All the points on the straight line are the solutions of this linear equation.

we can similarly find the solution set graphically for the linear inequalities in the below form

$ax+by+c< 0$

$ax+by+c> 0$

ax+by+c $\geq$ 0

ax+by+c $\leq$ 0

How to find the solution graphicaly for Linear inequalities in Two Variable.

1) Draw the graph of the equation obtained for the given inequality by replacing the inequality sign with an equal sign.

ie, $ax+by+c=0$

2) Use a dashed or dotted line if the problem involves a strict inequality, < or >.

3) Otherwise, use a solid line to indicate that the line itself constitutes part of the solution.

4) Pick a point lying in one of the half-planes determined by the line sketched in step 1 and substitute the values of x and y into the given inequality.

Use the origin whenever possible.

5) If the inequality is satisfied, the graph of the inequality includes the half-plane containing the test point. Otherwise, the solution includes the half-plane not containing the test point

Example

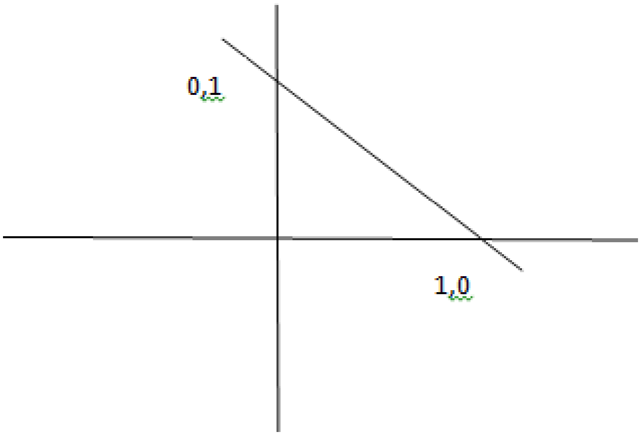
Determine the solution set for the inequality

x+y > 1

Solution

1) Draw the graph of the equation obtained for the given inequality by replacing the inequality sign with an equal sign.

i.e x+y=1



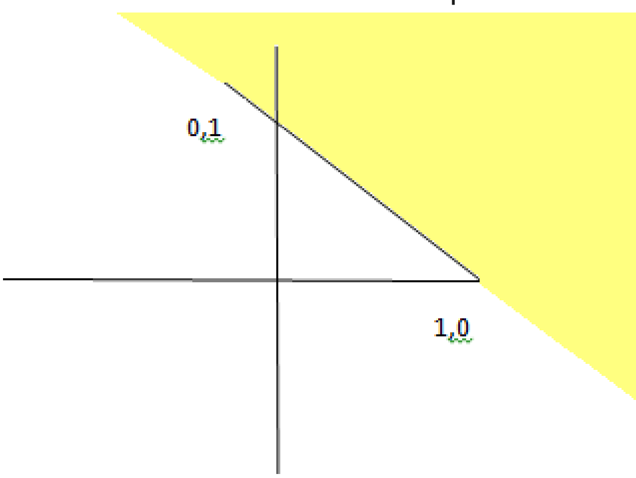
2) Pick the test point as origin (0,0), and put into the inequality

0+0 > 1

0> 1

Which is false

So the solution set is other half plane of the line



How to find the solution graphicaly for pair of Linear inequalities in Two Variable.

ax+by+c< 0

px+qy+c < 0

The solution set of a system of linear inequalities in two variables x and y is the set of all points (x, y) that

CHAPTER 6

LINEAR INEQUALITIES

Rules for solving inequalities

The following rules can be applied to any inequality

- Add or subtract the same number or expressions to both sides.
- Multiply or divide both sides by the same positive number.
- By multiplying or dividing with the same negative number, the inequality is reversed.
- $a<b$ implies $b>a$
- $a<b$ implies $-a>-b$
- $a<b$ implies $1/a > 1/b$
- $x^2 \leq a^2$ implies $x \leq a$ and $x \geq -a$

Recall

- $x = 0$ is y axis
- $y=0$ is x axis
- $x=k$ is a line parallel to y axis passing through (k,0)of x axis.
- $y=k$ is a line parallel to x axis and passing through (0,k)of y axis.

Procedure to solve a linear inequality

- Simplify both sides by removing group symbols and collecting like terms.
- Remove fractions by multiplying both sides by an appropriate factor.
- Isolate all variable terms on one side and all constants on the other side.
- Make the coefficient of the variable 1 and get the solution (whenever coefficient of x is negative multiply through out by -1 so that the ineqality is reversed.

Eg $-2x \geq 5$

$2x \leq -5$

$x \leq -5/2$

$x \leq -2.5$ therefore solution is $(-\infty,-2.5]$

Note: If **$x\geq 0$, $y\geq 0$** is given in question every point in the shaded region in the **first quadrant** including the points on the line and the axis, represents the solution of the given system of ineqalities.

Method to find Graphical Solution

- Draw lines corresponding to each equation treating it as equality
- Find the Feasible Region – intersection of all the inequalities

Steps to find Feasible region

Step 1: Take any point on the left (down) or right (up) of the line and substitute that in the given inequality.

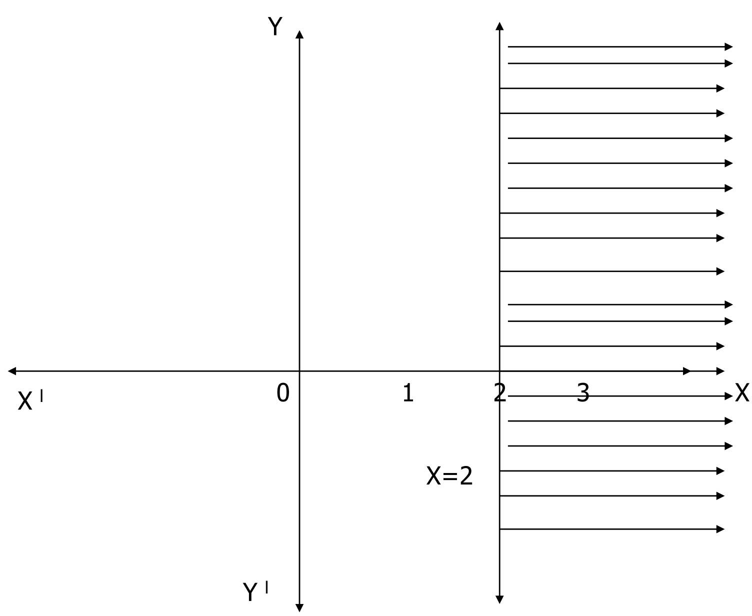
Step 2: If the point satisfies the inequality, the region containing that point is the required region. Otherwise opposite region is the required region.

Step 3: If inequality is of the type ≥ , ≤ then the points on the line are also included in the solution region, so draw dark lines.

Step 4: If inequality is of the type >, < then the points on the line are not included in the solution region, so draw dotted lines

Note: For the following in equation $x \geq 2$, substitute the point (0,0) (left side) in $x \geq 2$. Then we get $0 \geq 2$ which is false. Therefore right side is the required region.

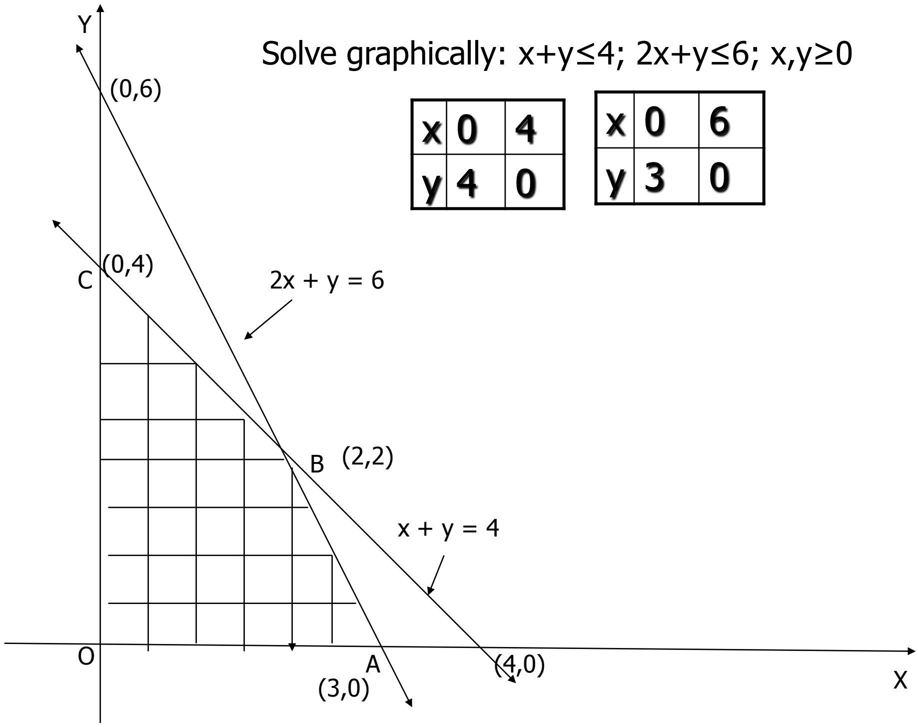
Shade the region corresponding to the inequation $x \geq 2$



Solve graphically: $x+y\le4$; $2x+y\le6$; $x,y\ge0$

x	0	4
y	4	0

x	0	6
y	3	0



Solve graphically

$x + y \le 400$

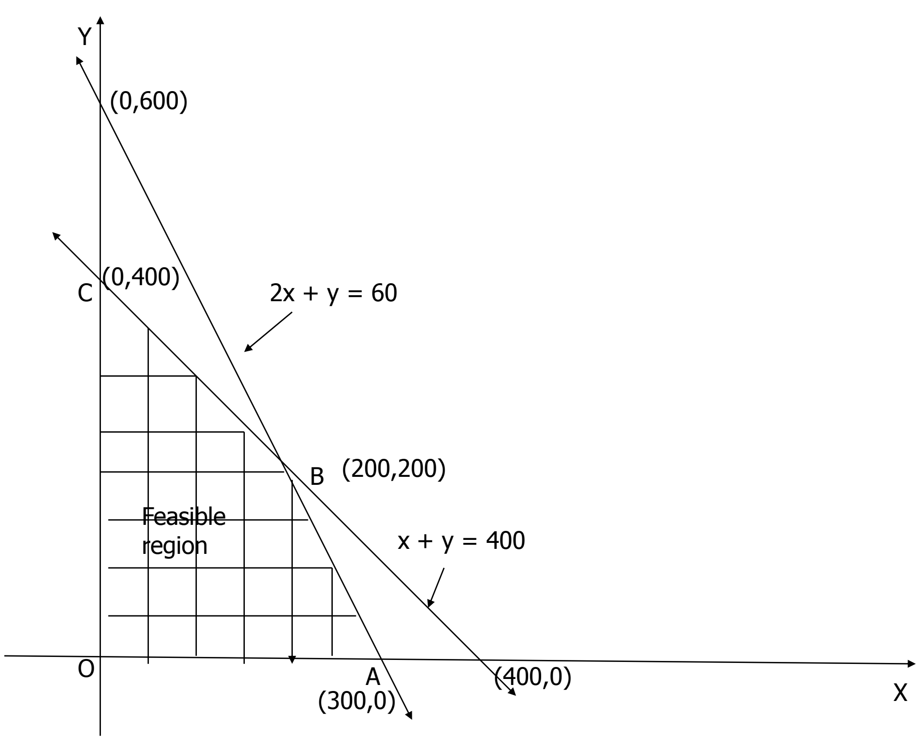
$2x+y \le 600$

Consider x+y=400

x	0	400
y	400	0

Consider 2x+y=600

X	0	300
y	600	0



$x + y \leq 20$

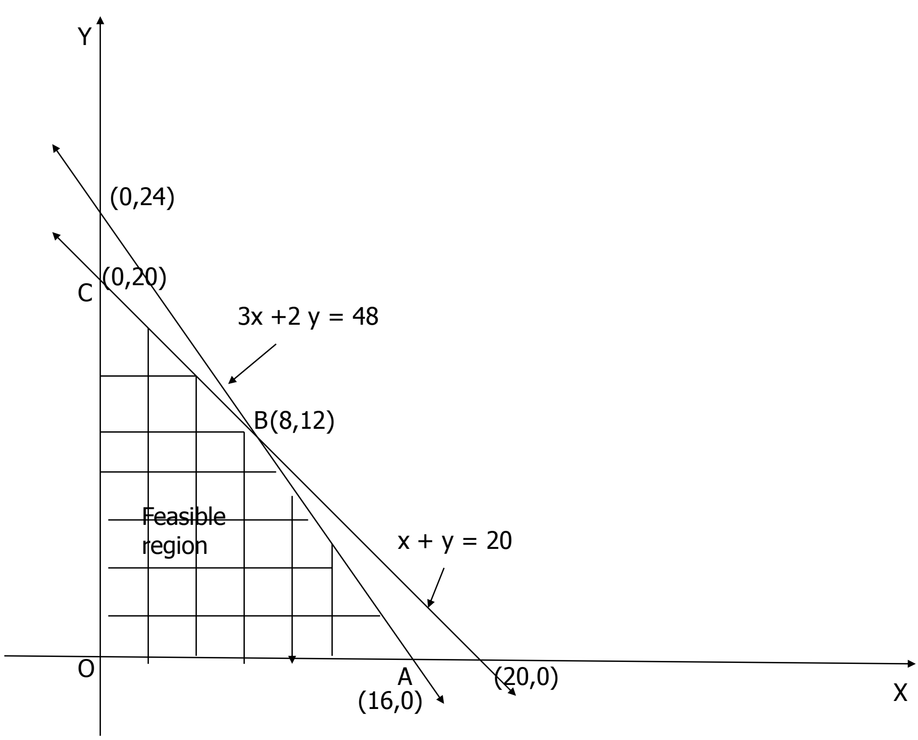
$360x + 240y \leq 5760$

Consider x+y=20

x	0	20
y	20	0

Consider 3x+2y=48 (Dividing throughout by 120)

X	0	16
y	24	0



Solve graphically:

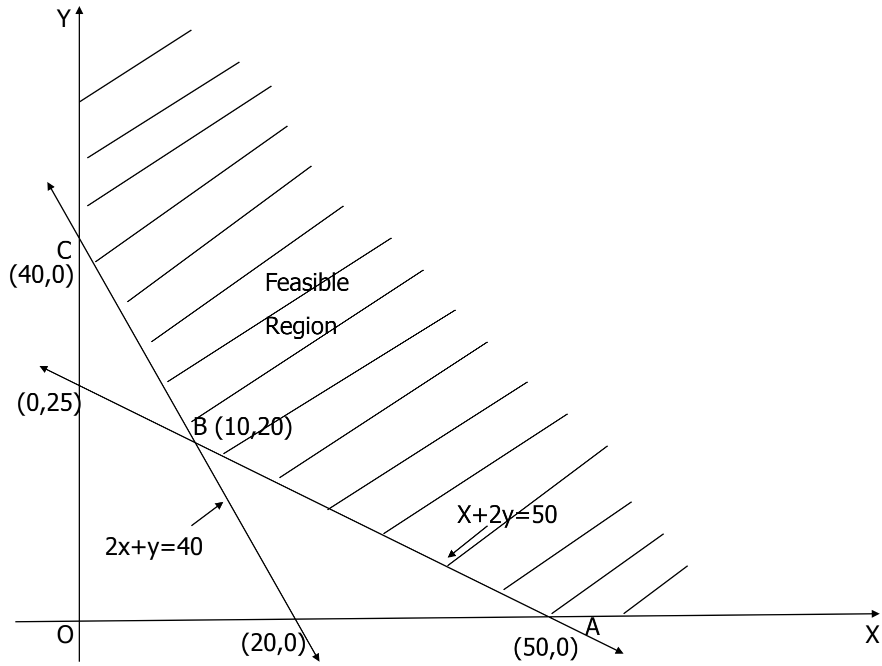
- $200X + 100Y \geq 4000$
- $X + 2Y \geq 50$

Consider 2x+y=40

X	0	20
y	40	0

Consider x+2y=40

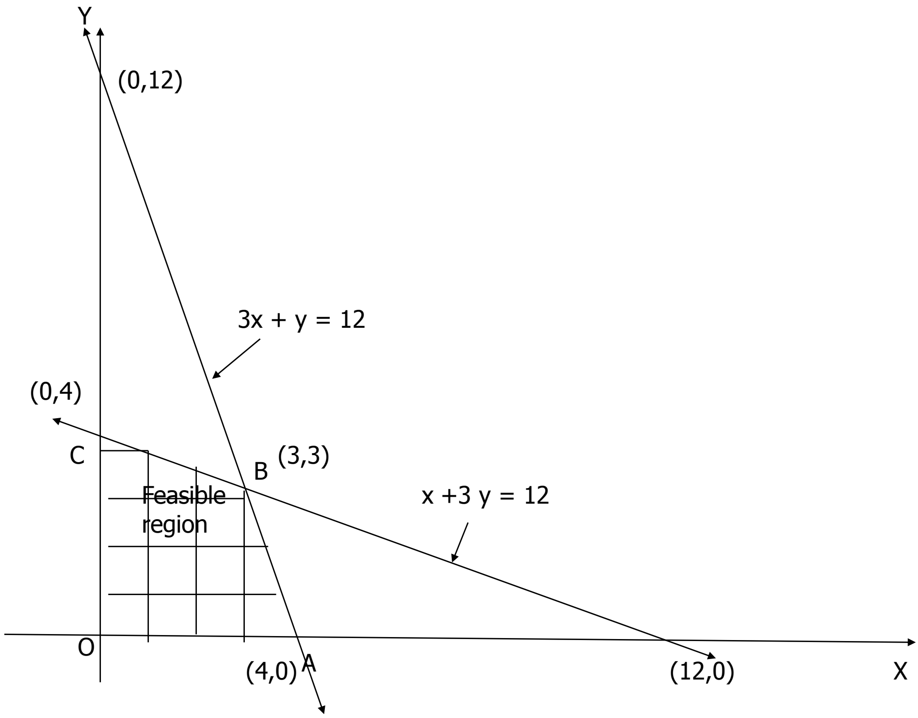
X	0	50
y	25	0



Solve graphically:

$x+3y \le 12$

$3x+y \le 12$



Solve graphically:

$x/10 + y/20 \geq 14$

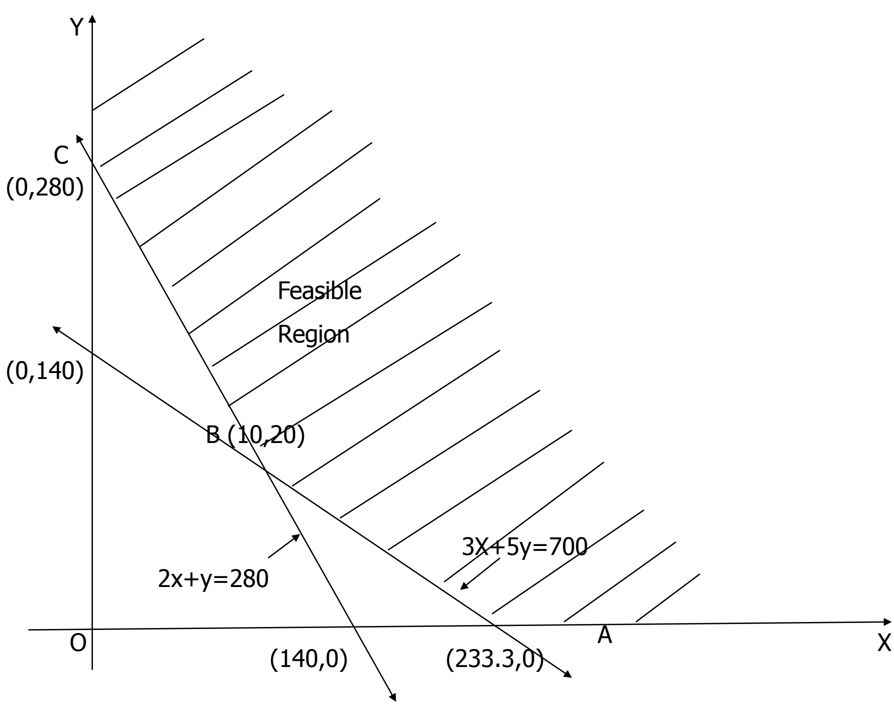
$3/50\, x + 1/10\, y \geq 14$

Consider 2x+y=280

X	0	140
y	280	0

Consider 3x+5y=700

X	0	233.3
y	140	0



Ex 6.1

1to 4 (1 mark)

11 to 15, 16**, 18**, 20**, 22**, 24**, 25**,26**

eg 15**

Ex 6.3

(8 to 15)**

eg 16**

eg17**

Misc Ex

4*, 5*,6*, 8*, 9*, 10*, (11 to 14)**

EXTRA /HOT QUESTIONS

1) Solve i) $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$

   ii) $\frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$

   iii) $\frac{2-3x}{5} < \frac{1-x}{3} < \frac{3+4x}{2}$

2) Solve graphically
   i) $2x+y \geq 3$, $x-2y \leq -1$, $x\geq0$, $y\geq0$
   ii) $x+4y \leq 4$, $2x+3y \leq 6$, $x\geq0$, $y\geq0$
   iii) $x+y \geq 1$, $x \leq 5$, $y \leq 4$, $2x +3y \leq 12$, $x\geq0$, $y\geq0$
3) The water acidity in a pool is considered normal when the average pH reading of 3 daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal
4) In the first 4 papers each of hundred marks Ravi got 90, 75, 73, 85 marks. If he wants an average of greater than or equal to 75 marks and less than 80 marks, find the range of marks she should score in the fifth paper.
5) The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm find the minimum length of the shortest side.

Answers

1) i) $(4,\infty)$

ii) $(-\infty, 63/10]$

iii) $(1/4, \infty)$

3 Between 7.77 and 8.67

4 More than or equal to 52 but less than 77

5 9 cm